SORU $I = \int \dfrac{\cos x\,dx}{2\cos x + \sin x + 3} = ?$

$\tan\frac{x}{2} = t$ dersek

$\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx = dt$

$dx = \dfrac{2dt}{1+t^2}$

$\sin\frac{x}{2} = \dfrac{t}{\sqrt{1+t^2}}$

$\cos\frac{x}{2} = \dfrac{1}{\sqrt{1+t^2}}$

$\sin x = \dfrac{2t}{1+t^2}$

$\cos x = \dfrac{1-t^2}{1+t^2}$

$$I = \int \dfrac{\dfrac{1-t^2}{1+t^2}\cdot\dfrac{2dt}{1+t^2}}{2\cdot\dfrac{1-t^2}{1+t^2} + \dfrac{2t}{1+t^2} + 3}$$

$$= 2\int \frac{1-t^2}{(1+t^2)(t^2+2t+5)}\,dt$$

$$\frac{1-t^2}{(1+t^2)(t^2+2t+5)} = \frac{At+B}{1+t^2} + \frac{Ct+D}{(t^2+2t+5)}$$

$$1-t^2 = (A+C)t^3 + (2A+B+D)t^2 + (5A+2B+C)t + 5B+D$$

$$\left\{\begin{array}{ll} (1) & A+C=0 \\ (2) & 2A+B+D=-1 \\ (3) & 5A+2B+C=0 \\ (4) & 5B+D=1 \end{array}\right.$$

$C=-A$

$4A+2B=0 \Rightarrow \boxed{2A+B=0}$

$\boxed{2A-4B=-2}$

$D=1-5B$

$2A+B+1-5B=-1$

$D=1-2=-1$

$B=\frac{2}{5}$

$D=-1$

$A=-\frac{1}{5}$

$C=\frac{1}{5}$

$$I = 2\int \frac{-\frac{1}{5}t+\frac{2}{5}}{1+t^2}\,dt + 2\int \frac{\frac{1}{5}t-1}{t^2+2t+5}\,dt$$

$$I = \frac{2}{5}\int \frac{-t+2}{1+t^2}\,dt + \frac{2}{5}\int \frac{t-5}{t^2+2t+5}\,dt$$

$$\boxed{I=\frac{2}{5}\left[-\frac{1}{2}\ln(1+t^2)+2\arctan t\right]+\frac{2}{5}I_1+C}$$

$$I_1=\frac{1}{2}\int\frac{2t+2-2-10}{t^2+2t+5}dt=\frac{1}{2}\int\frac{(2t+2)dt}{t^2+2t+5}-\frac{12}{2}\int\frac{dt}{(t^2+2t+5)}$$

$$I_1=\frac{1}{2}\ln(t^2+2t+5)-6\int\frac{dt}{(t+1)^2+4}$$

$$\boxed{I_1=\frac{1}{2}\ln(t^2+2t+5)-\frac{6}{2}\cdot\arctan\frac{t+1}{2}}$$

$$I=-\frac{1}{5}\ln(1+t^2)+\frac{4}{5}\arctan t+\frac{1}{5}\ln(t^2+2t+5)-\frac{6}{5}\arctan\frac{t+1}{2}+C$$

$\tan\frac{x}{2}=t$ yerine yazılır.