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New extensions of (2+1)-dimensional BLMP models with soliton solutions

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Abstract

Searching for soliton solutions of NPDEs is one of the most interesting and important areas of science in the field of nonlinear phenomena. Soliton is a localized wave with exponential wings or is a localized wave with an infinite support. In this work, we study two extensions of (2+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation. Based on the simplified Hirota's method (HM) and the Cole-Hopf transformation (CHT) method, new multiple front wave solutions are obtained for both versions.

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Key words: Boiti-Leon-Manna-Pempinelli equation; Hirota's method; Multiple soliton solutions.

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1 Introduction

Soliton is a nonlinear wave with two important properties [1]. First, soliton is a localized wave which propagates without change of its shape, velocity, etc. Second, soliton is a stable localized wave against mutual collisions which retains its identities. From the second property, we conclude that soliton has a property of a particle.
Finding multiple-soliton solutions of NPDEs play a very important role in treating nonlinear phenomena. Plasma physics, nonlinear optics, optical fibers, biology, fluid state physics, quantum physics, hydrodynamics, etc, just show some different fields of nonlinear sciences which solitons have important significant.

There are many effective methods have been used to obtain solitary wave solutions of nonlinear PDEs. The multiple exp-function [2], the three wave [3], the extended homoclinic test [4], the algebraic-geometric [5, 6], the inverse scattering [7, 8], the Hirota's bilinear [9], the Bäcklund transformation [10], and others. Multiple solitary wave solutions consist of several solitons [11]. Recently, various effective methods have been developed to find the multi-soliton solutions of NPDEs, such as the Hirota bilinear [12, 13], the homoclinic test approach [14, 15, 16], multiple exp-function [17, 18, 19] and other methods [20, 21, 22, 23].

Hirota's bilinear method has a powerful procedure which also produces multiple soliton solutions [24]. In this paper, the following BLMP equation, which was derived by Boiti et al. [25], is investigated

$$u_{yt} + u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y = 0. \tag{1}$$

Boiti et al. obtained this equation during their research on KdV equation through weak Lax pairs relations [25]. This equation describes the (2+1)-dimensional interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis [26, 27].
Based on the binary Bell polynomials, the bilinear form of the BLMP equation is found in [26]. For $y = x$, and by integrating the resulting equation in (1), the equation is reduced to the KdV equation [28]. Besides, some stair and step soliton solutions are obtained by Darvishi et al. [29] for (1). Solutions of (2+1)-dimensional BLMP equation using Wronskian techniques are obtained in [30]. For this equation, the variable separable solutions obtained in [31]. New solutions were obtained using the extended homoclinic testing [32].
We first study a new extension of (2+1)-dimensional BLMP equation that reads

$$(u_{yt} + u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y)_x = -\alpha^2\, u_{yyy}, \tag{2}$$

where the BLMP equation parameter α^2 is a real constant. Also, we introduce a new extension of BLMP equation as

$$(u_{yt} + u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y)_x = -\alpha^2\, u_{yyy} - \beta^2\, u_{xxx}, \tag{3}$$

where β^2 is a real constant. We will obtain solitary wave solutions for both equations (2) and (3). To do this, the HM and the CHT are used.
The remaining part of the paper is further arranged as follows: In Section 2, we introduce the algorithms of the Hirota's direct method. In Section 3 and 4, the this method is applied to find the soliton solutions. Finally, a conclusion is reached in Section 5.

2 Analysis of the method

Hirota's direct method can be used to obtain N-soliton solutions for completely integrable equations [24, 33, 34]. Further, soliton solutions are just polynomials of exponentials which is proved by many researchers which this will be also verified in this paper. In order to solve a nonlinear PDE, we first substitute

$$u(x, y, t) = \mathrm{e}^{kx+my-ct}, \tag{4}$$

into the linear terms of any equation under discussion to determine the relation between the parameters k and c. After that, we substitute the CHT

$$u(x, y, t) = R\,(\ln f(x, y, t))_{xx}, \tag{5}$$

where to determine R in (5), we define an auxiliary function f. Form of this auxiliary function depends on the solution which we look for. For example, the single soliton solution:

$$f(x, y, t) = 1 + C_1\, f_1(x, y, t) = 1 + C_1\, \mathrm{e}^{\theta_1}. \tag{6}$$

Next, we must apply the HM. The HM which have summarized in [24, 33]:

(i) To find the relation between k_i and c_i, we use

$$u(x, y, t) = \mathrm{e}^{\theta_i} \quad , \quad \theta_i = k_i\, x + m_i\, y - c_i\, t. \tag{7}$$

(ii) For single soliton, our auxiliary function is

$$f = 1 + C_1\, \mathrm{e}^{\theta_1}, \tag{8}$$

to determine R in (5).
(iii) Similarly for two-soliton solutions:

$$f = 1 + C_1\, \mathrm{e}^{\theta_1} + C_2\, \mathrm{e}^{\theta_2} + C_1\, C_2\, a_{12}\, \mathrm{e}^{\theta_1+\theta_2}, \tag{9}$$

to determine the phase shift coefficient a_{12}, this can be generalized for a_{ij}, $1 \leq i < j \leq 3$.
(iv) For three-soliton solutions:

$$\begin{aligned} f = 1 + C_1\, \mathrm{e}^{\theta_1} + C_2\, \mathrm{e}^{\theta_2} + C_3\, \mathrm{e}^{\theta_3} + C_1\, C_2\, a_{12}\, \mathrm{e}^{\theta_1+\theta_2} \\ + C_1\, C_3\, a_{13}\, \mathrm{e}^{\theta_1+\theta_3} + C_2\, C_3\, a_{23}\, \mathrm{e}^{\theta_2+\theta_3} \\ + C_1\, C_2\, C_3\, b_{123}\, \mathrm{e}^{\theta_1+\theta_2+\theta_3}, \end{aligned} \tag{10}$$

to determine b_{123}. Pekcan in [35] proved that $b_{123} = a_{12}\, a_{23}\, a_{13}$, then the equation gives rise to three-soliton solutions.
In the following sections, we will apply the aforementioned steps of HM on the extended BLMP equations (2) and (3). For $C_1 = C_2 = C_3 = 1$, multiple soliton solutions are obtained. While $C_1 = C_2 = C_3 = -1$, multiple singular soliton solutions are obtained.

3 The first extended BLMP equation

In this part, we investigate explicit formulas of soliton solutions for the first extension of (2+1)-dimensional BLMP equation as

$$(u_{yt} + u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y)_x = -\alpha^2\, u_{yyy}. \tag{11}$$

3.1 Multiple soliton solutions

We follow steps (i)-(iv) of Hirota's method for Eq. (11). We first consider $C_1 = C_2 = C_3 = 1$.

$$u\,(x, y, t) = \mathrm{e}^{\theta_i}, \quad \theta_i = k_i x + m_i y - w_i t, \tag{12}$$

into the linear terms of (11) yields

$$w_i = \frac{\alpha^2 m_i^2 + k_i^4}{k_i}, \qquad i = 1, 2, \ldots, N, \tag{13}$$

θ_i becomes

$$\theta_i = k_i x + m_i y - \frac{\alpha^2 m_i^2 + k_i^4}{k_i}\, t, \qquad i = 1, 2, \ldots, N. \tag{14}$$

To determine R, we substitute

$$u(x, y, t) = R\,(\ln f(x, y, t))_x, \tag{15}$$

where

$$f(x,y,t)=1+\mathrm{e}^{k_1x+m_1y-\frac{\alpha^2m_1^2+k_1^4}{k_1}t},$$

into (11) which by solving it we obtain $R=-2$.
From these we obtain the following single-kink soliton solution for (11)

$$u\left(x,y,t\right)=\frac{2\,k_1\mathrm{e}^{k_1x+m_1y-\frac{\left(\alpha^2m_1^2+k_1^4\right)t}{k_1}}}{1+\mathrm{e}^{k_1x+m_1y-\frac{\left(\alpha^2m_1^2+k_1^4\right)t}{k_1}}}. \tag{16}$$

For two-kink soliton solutions, auxiliary function:

$$f(x,y,t)=1+\mathrm{e}^{\theta_1}+\mathrm{e}^{\theta_2}+a_{12}\,\mathrm{e}^{\theta_1+\theta_2}, \tag{17}$$

into (11), where θ_1 and θ_2 are given in (14). This results in

$$a_{12}=\frac{\alpha^2(m_1+m_2)(k_1m_2-k_2m_1)^2-3\,k_1^2k_2^2\left(k_1^2-k_2^2\right)(m_1-m_2)}{\alpha^2(m_1+m_2)(k_1m_2-k_2m_1)^2-3\,k_1^2k_2^2(k_1+k_2)^2(m_1+m_2)}. \tag{18}$$

This can be generalized for the phase shifts by

$$a_{ij}=\frac{\alpha^2(m_i+m_j)(k_im_j-k_jm_i)^2-3\,k_i^2k_j^2\left(k_i^2-k_j^2\right)(m_i-m_j)}{\alpha^2(m_i+m_j)(k_im_j-k_jm_i)^2-3\,k_i^2k_j^2(k_i+k_j)^2(m_i+m_j)}, \tag{19}$$

where $1\leq i<j\leq 3$. The auxiliary function $f(x,y,t)$ of two-kink solutions:

$$\begin{aligned}f\left(x,y,t\right)=1&+\mathrm{e}^{k_1x+m_1y-\frac{\alpha^2m_1^2+k_1^4}{k_1}t}+\mathrm{e}^{k_2x+m_2y-\frac{\alpha^2m_2^2+k_2^4}{k_2}t}\\&+\frac{\alpha^2\left(m_1+m_2\right)\left(k_1m_2-k_2m_1\right)^2-3\,k_1^2k_2^2\left(k_1^2-k_2^2\right)\left(m_1-m_2\right)}{\alpha^2\left(m_1+m_2\right)\left(k_1m_2-k_2m_1\right)^2-3\,k_1^2k_2^2\left(k_1+k_2\right)^2\left(m_1+m_2\right)}\\&\times\mathrm{e}^{(k_1+k_2)x+(m_1+m_2)y-(\frac{\alpha^2m_1^2+k_1^4}{k_1}+\frac{\alpha^2m_2^2+k_2^4}{k_2})\,t}.\end{aligned}$$

Setting

$$\begin{aligned}f(x,y,t)=1+\mathrm{e}^{\theta_1}+\mathrm{e}^{\theta_2}+\mathrm{e}^{\theta_3}+a_{12}\,\mathrm{e}^{\theta_1+\theta_2}+a_{13}\,\mathrm{e}^{\theta_1+\theta_3}+a_{23}\,\mathrm{e}^{\theta_2+\theta_3}\\+a_{12}\,a_{23}\,a_{13}\,\mathrm{e}^{\theta_1+\theta_2+\theta_3},\end{aligned} \tag{20}$$

determines three-kink solutions, explicitly. Using the values of $\theta_i, i = 1, 2, 3$ in (14), Eq. (20) changes to

$$f\left(x,y,t\right) = 1 + \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^2 + k_1^4}{k_1} t} + \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^2 + k_2^4}{k_2} t} + \mathrm{e}^{k_3 x + m_3 y - \frac{\alpha^2 m_3^2 + k_3^4}{k_3} t}$$

$$+ a_{12} \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_2^2 + k_2^4}{k_2})\, t}$$

$$+ a_{13} \mathrm{e}^{(k_1+k_3)x + (m_1+m_3)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3})\, t}$$

$$+ a_{23} \mathrm{e}^{(k_2+k_3)x + (m_2+m_3)y - (\frac{\alpha^2 m_2^2 + k_2^4}{k_2} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3})\, t}$$

$$+ b_{123} \mathrm{e}^{(k_1+k_2+k_3)x + (m_1+m_2+m_3)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_2^2 + k_2^4}{k_2} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3})\, t},$$

proceed as before to find that

$$b_{123} = a_{12}\, a_{13}\, a_{23},$$

for the three-kink solutions, we substitute the last result for $f(x, y, t)$ into (15). The higher level soliton solutions, for $N \geq 4$ can be obtained in a similar manner. Therefore, we conclude that the (2+1)-dimensional BLMP equation (11) is completely integrable. Hence, as a result, the equation gives rise to have multiple-soliton solutions of any order.

3.2 Multiple singular soliton solutions

For $C_1 = C_2 = C_3 = -1$, the multiple singular-kink solutions will be determined. In what follows we set only the results and the auxiliary functions that we used, because the procedure is similar to the procedure of previous parts. Auxiliary function for single singular-kink solutions:

$$f(x,y,t) = 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^2 + k_1^4}{k_1} t},$$

to find that

$$u\left(x,y,t\right) = -\frac{2\, k_1 \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 m_1^2 + k_1^4\right) t}{k_1}}}{1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 m_1^2 + k_1^4\right) t}{k_1}}}. \tag{21}$$

Setting the auxiliary function

$$f(x,y,t) = 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^2 + k_1^4}{k_1} t} - \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^2 + k_2^4}{k_2} t}$$

$$+ \frac{\alpha^2 (m_1+m_2)(k_1 m_2 - k_2 m_1)^2 - 3 k_1^2 k_2^2 (k_1^2 - k_2^2)(m_1 - m_2)}{\alpha^2 (m_1+m_2)(k_1 m_2 - k_2 m_1)^2 - 3 k_1^2 k_2^2 (k_1+k_2)^2 (m_1+m_2)}$$

$$\times \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_2^2 + k_2^4}{k_2}) t},$$

in (15) gives two-king soliton soltions. Finally, for the three-kink soliton solutions, the auxiliary function:

$$f(x,y,t) = 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^2 + k_1^4}{k_1} t} - \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^2 + k_2^4}{k_2} t} - \mathrm{e}^{k_3 x + m_3 y - \frac{\alpha^2 m_3^2 + k_3^4}{k_3} t}$$

$$+ a_{12} \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_2^2 + k_2^4}{k_2}) t}$$

$$+ a_{13} \mathrm{e}^{(k_1+k_3)x + (m_1+m_3)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3}) t}$$

$$+ a_{23} \mathrm{e}^{(k_2+k_3)x + (m_2+m_3)y - (\frac{\alpha^2 m_2^2 + k_2^4}{k_2} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3}) t}$$

$$+ b_{123} \mathrm{e}^{(k_1+k_2+k_3)x + (m_1+m_2+m_3)y - (\frac{\alpha^2 m_1^2 + k_1^4}{k_1} + \frac{\alpha^2 m_2^2 + k_2^4}{k_2} + \frac{\alpha^2 m_3^2 + k_3^4}{k_3}) t},$$

into (15), to find that

$$b_{123} = -a_{12}\, a_{13}\, a_{23}.$$

This shows that multiple singular soliton solutions can be obtained for this model.

4 The second extended BLMP equation

In this section, explicit formulas of soliton solutions for the second extension of (2+1)-dimensional BLMP equation:

$$(u_{yt} + u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y)_x = -\alpha^2\, u_{yyy} - \beta^2\, u_{xxx}. \tag{22}$$

are investigated.

4.1 Multiple soliton solutions

To follow steps (i)-(iv) of Hirota's method, we first consider $C_1 = C_2 = C_3 = 1$.

$$u\,(x,y,t) = \mathrm{e}^{\theta_i}, \quad \theta_i = k_i x + m_i y - w_i t, \tag{23}$$

into the linear terms of (22) gives

$$w_i = \frac{\alpha^2 m_i^3 + \beta^2 k_i^3 + k_i^4 m_i}{k_1 m_i}, \qquad i = 1, 2, \ldots, N. \tag{24}$$

From (24) the following values are obtained for θ_i

$$\theta_i = k_i x + m_i y - \frac{\alpha^2 m_i^3 + \beta^2 k_i^3 + k_i^4 m_i}{k_i m_i}\, t, \qquad i = 1, 2, \ldots, N. \tag{25}$$

To determine R, we substitute

$$u(x,y,t) = R\,(\ln f(x,y,t))_x, \tag{26}$$

with

$$f(x,y,t) = 1 + \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1}\, t},$$

into (22) that by solving the resulting equation we obtain $R = -2$.
The single-kink soliton solution is:

$$u\,(x,y,t) = -\frac{2\,k_1 \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1\right) t}{k_1 m_1}}}{1 + \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1\right) t}{k_1 m_1}}}. \tag{27}$$

the auxiliary function for two-kink soliton solutions:

$$f(x,y,t) = 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + a_{12}\,\mathrm{e}^{\theta_1 + \theta_2}, \tag{28}$$

into (22), where θ_1 and θ_2 are given in (25). This yields

$$a_{12} = \frac{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\,k_1^2 k_2^2 m_1 m_2 \left(k_1^2 - k_2^2\right)(m_1 - m_2)}{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\,k_1^2 k_2^2 m_1 m_2 (k_1 + k_2)^2 (m_1 + m_2)}. \tag{29}$$

We can generalize this for the phase shifts by

$$a_{ij} = \frac{(k_i m_j - k_j m_i)^2 \left(\left(k_i^2 k_j + k_i k_j^2\right)\beta^2 + \left(m_i^2 m_j + m_i m_j^2\right)\alpha^2\right) - 3\,k_i^2 k_j^2 m_i m_j \left(k_i^2 - k_j^2\right)(m_i - m_j)}{(k_i m_j - k_j m_i)^2 \left(\left(k_i^2 k_j + k_i k_j^2\right)\beta^2 + \left(m_i^2 m_j + m_i m_j^2\right)\alpha^2\right) - 3\,k_i^2 k_j^2 m_i m_j (k_i + k_j)^2 (m_i + m_j)}, \tag{30}$$

where $1 \leq i < j \leq 3$. The auxiliary function $f(x,y,t)$ of the two-kink soliton solutions:

$$f\left(x,y,t\right) = 1 + \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1}\, t} + \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2}\, t}$$

$$+ \frac{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\, k_1^2 k_2^2 m_1 m_2 \left(k_1^2 - k_2^2\right)(m_1 - m_2)}{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\, k_1^2 k_2^2 m_1 m_2 (k_1 + k_2)^2 (m_1 + m_2)}$$

$$\times \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2})\, t}.$$

For the three-kink soliton solutions explicitly,

$$\begin{aligned} f(x,y,t) = {} & 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + \mathrm{e}^{\theta_3} + a_{12}\, \mathrm{e}^{\theta_1+\theta_2} + a_{13}\, \mathrm{e}^{\theta_1+\theta_3} + a_{23}\, \mathrm{e}^{\theta_2+\theta_3} \\ & + a_{12}\, a_{23}\, a_{13}\, \mathrm{e}^{\theta_1+\theta_2+\theta_3}, \end{aligned} \tag{31}$$

or equivalently

$$\begin{aligned} f\left(x,y,t\right) = {} & 1 + \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1}\, t} + \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2}\, t} \\ & + \mathrm{e}^{k_3 x + m_3 y - \frac{\alpha^2 m_3^2 + k_3^4}{k_3}\, t} \\ & + a_{12} \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2})\, t} \\ & + a_{13} \mathrm{e}^{(k_1+k_3)x + (m_1+m_3)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t} \\ & + a_{23} \mathrm{e}^{(k_2+k_3)x + (m_2+m_3)y - (\frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t} \\ & + b_{123} \mathrm{e}^{(k_1+k_2+k_3)x + (m_1+m_2+m_3)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t}, \end{aligned}$$

and proceed as before to find that

$$b_{123} = a_{12}\, a_{13}\, a_{23},$$

For the three-kink soliton solutions, we substitute the last result for $f(x,y,t)$ into (26). The higher level soliton solutions, for $N \geq 4$ can be obtained in a similar manner. This confirms the fact that the (2+1)-dimensional BLMP equation (22) is completely integrable and gives rise to have multiple-soliton solutions of any order.

4.2 Multiple singular soliton solutions

For $C_1 = C_2 = C_3 = -1$ determines the multiple singular-kink soliton solutions. The auxiliary function of single singular-kink soliton solutions:

$$f(x,y,t) = 1 - f_1(x,y,t) = 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 {m_1}^3 + \beta^2 {k_1}^3 + {k_1}^4 m_1\right) t}{k_1 m_1}},$$

to find that

$$u\,(x,y,t) = \frac{2\,k_1 \mathrm{e}^{k_1 x + m_1 y - \frac{\left(\alpha^2 {m_1}^3 + \beta^2 {k_1}^3 + {k_1}^4 m_1\right) t}{k_1 m_1}}}{1 - \mathrm{e}^{k_1 x + m_1 y - \frac{(\alpha^2 {m_1}^3 + \beta^2 {k_1}^3 + {k_1}^4 m_1) t}{k_1 m_1}}}. \tag{32}$$

The auxiliary function for two-kink soliton solutions:

$$f\,(x,y,t) = 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} t} - \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} t}$$

$$+ \frac{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\,k_1^2 k_2^2 m_1 m_2 \left(k_1^2 - k_2^2\right)(m_1 - m_2)}{(k_1 m_2 - k_2 m_1)^2 \left(\left(k_1^2 k_2 + k_1 k_2^2\right)\beta^2 + \left(m_1^2 m_2 + m_1 m_2^2\right)\alpha^2\right) - 3\,k_1^2 k_2^2 m_1 m_2 (k_1 + k_2)^2 (m_1 + m_2)}$$

$$\times \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2}\, t},$$

in (26). Finally, in order to obtain three-kink soliton solutions, the auxiliary function:

$$\begin{aligned} f\,(x,y,t) = {} & 1 - \mathrm{e}^{k_1 x + m_1 y - \frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} t} \\ & - \mathrm{e}^{k_2 x + m_2 y - \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} t} - \mathrm{e}^{k_3 x + m_3 y - \frac{\alpha^2 m_3^2 + k_3^4}{k_3} t} \\ & + a_{12} \mathrm{e}^{(k_1+k_2)x + (m_1+m_2)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2})\, t} \\ & + a_{13} \mathrm{e}^{(k_1+k_3)x + (m_1+m_3)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t} \\ & + a_{23} \mathrm{e}^{(k_2+k_3)x + (m_2+m_3)y - (\frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t} \\ & + b_{123} \mathrm{e}^{(k_1+k_2+k_3)x + (m_1+m_2+m_3)y - (\frac{\alpha^2 m_1^3 + \beta^2 k_1^3 + k_1^4 m_1}{k_1 m_1} + \frac{\alpha^2 m_2^3 + \beta^2 k_2^3 + k_2^4 m_2}{k_2 m_2} + \frac{\alpha^2 m_3^3 + \beta^2 k_3^3 + k_3^4 m_3}{k_3 m_3})\, t}, \end{aligned}$$

into (26) which gives

$$b_{123} = -a_{12}\, a_{13}\, a_{23}.$$

Therefore, we could obtain multiple singular soliton solutions for this model as well.

5 Conclusion

In this paper, the HM and the Cole-Hopf method are used to carry out the integrability of new extensions of (2+1)-dimensional BLMP equation. Various kinds of solutions are obtained for these extensions. We used the simplified Hirota's method and the CHM to formally derive the multiple soliton solutions for the extended equations. It can be concluded that HM with CHT is a powerful mathematical tool for a wide variety of another extensions of integrable multidimensional equations in different scientific areas. The results obtained in this study are helpful to explain some physical meanings of some nonlinear dynamics models in in waves propagation. It has been noticed that this method is algorithmic and computational, and easy to implementable.

6 Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

7 Competing interests

The authors declare that they have no competing interests.

8 Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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