

MAT 5120 - ADVANCED ALGEBRA - 2020-2021 SPRING

HOMEWORK ASSIGNMENT 3

DUE MAY 6TH 2021

There are 10 questions each worth 10 points.

(1) Determine the conjugacy classes and the number of elements in each conjugacy class for the groups D_8 , Q_8 and A_4 . Show your work.

(2) Determine the conjugacy classes and the number of elements in each conjugacy class for the groups S_6 and A_6 . Show your work.

(3) Show that A_5 has no subgroup of order 15, 20 or 30. (Hint: Consider the action of A_5 on the left cosets of a subgroup.)

(4) Let G be a group.

(a) Show that $Z(G)$ is characteristic in G .

(b) Show that $\text{Inn}(G)$ is a normal subgroup of $\text{Aut}(G)$.

(5) Let $H \leq K \leq G$ be groups.

(a) Show that if H is characteristic in K and K is normal in G , then H is normal in G .

(b) Show that if H is characteristic in K and K is characteristic in G , then H is characteristic in G .

(c) Give an example to show that if H is normal in K and K is characteristic in G , then H need not be normal in G .

(6) Let $H \leq K \leq G$ be groups. Show that if H is characteristic in G and K/H is characteristic in G/H , then K is characteristic in G .

(7) Show that there is no simple group of order 28, 56, or 312.

(8) Show that a group of order $231 = 3 \cdot 7 \cdot 11$ must have

(a) a normal Sylow 7-subgroup,

(b) a normal Sylow 11-subgroup in the center.

(9) For $p \in \{2, 3, 5\}$, find $n_p(S_5)$.

(10) Classify all abelian groups of order 4320 by writing the invariant factor decomposition of each group.