EXAMPLE 1

Humans are able to control their heat production rate and heat loss rate to maintain a nearly constant core temperature of T_c = 37°C under a wide range of environmental conditions. This process is called *thermoregulation*. From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature, T_i = 35°C = 308 K. Consider a person with a skin/fat layer of thickness L = 3 mm and effective thermal conductivity k = 0.3 W/m K. The person has a surface area A = 1.8 m^2 and is dressed in a bathing suit. The emissivity of the skin is $\varepsilon = 0.95$.

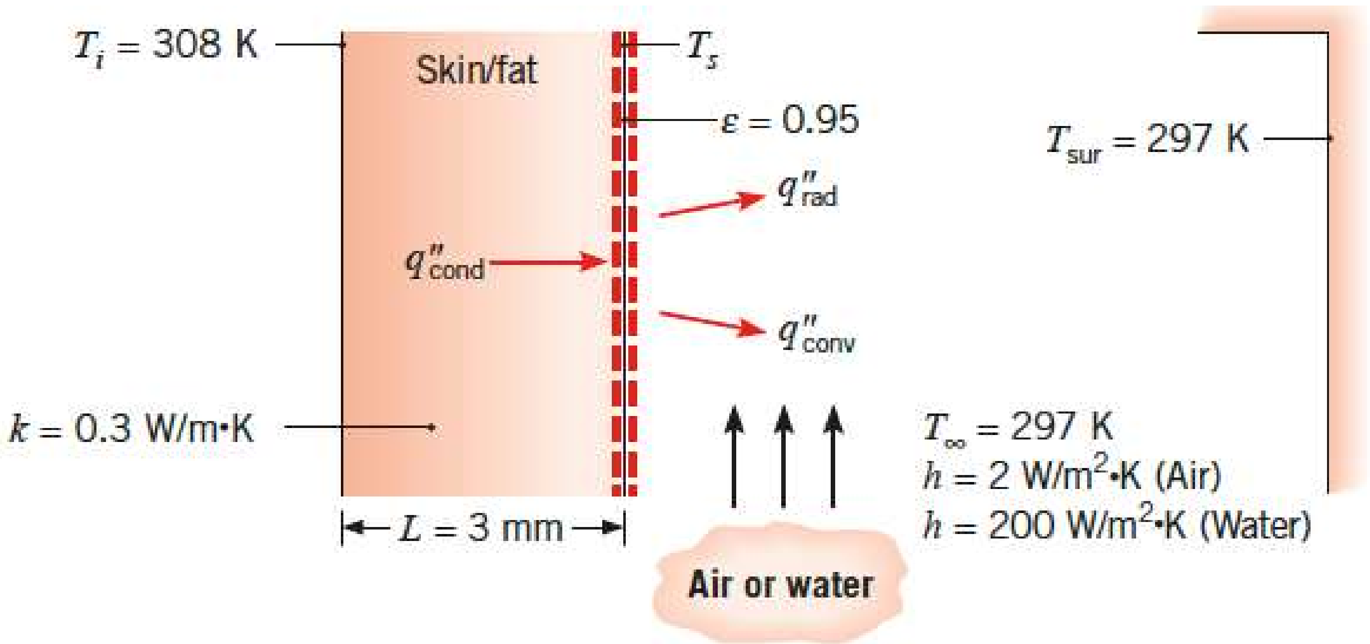
1. When the person is in still air at T_∞ = 297 K, what is the skin surface temperature and rate of heat loss to the environment? Convection heat transfer to the air is characterized by a free convection coefficient of h = 2 W/m^2K.
2. When the person is in water at T_∞ = 297 K, what is the skin surface temperature and heat loss rate? Heat transfer to the water is characterized by a convection coefficient of h = 200 W/m^2K.

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, emissivity, and surface area. Ambient conditions.

Find: Skin surface temperature and heat loss rate for the person in air and the person in water.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat layer.
3. Thermal conductivity is uniform.
4. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
5. Liquid water is opaque to thermal radiation.
6. Bathing suit has no effect on heat loss from body.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

Analysis:

1. The skin surface temperature may be obtained by performing an energy balance at the skin surface.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

It follows that, on a unit area basis,

$$\dot{q}''_{cond} - \dot{q}''_{conv} - \dot{q}''_{rad} = 0$$

or, rearranging and substituting,

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{surr}^4)$$

The only unknown is T_s, but we cannot solve for it explicitly because of the fourth-power dependence of the radiation term. Therefore, we must solve the equation iteratively, which can be done by hand or by using some other equation solver. To expedite a hand solution, we write the radiation heat flux in terms of the radiation heat transfer coefficient:

$$q_{rad} = h_r A(T_s - T_{sur})$$

$$h_r \equiv \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$$

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + h_r(T_s - T_{surr})$$

Solving for T_s, with $T_{sur} = T_\infty$, we have

$$T_s = \frac{\frac{kT_i}{L} + (h + h_r)T_\infty}{\frac{k}{L} + (h + h_r)}$$

We estimate h_r using Equation $h_r \equiv \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$ with a guessed value of T_s = 305 K and T_∞ = 297 K, to yield h_r = 5.9 W/m² K. Then, substituting numerical values into the preceding equation, we find

$$T_s = \frac{\frac{0.3\frac{W}{m}\cdot K x\ 308\ K}{3x10^{-3}\ m} + (2+5.9)\frac{W}{m^2\cdot K} x\ 297\ K}{\frac{0.3\frac{W}{m}\cdot K}{3x10^{-3}\ m} + (2+5.9)\frac{W}{m^2\cdot K}} = 307.2\ K$$

With this new value of T_s, we can recalculate h_r and Ts, which are unchanged. Thus the skin temperature is 307.2 K ≅ 34 C.
The rate of heat loss can be found by evaluating the conduction through the skin/fat layer:

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\frac{W}{m}\cdot K\ x\ 1.8\ m^2\ x\ \frac{(308 - 307.2)\ K}{3x10^{-3}\ m} = 146\ W$$

2. Since liquid water is opaque to thermal radiation, heat loss from the skin surface is by convection only. Using the previous expression with $h_r = 0$, we find

$$T_s = \frac{\dfrac{0.3\frac{W}{m}\cdot Kx\ 308\ K}{3x10^{-3}\ m} + 200\ \dfrac{W}{m^2\cdot K}x\ 297\ K}{\dfrac{0.3\frac{W}{m}\cdot K}{3x10^{-3}\ m} + 200\ \dfrac{W}{m^2\cdot K}} = 300.7\ K$$

and

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\frac{W}{m}\cdot K\ x\ 1.8\ m^2\ x\ \frac{(308 - 300.7)\ K}{3x10^{-3}\ m} = 1320\ W$$

Comments:

1. When using energy balances involving radiation exchange, the temperatures appearing in the radiation terms must be expressed in kelvins, and it is good practice to use kelvins in all terms to avoid confusion.
2. In part 1, heat losses due to convection and radiation are 37 W and 109 W, respectively. Thus, it would not have been reasonable to neglect radiation. Care must be taken to include radiation when the heat transfer coefficient is small (as it often is for natural convection to a gas), even if the problem statement does not give any indication of its importance.
3. A typical rate of metabolic heat generation is 100 W. If the person stayed in the water too long, the core body temperature would begin to fall. The large heat loss in water is due to the higher heat transfer coefficient, which in turn is due to the much larger thermal conductivity of water compared to air.
4. The skin temperature of 34^{o}C in part 1 is comfortable, but the skin temperature of 28^{o}C in part 2 is uncomfortably cold.

EXAMPLE 2

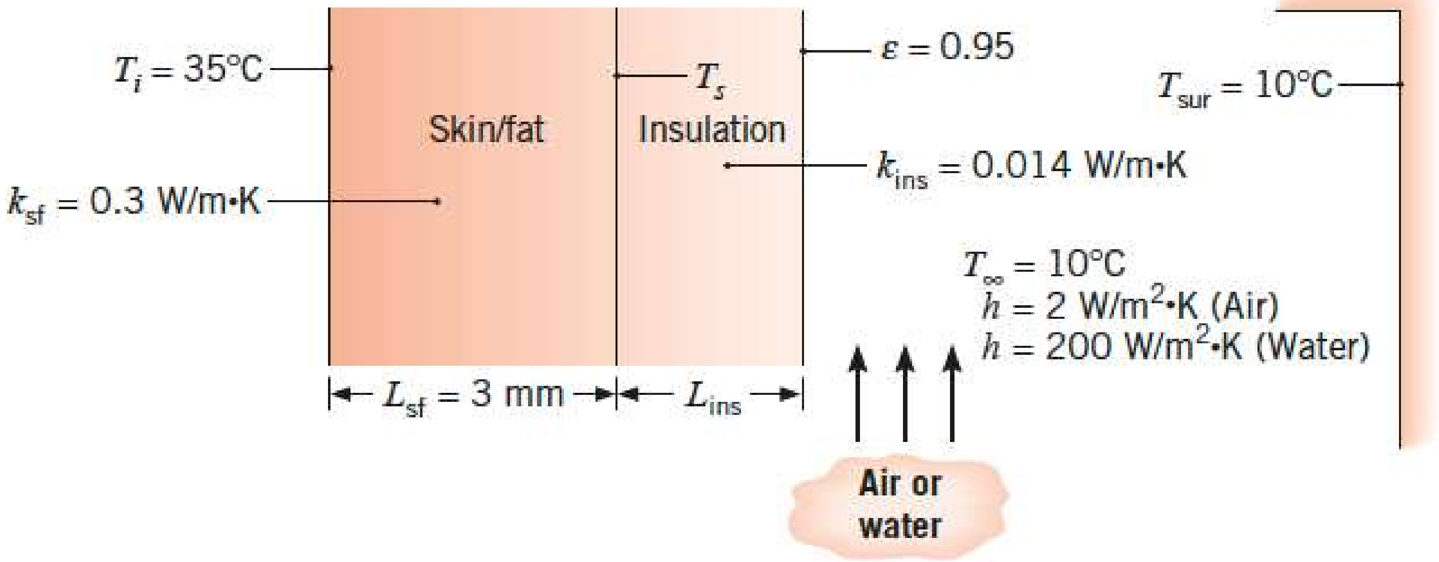
In Example 1, we calculated the heat loss rate from a human body in air and water environments. Now we consider the same conditions except that the surroundings (air or water) are at 10^{o}C. To reduce the heat loss rate, the person wears special sporting gear (snow suit and wet suit) made from a nanostructured silica aerogel insulation with an extremely low thermal conductivity of 0.014 W/m K. The emissivity of the outer surface of the snow and wet suits is 0.95. What thickness of aerogel insulation is needed to reduce the heat loss rate to 100 W (a typical metabolic heat generation rate) in air and water? What are the resulting skin temperatures?

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, and surface area. Thermal conductivity and emissivity of snow and wet suits. Ambient conditions.

Find: Insulation thickness needed to reduce heat loss rate to 100 W and corresponding skin temperature.

Schematic:

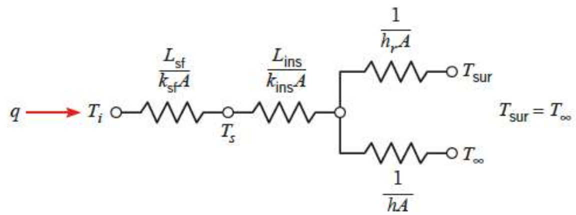


Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat and insulation layers.
3. Contact resistance is negligible.
4. Thermal conductivities are uniform.
5. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
6. Liquid water is opaque to thermal radiation.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

Analysis:

The thermal circuit can be constructed by recognizing that resistance to heat flow is associated with conduction through the skin/fat and insulation layers and convection and radiation at the outer surface. Accordingly, the circuit and the resistances are of the following form (with $h_r = 0$ for water):



The total thermal resistance needed to achieve the desired heat loss rate is found from

$$R_{tot} = \frac{T_i - T_\infty}{q} = \frac{(35 - 10)\ K}{100\ W} = 0.25\ K/W$$

The total thermal resistance between the inside of the skin/fat layer and the cold surroundings includes conduction resistances for the skin/fat and insulation layers and an effective resistance associated with convection and radiation, which act in parallel.

Hence,

$$R_{tot} = \frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_rA}\right)^{-1} = \frac{1}{A}\left(\frac{L_{sf}}{k_{sf}} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h + h_r}\right)$$

Air

The radiation heat transfer coefficient is approximated as having the same value as in Example 1: h_r = 5.9 W/m^2 K.

$$L_{ins} = k_{ins}\left(AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h + h_r}\right)$$

$$L_{ins} = 0.014\, W/m \cdot K\left(1.8\, m^2 x\, 0.25\, K/W - \frac{3x10^{-3}\, m}{0.3\, \frac{W}{m} \cdot K} - \frac{1}{(2 + 5.9)\, W/m^2 \cdot K}\right)$$

$$L_{ins} = 0.0044\, m = 4.4\, mm$$

Water

$$L_{ins} = k_{ins}\left(AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h}\right)$$

$$L_{ins} = 0.014\ W/m\cdot K\left(1.8\ m^2 x\ 0.25\ K/W - \frac{3x10^{-3}\ m}{0.3\frac{W}{m}\cdot K} - \frac{1}{200\ W/m^2\cdot K}\right)$$

$$L_{ins} = 0.0061\ m = 6.1\ mm$$

These required thicknesses of insulation material can easily be incorporated into the snow and wet suits.

The skin temperature can be calculated by considering conduction through the skin/fat layer:

$$q = k_{sf} A \frac{T_i - T_s}{L_{sf}}$$

or solving for T_s,

$$T_s = T_i - \frac{q L_{sf}}{k_{sf} A} = 35°\text{C} - \frac{100\ W\ x\ 3x10^{-3}\ m}{0.3 \frac{W}{m} \cdot K\ x\ 1.8\ m^2} = 34.4°\text{C}$$

The skin temperature is the same in both cases because the heat loss rate and skin/fat properties are the same.

Comments:

1. The nanostructured silica aerogel is a porous material that is only about 5% solid. Its thermal conductivity is less than the thermal conductivity of the gas that fills its pores. As explained in the context, the reason for this seemingly impossible result is that the pore size is only around 20 nm, which reduces the mean free path of the gas and hence decreases its thermal conductivity.
2. By reducing the heat loss rate to 100 W, a person could remain in the cold environments indefinitely without becoming chilled. The skin temperature of 34.4^{o}C would feel comfortable.
3. In the water case, the thermal resistance of the insulation dominates and all other resistances can be neglected.
4. The convection heat transfer coefficient associated with the air depends on the wind conditions, and it can vary over a broad range. As it changes, so will the outer surface temperature of the insulation layer. Since the radiation heat transfer coefficient depends on this temperature, it will also vary.