

# Chapter 9

## One- and Two-Sample Estimation Problems

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# Section 9.1

## Introduction

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# Section 9.2

## Statistical Inference

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# Section 9.3

## Classical Methods of Estimation

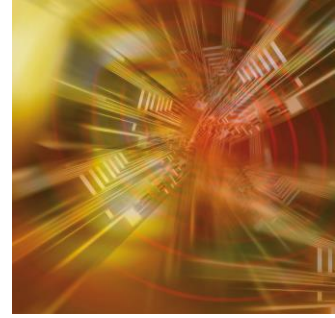
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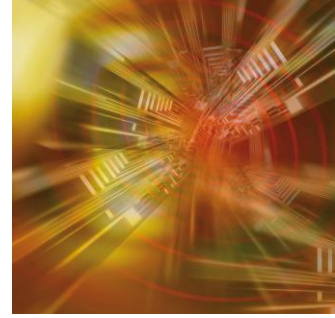
# Definition 9.1



A statistic  $\hat{\Theta}$  is said to be an **unbiased estimator** of the parameter  $\theta$  if

$$\mu_{\hat{\Theta}} = E(\hat{\Theta}) = \theta.$$

## Definition 9.2

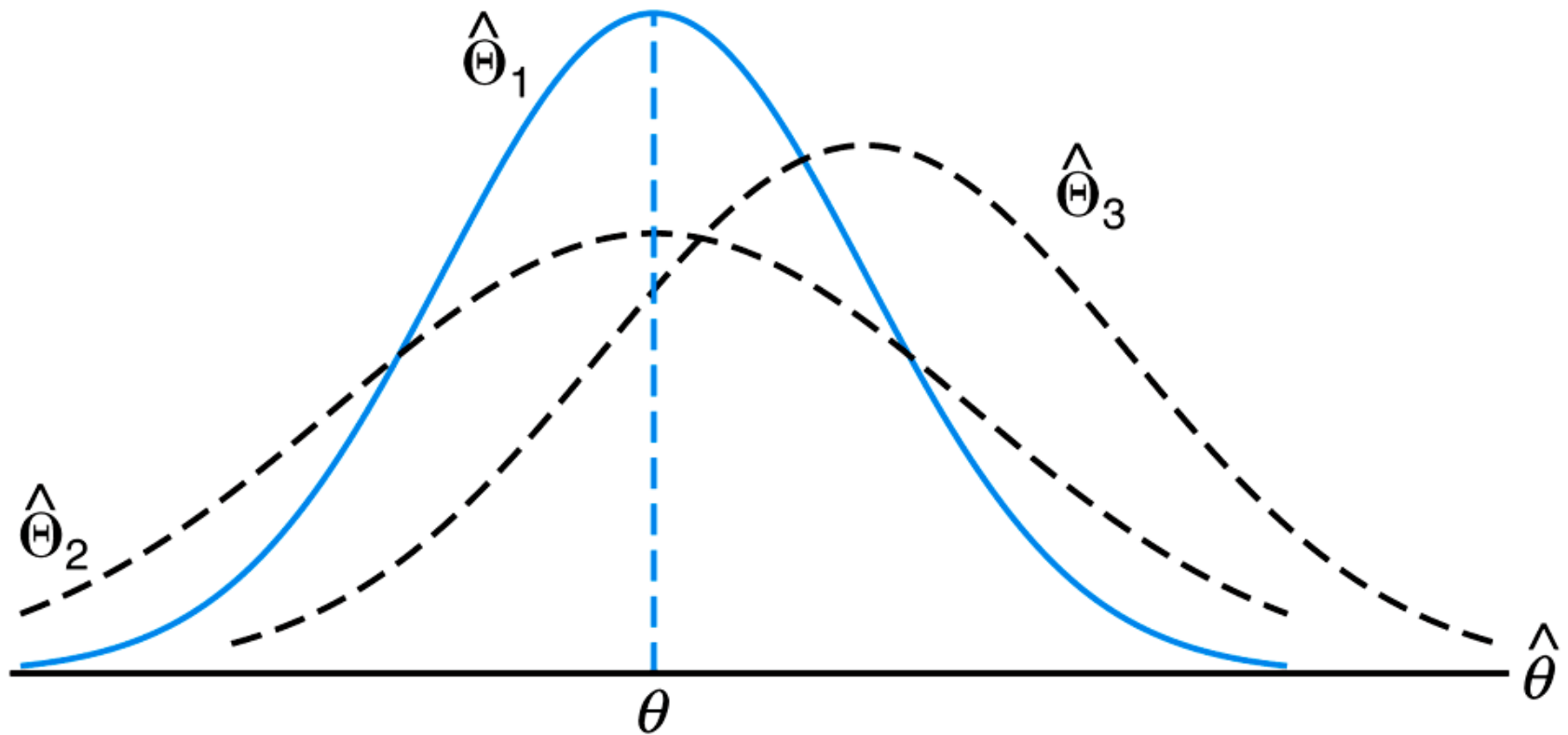


If we consider all possible unbiased estimators of some parameter  $\theta$ , the one with the smallest variance is called the **most efficient estimator** of  $\theta$ .





**Figure 9.1** Sampling distributions of different estimators of  $\theta$



# Section 9.4

## Single Sample: Estimating the Mean

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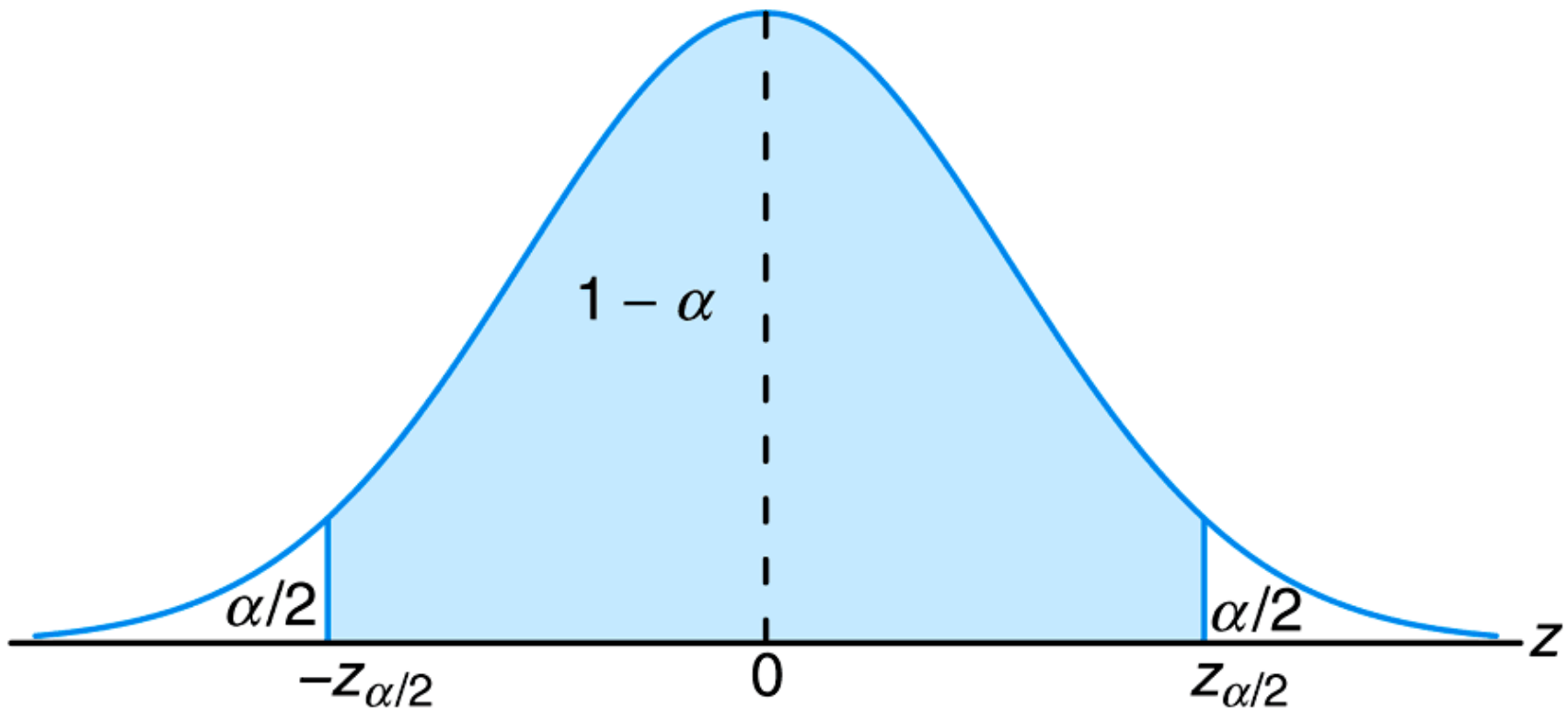
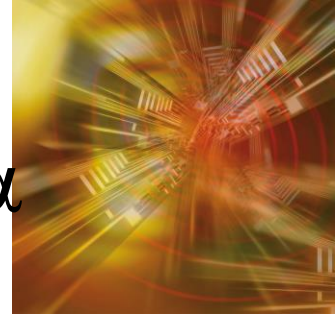
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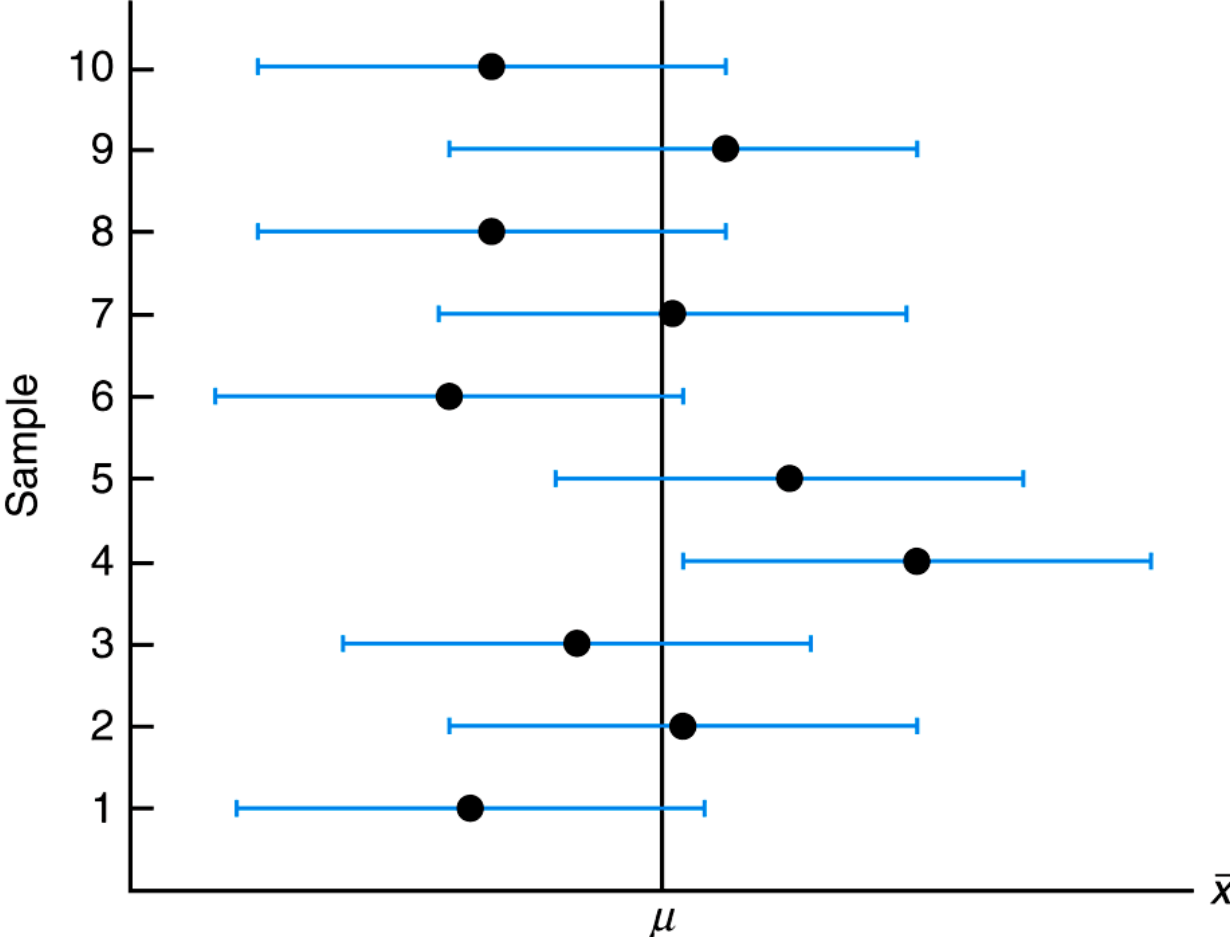


**Figure 9.2**  $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$

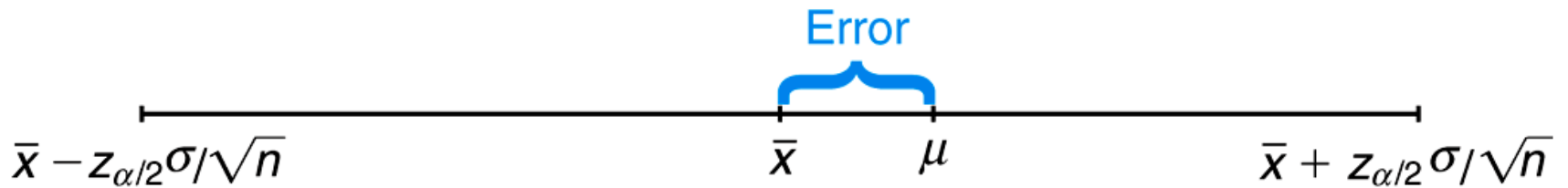




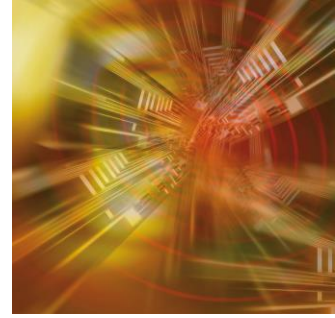
# Figure 9.3 Interval estimates of $\mu$ for different samples



# Figure 9.4 Error in estimating $\mu$ by $\bar{x}$

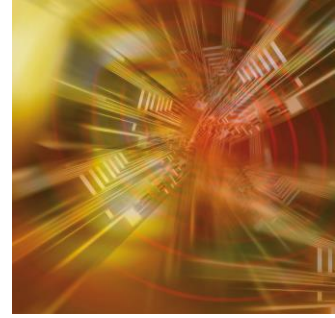


# Theorem 9.1



If  $\bar{x}$  is used as an estimate of  $\mu$ , we can be  $100(1 - \alpha)\%$  confident that the error will not exceed  $z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ .

# Theorem 9.2



If  $\bar{x}$  is used as an estimate of  $\mu$ , we can be  $100(1 - \alpha)\%$  confident that the error will not exceed a specified amount  $e$  when the sample size is

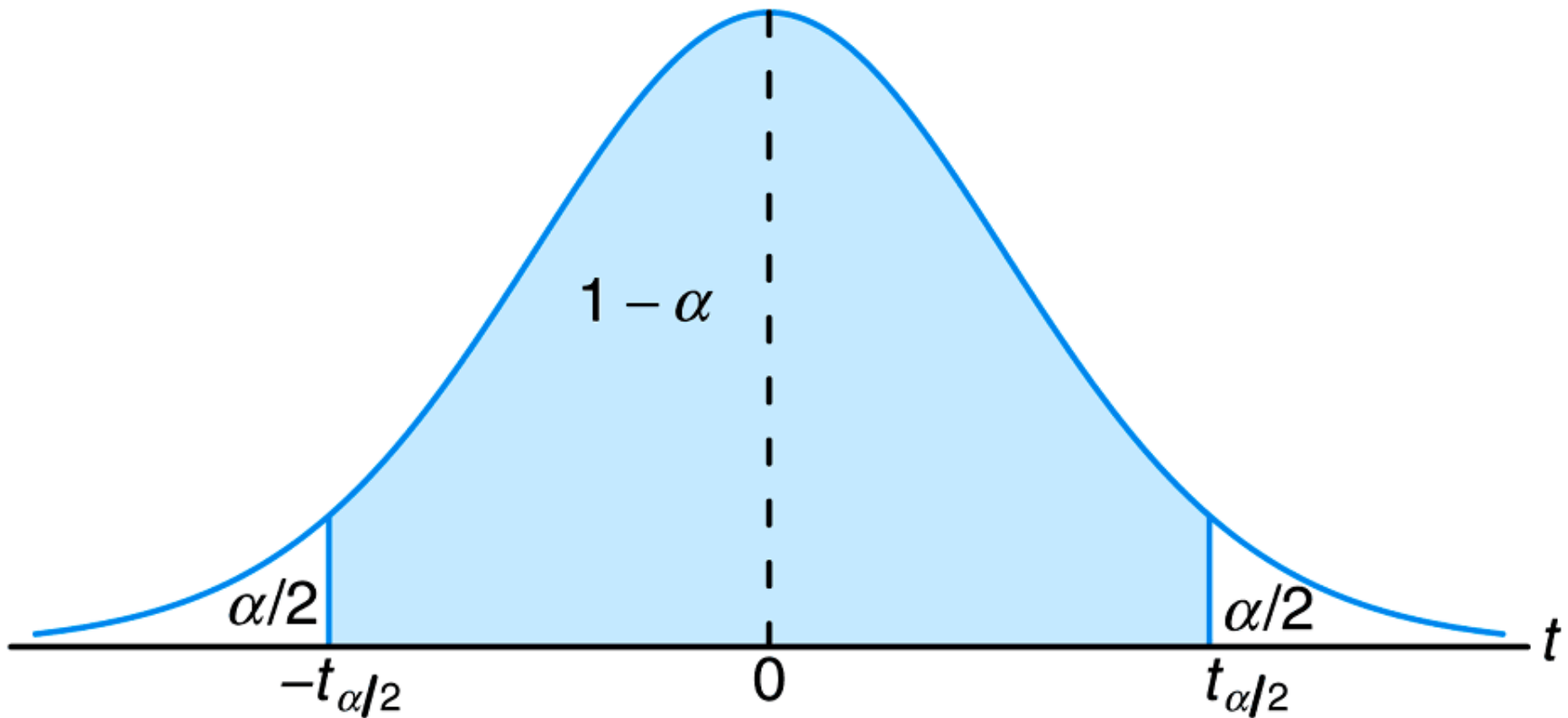
$$n = \left( \frac{z_{\alpha/2}\sigma}{e} \right)^2 .$$

Figure 9.5

$$P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha$$



$$P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha$$





# Section 9.5

## Standard Error of a Point Estimate

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# Section 9.6

## Prediction Intervals

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# Section 9.7

## Tolerance Limits

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# Section 9.8

## Two Samples: Estimating the Difference between Two Means

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# Section 9.9

## Paired Observations

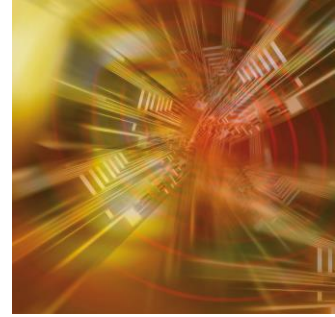
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# Table 9.1 Data for Example 9.13



Veteran	TCDD Levels in Plasma	TCDD Levels in Fat Tissue	$d_i$	Veteran	TCDD Levels in Plasma	TCDD Levels in Fat Tissue	$d_i$
1	2.5	4.9	-2.4	11	6.9	7.0	-0.1
2	3.1	5.9	-2.8	12	3.3	2.9	0.4
3	2.1	4.4	-2.3	13	4.6	4.6	0.0
4	3.5	6.9	-3.4	14	1.6	1.4	0.2
5	3.1	7.0	-3.9	15	7.2	7.7	-0.5
6	1.8	4.2	-2.4	16	1.8	1.1	0.7
7	6.0	10.0	-4.0	17	20.0	11.0	9.0
8	3.0	5.5	-2.5	18	2.0	2.5	-0.5
9	36.0	41.0	-5.0	19	2.5	2.3	0.2
10	4.7	4.4	0.3	20	4.1	2.5	1.6

Source: Schecter, A. et al. "Partitioning of 2,3,7,8-chlorinated dibenzo-*p*-dioxins and dibenzofurans between adipose tissue and plasma lipid of 20 Massachusetts Vietnam veterans," *Chemosphere*, Vol. 20, Nos. 7-9, 1990, pp. 954-955 (Tables I and II).



# Section 9.10

## Single Sample: Estimating a Proportion

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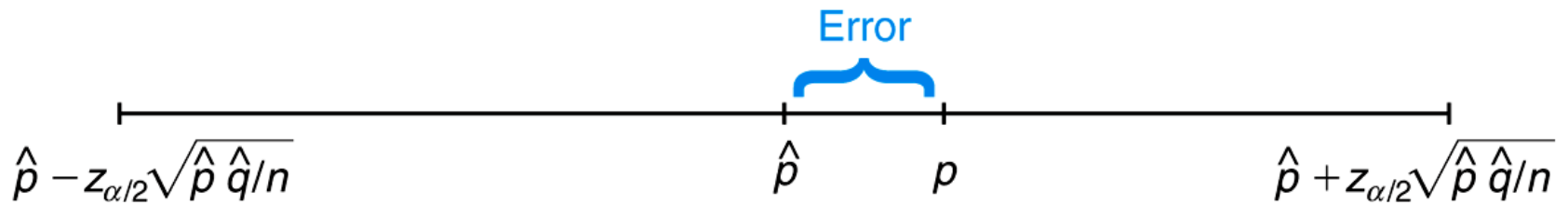
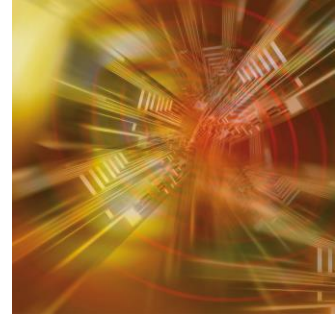
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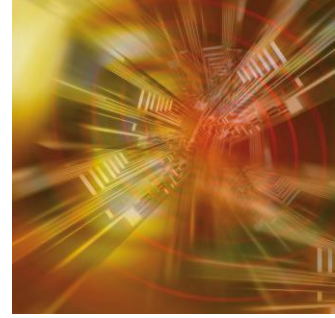
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# Figure 9.6 Error in estimating $p$ by

$\hat{p}$

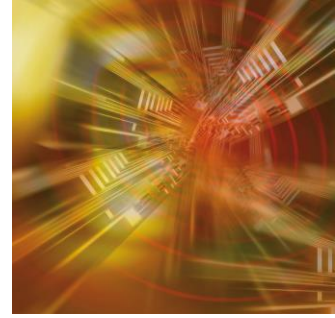


# Theorem 9.3



If  $\hat{p}$  is used as an estimate of  $p$ , we can be  $100(1 - \alpha)\%$  confident that the error will not exceed  $z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n}$ .

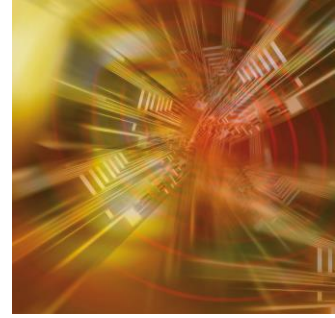
# Theorem 9.4



If  $\hat{p}$  is used as an estimate of  $p$ , we can be  $100(1 - \alpha)\%$  confident that the error will be less than a specified amount  $e$  when the sample size is approximately

$$n = \frac{z_{\alpha/2}^2 \hat{p} \hat{q}}{e^2}.$$

# Theorem 9.5



If  $\hat{p}$  is used as an estimate of  $p$ , we can be **at least**  $100(1 - \alpha)\%$  confident that the error will not exceed a specified amount  $e$  when the sample size is

$$n = \frac{z_{\alpha/2}^2}{4e^2}.$$

# Section 9.11

## Two Samples: Estimating the Difference between Two Proportions

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# Section 9.12

## Single Sample: Estimating the Variance

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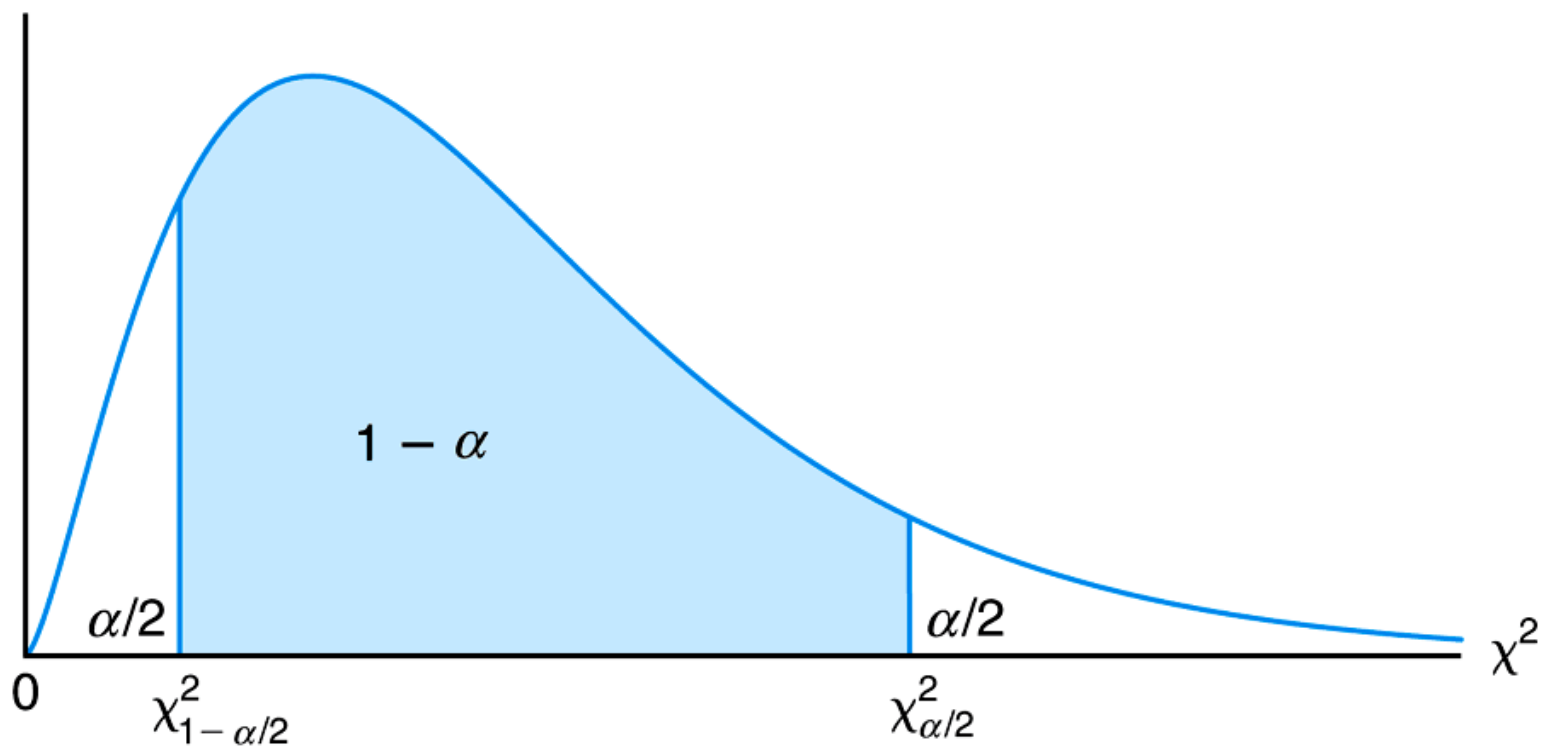
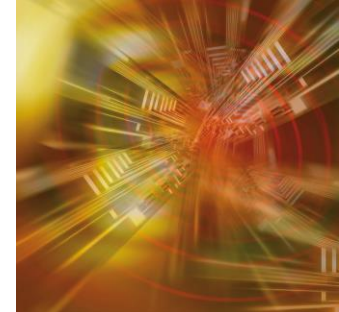
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**Figure 9.7**

$$P(\chi^2_{1-\alpha/2} < X^2 < \chi^2_{\alpha/2}) = 1 - \alpha$$



# Section 9.13

## Two Samples: Estimating the Ratio of Two Variances

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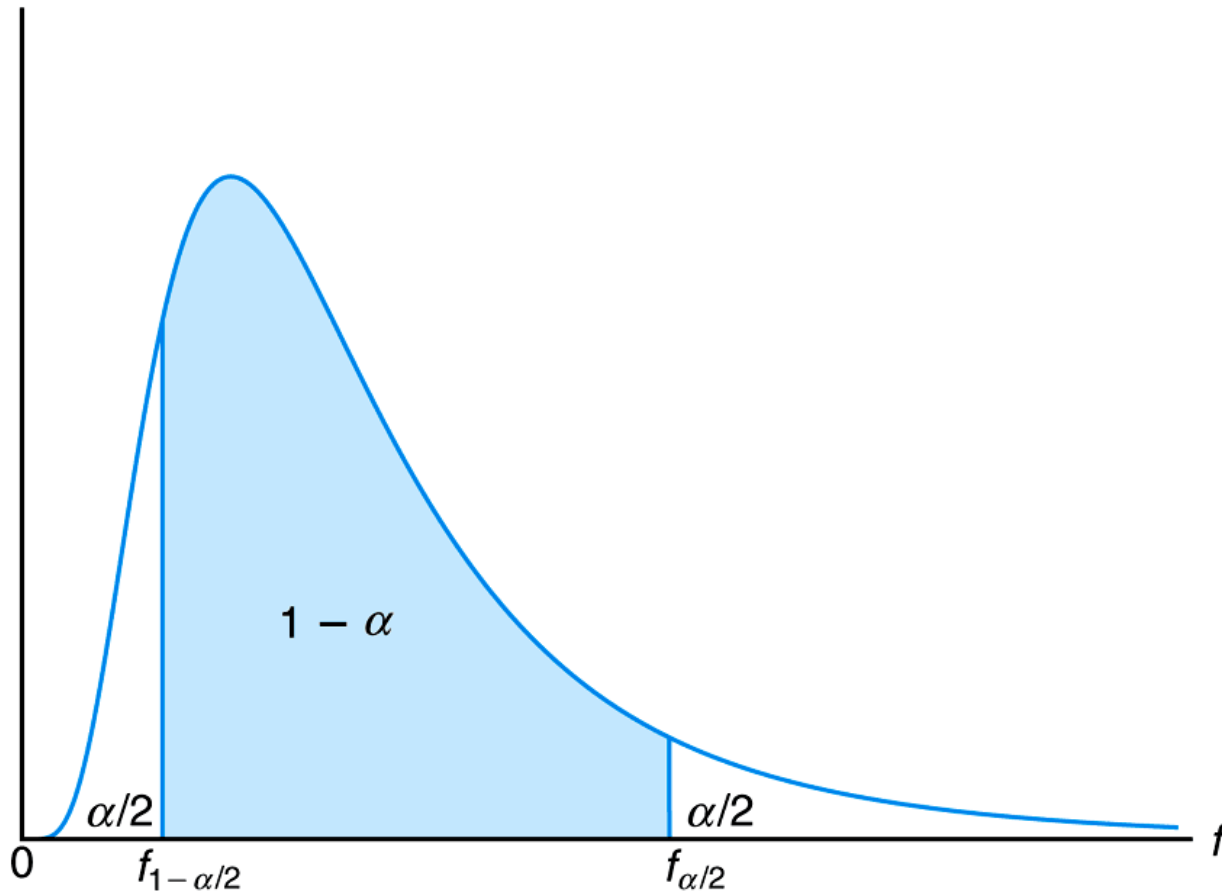
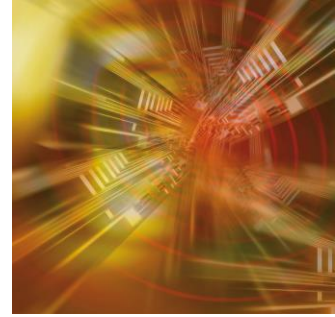
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# Figure 9.8

$$P[f_{1-\alpha/2}(v_1, v_2) < F < f_{\alpha/2}(v_1, v_2)] = 1 - \alpha$$



# Section 9.14

## Maximum Likelihood Estimation (Optional)

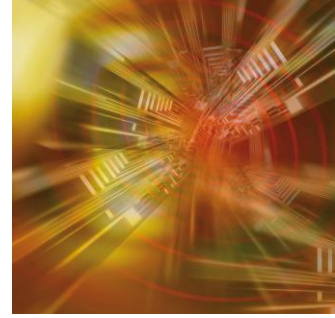
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# Definition 9.3



Given independent observations  $x_1, x_2, \dots, x_n$  from a probability density function (continuous case) or probability mass function (discrete case)  $f(\mathbf{x}; \theta)$ , the maximum likelihood estimator  $\hat{\theta}$  is that which maximizes the likelihood function

$$L(x_1, x_2, \dots, x_n; \theta) = f(\mathbf{x}; \theta) = f(x_1, \theta) f(x_2, \theta) \cdots f(x_n, \theta).$$



# Section 9.15

Potential  
Misconceptions  
and Hazards;  
Relationship to  
Material in Other  
Chapters

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