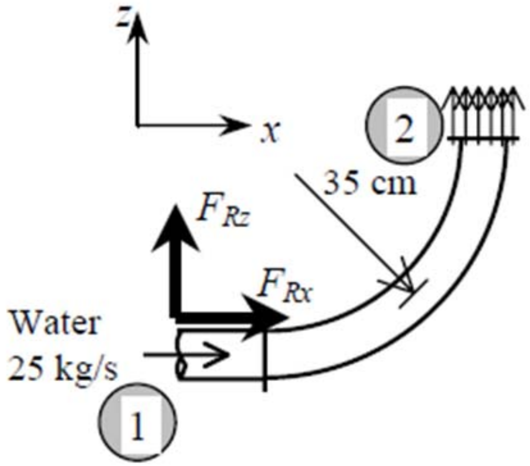
Newton's Laws and Conservation of Momentum
Sample Questions and Answers

EXAMPLE 6–22

A 90° elbow is used to direct water flow at a rate of *25 kg/s* in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be weightless.



Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be $1000\ kg/m^3$.

$\dot{m}_1 = \dot{m}_2 = \dot{m}$ 30kg/s

$\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\ \text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow P_1 - P_2 = \rho g(z_2 - z_1) \rightarrow P_{1,\text{gage}} = \rho g(z_2 - z_1)$$

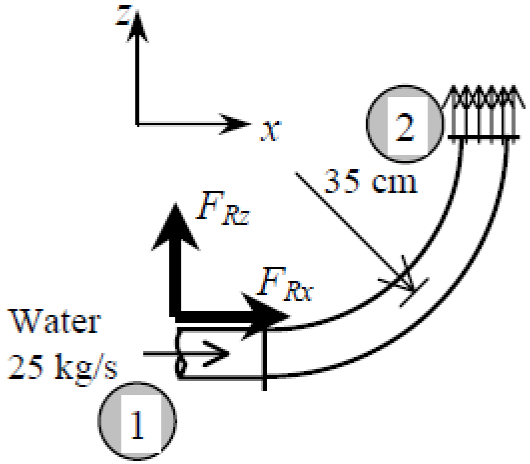
Substituting,

$$P_{1,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.35\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 3.434\ \text{kN/m}^2 = \mathbf{3.434\ kPa}$$

(b) The momentum equation for steady one-dimensional flow $\sum \vec{F} = \sum_{out} \beta\dot{m}\vec{V} - \sum_{in} \beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \beta\dot{m}(+V_1) = -\beta\dot{m}V$$
$$F_{Rz} = \beta\dot{m}(+V_2) = \beta\dot{m}V$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,



$$F_{Rx} = -\beta \dot{m} V - P_{1,\,gage} A_1$$

$$= -1.03(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) - (3434\ \text{N/m}^2)[\pi(0.1\ \text{m})^2/4]$$

$$= -109\ \text{N}$$

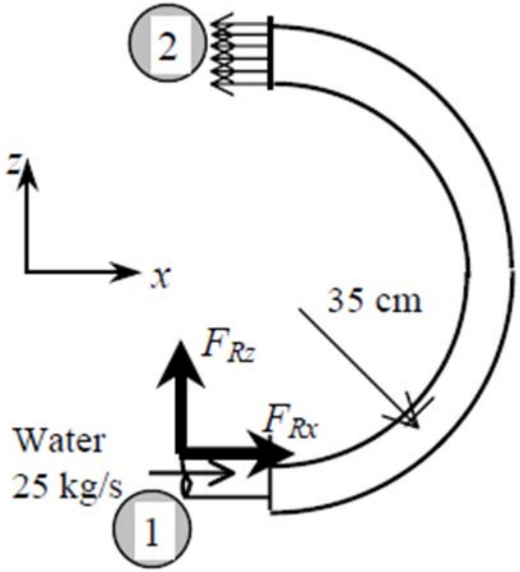
$$F_{Ry} = \beta \dot{m} V = 1.03(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 81.9\ \text{N}$$

and $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-109)^2 + 81.9^2} = \mathbf{136\ N},\quad \theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}} = \tan^{-1}\dfrac{81.9}{-109} = -37° = \mathbf{143°}$

Discussion Note that the magnitude of the anchoring force is 136 N, and its line of action makes 143° from the positive *x* direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

EXAMPLE 6–23

Repeat Prob. 6–22 for the case of another (identical) elbow being attached to the existing elbow so that the fluid makes a U-turn.



Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be *β = 1.03*.

Properties We take the density of water to be *1000 kg/m³*.

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\,\text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \rightarrow P_1-P_2=\rho g(z_2-z_1) \rightarrow P_{1,\text{ gage}}=\rho g(z_2-z_1)$$

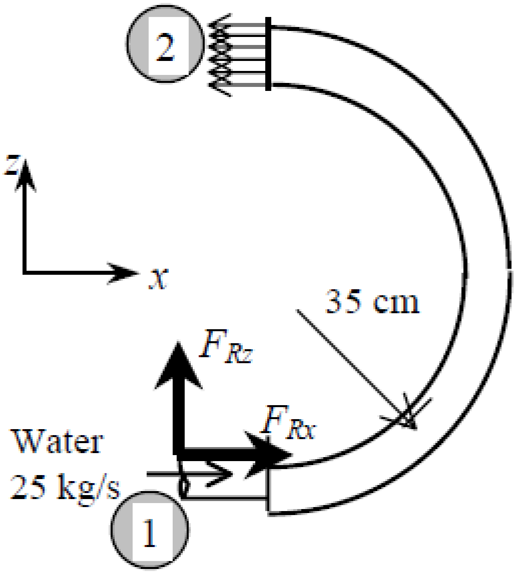
Substituting,

$$P_{1,\text{ gage}}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.70\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=6.867\text{ kN/m}^2=\mathbf{6.867\ kPa}$$

(b) The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=\beta\dot{m}(-V_2)-\beta\dot{m}(+V_1)=-2\beta\dot{m}V$$
$$F_{Rz}=0$$

Solving for F_{Rx} and substituting the given values,



$$F_{Rx} = -2\beta \dot{m} V - P_{1,\,gage} A_1$$

$$= -2 \times 1.03 (25\ \text{kg/s})(3.18\ \text{m/s}) \left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right) - (6867\ \text{N/m}^2)[\pi (0.1\ \text{m})^2 / 4]$$

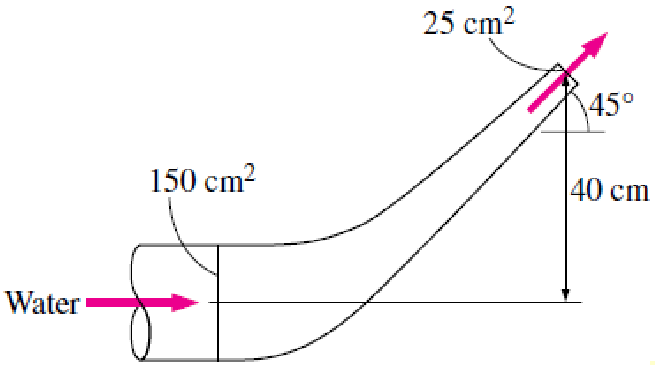
$$= -218\ \text{N}$$

and F_R = F_{Rx} = **- 218 N** since the y-component of the anchoring force is zero. Therefore, the anchoring force has a magnitude of 218 N and it acts in the negative *x* direction.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

EXAMPLE 6–24

A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle θ = 45° from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is *150 cm²* at the inlet and *25 cm²* at the exit. The elevation difference between the centers of the exit and the inlet is *40 cm*. The mass of the elbow and the water in it is *50 kg*. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.



Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\,\text{kg})(9.81\,\text{m/s}^2) = 490.5\,\text{N} = 0.4905\ \text{kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\,\text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\,\text{m}^2)} = 2.0\,\text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\,\text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\,\text{m}^2)} = 12\,\text{m/s}$$

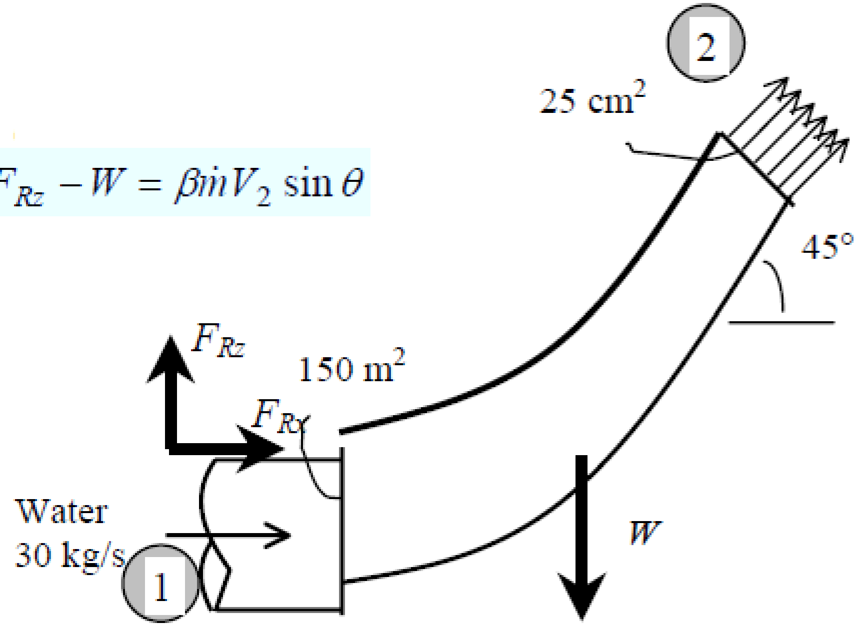
Taking the center of the inlet cross section as the reference level $(z_1 = 0)$ and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\,\text{gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

Substituting,

$$P_{1,\,\text{gage}} = (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\left(\frac{(12\,\text{m/s})^2 - (2\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + 0.4\right)\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 73.9\,\text{kN/m}^2 = 73.9\,\text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become



$$F_{Rx} + P_{1,\text{gage}} A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\text{ gage}} A_1$$

$$= 1.03(30 \text{ kg/s})[(12\cos 45° - 2) \text{ m/s}]\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$-(73.9 \text{ kN/m}^2)(0.0150 \text{ m}^2)$$

$$= -0.908 \text{ kN}$$

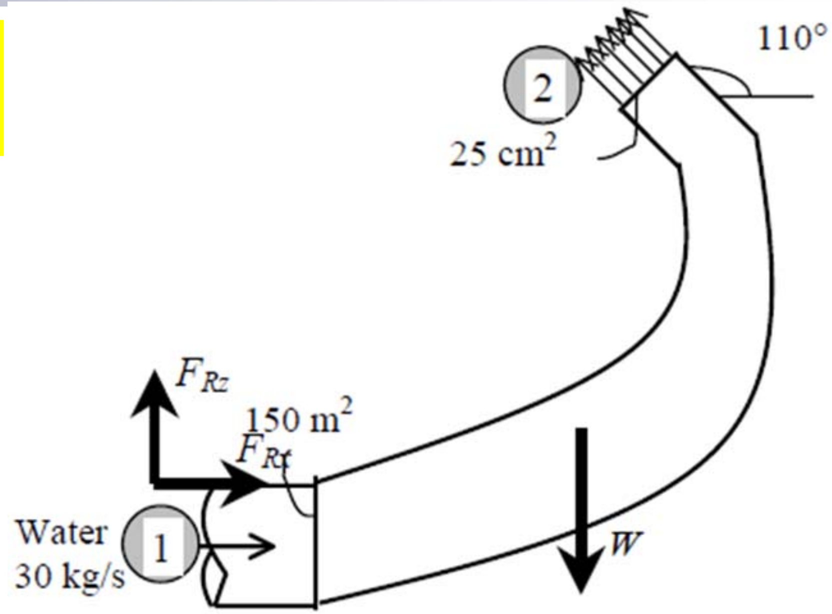
$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30 \text{ kg/s})(12\sin 45° \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) + 0.4905 \text{ kN} = 0.753 \text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\ kN}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = -\mathbf{39.7}°$$

Discussion Note that the magnitude of the anchoring force is *1.18 kN*, and its line of action makes *–39.7°* from *+x* direction. Negative value for F_{Rx} indicates the assumed direction is wrong.

EXAMPLE 6–25

Repeat Prob. 6–25 for the case of u 110°.



Properties We take the density of water to be *1000 kg/m^3*.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\ \text{kg})(9.81\ \text{m/s}^2) = 490.5\ \text{N} = 0.4905\ \text{kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\ \text{m}^2)} = 2.0\ \text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\ \text{m}^2)} = 12\ \text{m/s}$$

Taking the center of the inlet cross section as the reference level *(z_1 = 0)* and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \;\rightarrow\; P_1-P_2=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2-z_1\right) \;\rightarrow\; P_{1,\text{ gage}}=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2\right)$$

$$\text{or, } P_{1,\text{ gage}}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2-(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=73.9\text{ kN/m}^2=73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the *x*- and *y*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=\beta\dot{m}V_2\cos\theta-\beta\dot{m}V_1 \quad \text{and} \quad F_{Ry}-W=\beta\dot{m}V_2\sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta\dot{m}(V_2 \cos\theta - V_1) - P_{1,\text{ gage}} A_1$$

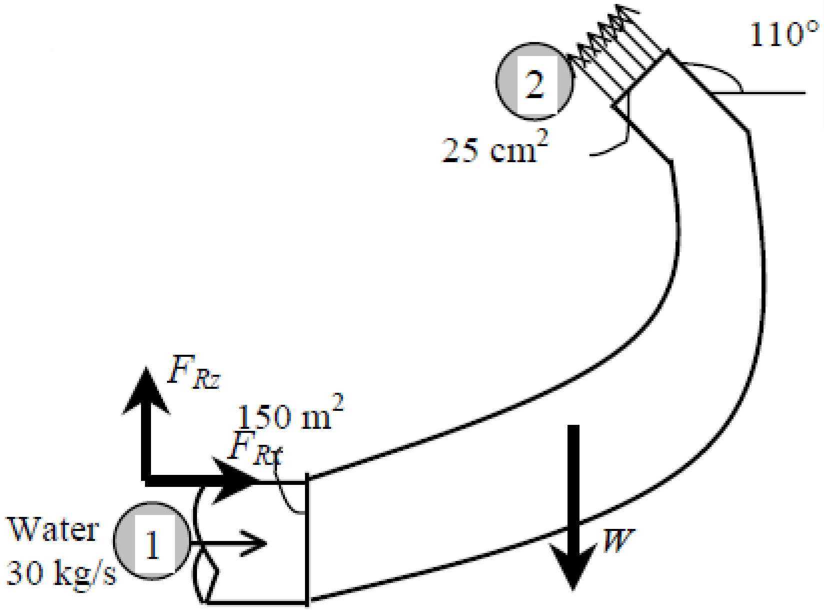
$$= 1.03(30 \text{ kg/s})[(12\cos 110° - 2) \text{ m/s}]\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) - (73.9 \text{ kN/m}^2)(0.0150 \text{ m}^2) = -1.297 \text{ kN}$$

$$F_{Rz} = \beta\dot{m}V_2 \sin\theta + W = 1.03(30 \text{ kg/s})(12\sin 110° \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) + 0.4905 \text{ kN} = 0.8389 \text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-1.297)^2 + 0.8389^2} = \mathbf{1.54\ kN}$$

and

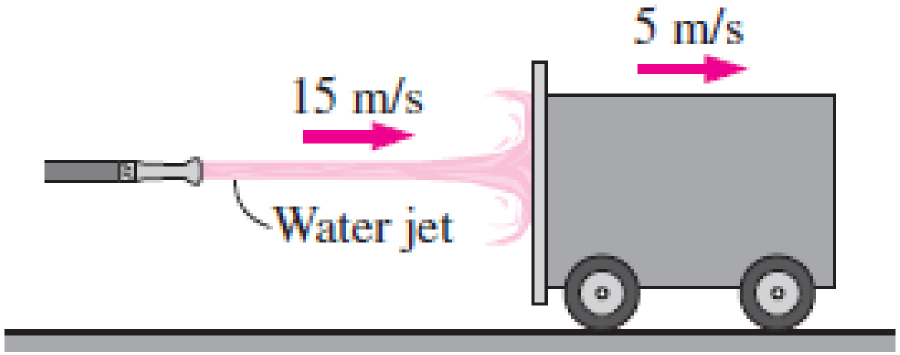
$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.8389}{-1.297} = \mathbf{-32.9°}$$



Discussion Note that the magnitude of the anchoring force is 1.54 kN, and its line of action makes –32.9° from +*x* direction. Negative value for F_{Rx} indicates assumed direction is wrong, and should be reversed.

EXAMPLE 6–26

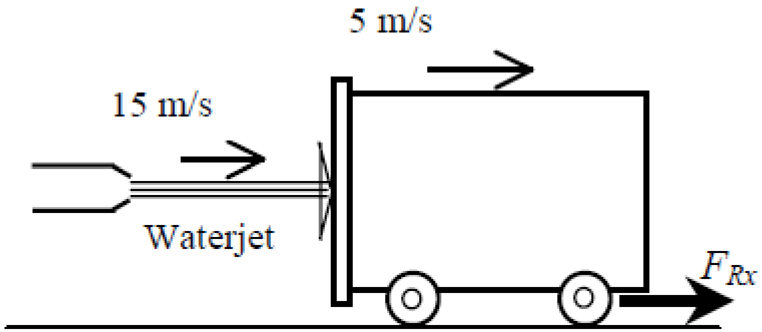
Water accelerated by a nozzle to 15 m/s strikes the vertical back surface of a cart moving horizontally at a constant velocity of 5 m/s in the flow direction. The mass flow rate of water is 25 kg/s. After the strike, the water stream splatters off in all directions in the plane of the back surface. (a) Determine the force that needs to be applied on the brakes of the cart to prevent it from accelerating. (b) If this force were used to generate power instead of wasting it on the brakes, determine the maximum amount of power that can be generated.



Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** Fiction during motion is negligible. **5** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal. **6** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the cart and the jet is

$$V_r = V_{\text{jet}} - V_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$



Therefore, we can assume the cart to be stationary and the jet to move with a velocity of *10 m/s*. The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m} V_r$$

We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m} V_r = -(25 \text{ kg/s})(+10 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{-250 \text{ N}}$$

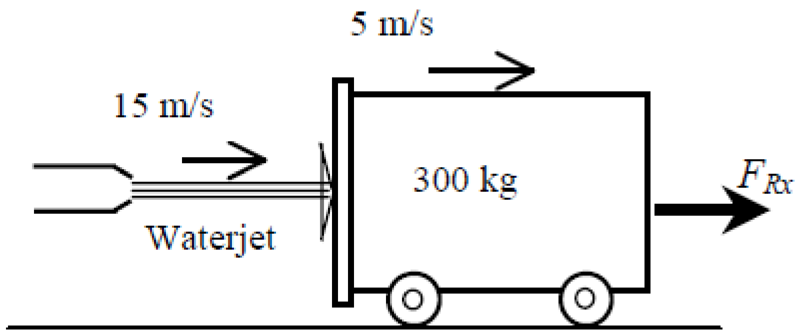
The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} V_{\text{cart}} = (250 \text{ N})(5 \text{ m/s})\left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right) = \mathbf{1.25 \text{ kW}}$$

Discussion Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

EXAMPLE 6-27

Reconsider Prob. 6–27. If the mass of the cart is 300 kg and the brakes fail, determine the acceleration of the cart when the water first strikes it. Assume the mass of water that wets the back surface is negligible.



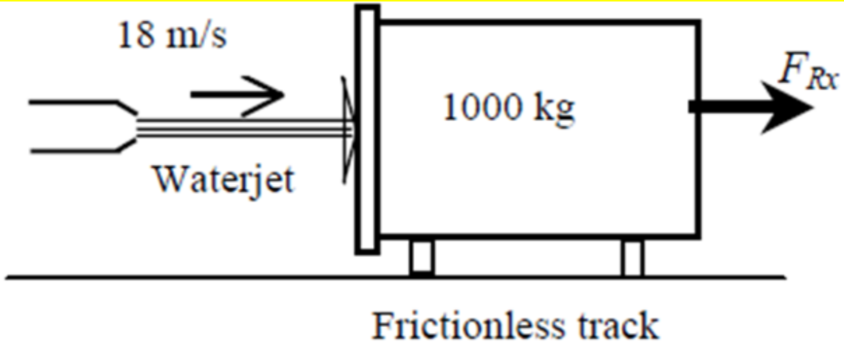
Analysis The braking force was determined in previous problem to be 250 N. When the brakes fail, this force will propel the cart forward, and the accelerating will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{250\ \text{N}}{300\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.833\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.

EXAMPLE 6–28

A horizontal 5-cm-diameter water jet with a velocity of 18 m/s impinges normally upon a vertical plate of mass 1000 kg. The plate is held in a nearly frictionless track and is initially stationary. When the jet strikes the plate, the plate begins to move in the direction of the jet. The water always splatters in the plane of the retreating plate. Determine (a) the acceleration of the plate when the jet first strikes it (time = 0), (b) the time it will take for the plate to reach a velocity of 9 m/s, and (c) the plate velocity 20 s after the jet first strikes the plate. Assume the velocity of the jet relative to the plate remains constant.



Properties We take the density of water to be *1000 kg/m*^{3}.
Analysis (a) We take the vertical plate on the frictionless track as the control volume, and the direction of flow as the positive direction of x axis.
The mass flow rate of water in the jet is

$$\dot{m} = \rho VA = (1000\ \text{kg/m}^3)(18\ \text{m/s})[\pi(0.05\ \text{m})^2 / 4] = 35.34\ \text{kg/s}$$

The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{Rx} = -\dot{m} V$$

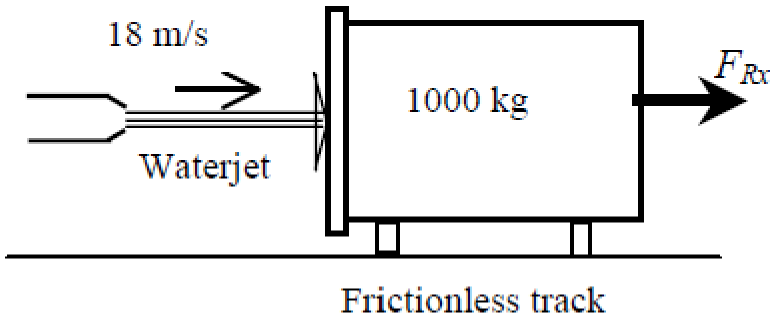
where F_{Rx} is the reaction force required to hold the plate in place. When the plate is released, an equal and opposite impulse force acts on the plate, which is determined to

$$F_{\text{plate}} = -F_{Rx} = \dot{m} V = (35.34\ \text{kg/s})(18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 636\ \ \text{N}$$

Then the initial acceleration of the plate becomes

$$a = \frac{F_{\text{plate}}}{m_{\text{plate}}} = \frac{636\ \text{N}}{1000\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \ \text{N}}\right) = \mathbf{0.636\ m/s^2}$$

This acceleration will remain constant during motion since the force acting on the plate remains constant.



(b) Noting that *a = dV/dt = ΔV/Δt* since the acceleration a is constant, the time it takes for the plate to reach a velocity of 9 m/s is

$$\Delta t = \frac{\Delta V_{\text{plate}}}{a} = \frac{(9-0)\ \text{m/s}}{0.636\ \text{m/s}^2} = \mathbf{14.2\ s}$$

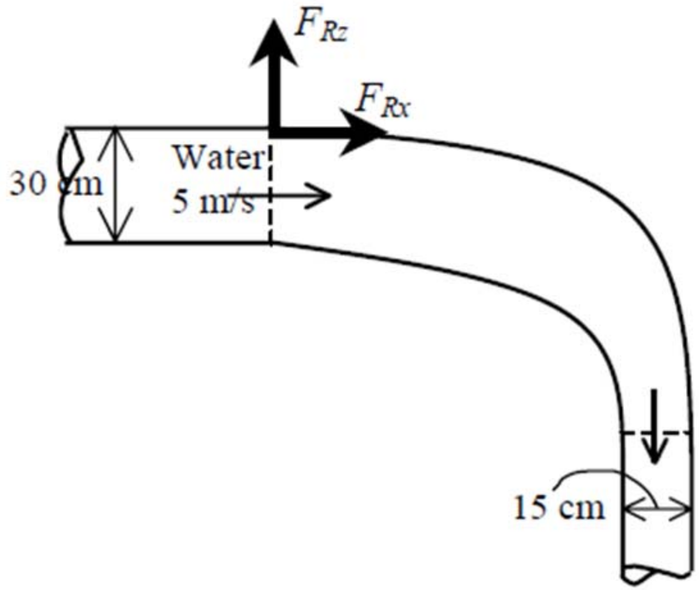
(c) Noting that *a = dV/dt* and thus *dV = adt* and that the acceleration a is constant, the plate velocity in 20 s becomes

$$V_{\text{plate}} = V_{0,\text{ plate}} + a\Delta t = 0 + (0.636\ \text{m/s}^2)(20\ \text{s}) = \mathbf{12.7\ m/s}$$

Discussion The assumption that the relative velocity between the water jet and the plate remains constant is valid only for the initial moments of motion when the plate velocity is low unless the water jet is moving with the plate at the same velocity as the plate.

EXAMPLE 6–29

Water flowing in a horizontal 30-cm-diameter pipe at 5 m/s and 300 kPa gage enters a 90° bend reducing section, which connects to a 15-cm-diameter vertical pipe. The inlet of the bend is 50 cm above the exit. Neglecting any frictional and gravitational effects, determine the net resultant force exerted on the reducer by the water. Take the momentum-flux correction factor to be 1.04.



Properties We take the density of water to be *1000 kg/m³*.

Analysis We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 353.4$ kg/s. Noting that $\dot{m} = \rho AV$, the mass flow rate of water and its outlet velocity are

$$\dot{m} = \rho V_1 A_1 = \rho V_1 (\pi D_1^2 / 4) = (1000\ \text{kg/m}^3)(5\ \text{m/s})[\pi (0.3\ \text{m})^2 / 4] = 353.4\ \text{kg/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m}}{\rho \pi D_2^2 / 4} = \frac{353.4\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi (0.15\ \text{m})^2 / 4]} = 20\ \text{m/s}$$

The Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_2=P_1+\rho g\left(\frac{V_1^2-V_2^2}{2g}+z_1-z_2\right)$$

Substituting, the gage pressure at the outlet becomes

$$P_2=(300\,\text{kPa})+(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\left(\frac{(5\,\text{m/s})^2-(20\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}+0.5\right)\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right)=117.4\,\text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=0-\beta\dot{m}V_1$$
$$F_{Rz}-P_{2,\text{gage}}A_2=\beta\dot{m}(-V_2)-0$$

Note that we should not forget the negative sign for forces and velocities in the negative x or z direction. Solving for FRx and FRz, and substituting the given values,

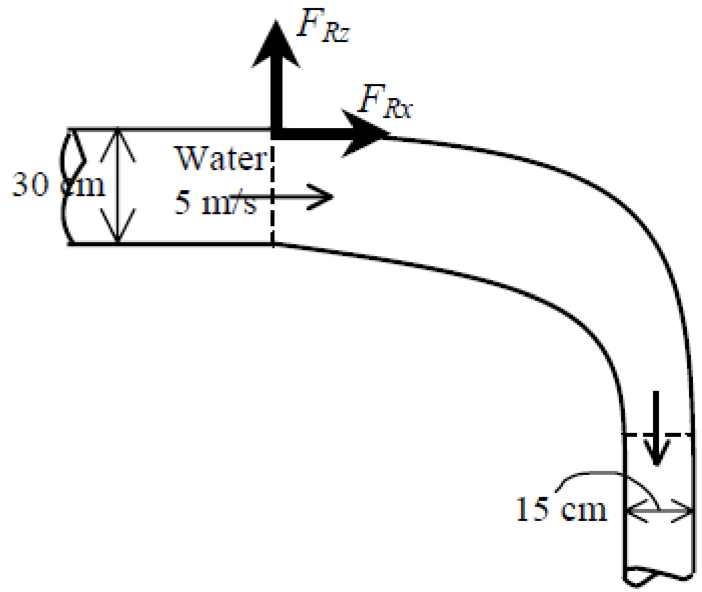
$$F_{Rx} = -\beta \dot{m} V_1 - P_{1,\,gage} A_1 = -1.04(353.4 \text{ kg/s})(5 \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) - (300 \text{ kN/m}^2)\frac{\pi(0.3 \text{ m})^2}{4} = -23.0 \text{ kN}$$

$$F_{Rz} = -\beta \dot{m} V_2 + P_{2,\,gage} A_1 = -1.04(353.4 \text{ kg/s})(20 \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) + (117.4 \text{ kN/m}^2)\frac{\pi(0.15 \text{ m})^2}{4} = -5.28 \text{ kN}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-23.0)^2 + (-5.28)^2} = \mathbf{23.6\ kN}$$

$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{-5.28}{-23.0} = \mathbf{12.9°}$$

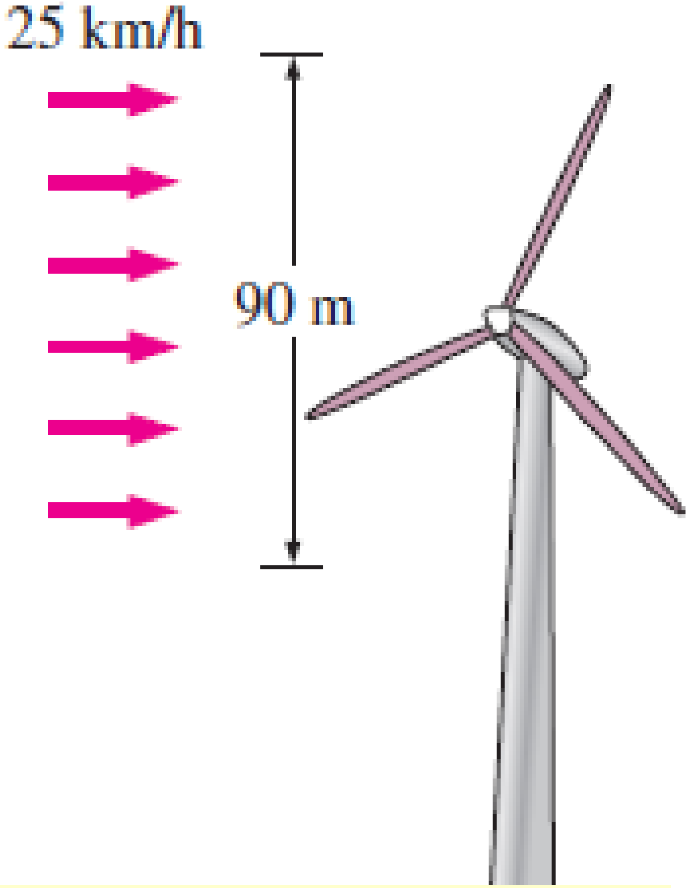


Discussion The magnitude of the anchoring force is 23.6 kN, and its line of action makes 12.9° from +x direction. Negative values for F_{Rx} and F_{Ry} indicate that the assumed directions are wrong, and should be reversed.

EXAMPLE 6-30

Commercially available large wind turbines have blade span diameters as large as 100 m and generate over 3 MW of electric power at peak design conditions.

Consider a wind turbine with a 90-m blade span subjected to 25-km/h steady winds. If the combined turbine– generator efficiency of the wind turbine is 32 percent, determine (a) the power generated by the turbine and (b) the horizontal force exerted by the wind on the supporting mast of the turbine. Take the density of air to be *1.25 kg/m³*, and disregard frictional effects.



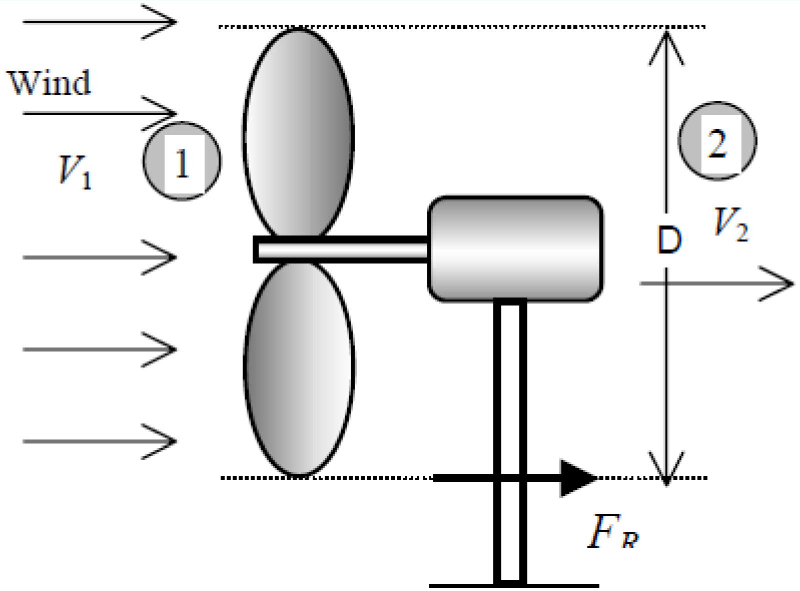
Properties The density of air is given to be $1.25\ kg/m^3$.

Analysis (a) The power potential of the wind is its kinetic energy, which is $\dot{m}V^2/2$ per unit mass, and for a given mass flow rate:

$$V_1 = (25\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 6.94\ \text{m/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (1.25\ \text{kg/m}^3)(6.94\ \text{m/s}) \frac{\pi (90\ \text{m})^2}{4} = 55{,}200\ \text{kg/s}$$

$$\dot{W}_{\max} = \dot{m} ke_1 = \dot{m} \frac{V_1^2}{2} = (55{,}200\ \text{kg/s}) \frac{(6.94\ \text{m/s})^2}{2} \left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2} \right) \left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}} \right) = 1330\ \text{kW}$$



Then the actual power produced becomes

$$\dot{W}_{\text{act}} = \eta_{\text{wind turbine}} \dot{W}_{\max} = (0.32)(1330\ \text{kW}) = \mathbf{426\ kW}$$

(b) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Therefore,

$$\dot{m}ke_2 = \dot{m}ke_1(1-\eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1-\eta_{\text{wind turbine}})$$

or

$$V_2 = V_1\sqrt{1-\eta_{\text{wind turbine}}} = (6.94\text{ m/s})\sqrt{1-0.32} = 5.72\text{ m/s}$$

We choose the control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet, and the entire control surface is at the atmospheric pressure. The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum_{out}\beta\dot{m}\vec{V} - \sum_{in}\beta\dot{m}\vec{V}$. Writing it along the x-direction (without forgetting the negative sign for forces and velocities in the negative x-direction) and assuming the flow velocity through the turbine to be equal to the wind velocity give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1) = (55{,}200\text{ kg/s})(5.72-6.94\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = -\mathbf{67.3\ kN}$$

The negative sign indicates that the reaction force acts in the negative x direction, as expected.

Discussion This force acts on top of the tower where the wind turbine is installed, and the bending moment it generates at the bottom of the tower is obtained by multiplying this force by the tower height.

EXAMPLE 6–31

Firefighters are holding a nozzle at the end of a hose while trying to extinguish a fire. If the nozzle exit diameter is *6 cm* and the water flow rate is *5 m^3/min*, determine (*a*) the average water exit velocity and (*b*) the horizontal resistance force required of the firefighters to hold the nozzle.

Properties We take the density of water to be *1000 kg/m^3*.

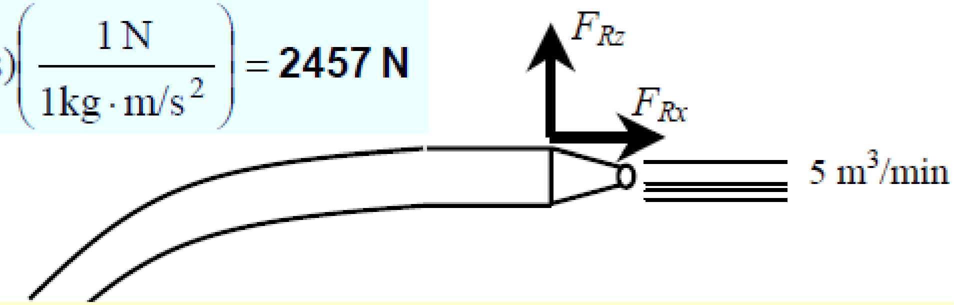
Analysis (a) We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and outlets horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction), and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction). The average outlet velocity and the mass flow rate of water are determined from

$$V = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi D^2/4} = \frac{5\ \text{m}^3/\text{min}}{\pi(0.06\ \text{m})^2/4} = 1768\ \text{m/min} = \mathbf{29.5\ m/s}$$

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(5\ \text{m}^3/\text{min}) = 5000\ \text{kg/min} = 83.3\ \text{kg/s}$$

(b) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$. We let horizontal force applied by the firemen to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction gives

$$F_{Rx} = \dot{m}V_e - 0 = \dot{m}V = (83.3\ \text{kg/s})(29.5\ \text{m/s})\left(\frac{1\ \text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{2457\ N}$$

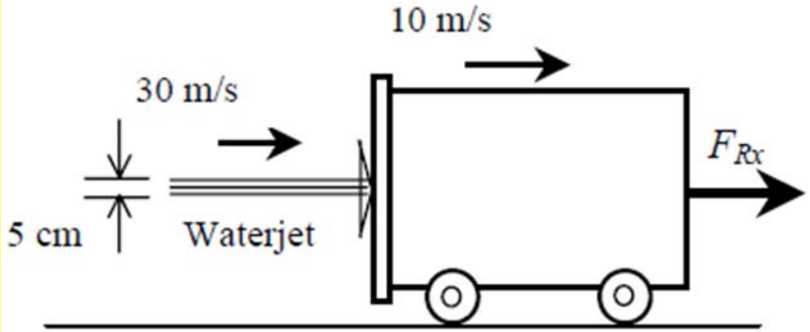


Therefore, the firemen must be able to resist a force of 2457 N to hold the nozzle in place.

Discussion The force of 2457 N is equivalent to the weight of about 250 kg. That is, holding the nozzle requires the strength of holding a weight of 250 kg, which cannot be done by a single person. This demonstrates why several firemen are used to hold a hose with a high flow rate.

EXAMPLE 6-32

A 5-cm-diameter horizontal jet of water with a velocity of 30 m/s strikes a flat plate that is moving in the same direction as the jet at a velocity of 10 m/s. The water splatters in all directions in the plane of the plate. How much force does the water stream exert on the plate?



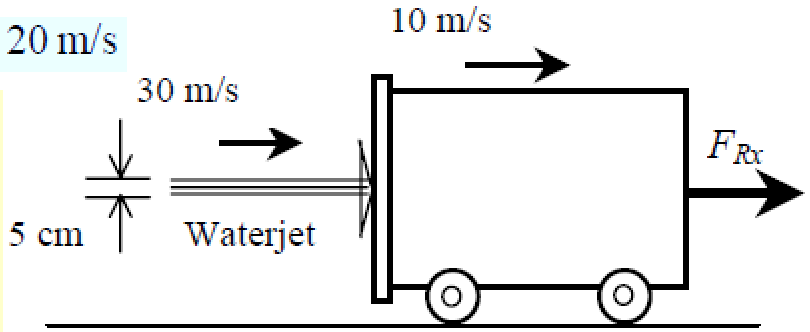
Properties We take the density of water to be *1000 kg/m³*.

Analysis We take the plate as the control volume, and the flow direction as the positive direction of *x* axis. The mass flow rate of water in the jet is 6-20

$$\dot{m} = \rho V_{jet} A = \rho V_{jet} \frac{\pi D^2}{4} = (1000\ \text{kg/m}^3)(30\ \text{m/s}) \frac{\pi (0.05\ \text{m})^2}{4} = 58.9\ \text{kg/s}$$

The relative velocity between the plate and the jet is

$$V_r = V_{jet} - V_{plate} = 30 - 10 = 20 \text{ m/s}$$



Therefore, we can assume the plate to be stationary and the jet to move with a velocity of 20 m/s. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$. We let the horizontal reaction force applied to the plate in the negative x direction to counteract the impulse of the water jet be F_{Rx}. Then the momentum equation along the *x* direction gives

$$-F_{Rx} = 0 - \dot{m}V_i \quad \rightarrow \quad F_{Rx} = \dot{m}V_r = (58.9 \text{ kg/s})(20 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{kg} \cdot \text{m/s}^2}\right) = \mathbf{1178\ N}$$

Therefore, the water jet applies a force of 1178 N on the plate in the direction of motion, and an equal and opposite force must be applied on the plate if its velocity is to remain constant.

Discussion Note that we used the relative velocity in the determination of the mass flow rate of water in the momentum analysis since water will enter the control volume at this rate. (In the limiting case of the plate and the water jet moving at the same velocity, the mass flow rate of water relative to the plate will be zero since no water will be able to strike the plate).