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Concluding Gibrat's law with Turkish firm data

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HIGHLIGHTS

- An unprecedented balanced panel of 953,177 firms in 12 years is utilized.
- Varying evidence on the Gibrat's Law is due to incomplete data and/or modeling.
- The original model has been improved by promoting heterogeneity among firms.
- Tail distribution of firms and modified Law have been incorporated the model.
- The Gibrat's Law is rejected conclusively.

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ABSTRACT

In this study, the validity of the Gibrat's Law, i.e. the Law of the Proportionate Effect, is tested using a balanced panel of nearly one million firms in Turkey between 2005–2016. A random parameters model is considered to make use of the heterogeneity of individual firms. Unlike the somehow weak results of the rejecting studies from the literature, Gibrat's Law is strongly rejected with the random slope and random intercept model utilizing a complete dataset. Employing the distributional properties of firms and modified Gibrat's Law, which is being used to justify the recent revival of the Law in the literature, did not alter the result.

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1. Introduction

Gibrat's Law is a simple but strong assumption that firm growth is a purely random process [1]. Also referred as the Law of the Proportionate Effect, or Proportionate Growth Theory, i.e. the change of a firm size is proportional to its size, it was built upon constant elasticity cost or demand functions. The original idea of the proportionate growth in [1] was not solely focusing on the business firms. The fundamental aim was to define another type of skew distribution, alternative to the Laplace's. The Law of the Proportionate Effect were claimed to differ from the Laplace's Law in its elements' larger magnitude of variation, namely wide range of firm sizes measured by their employees, despite the narrow variance among the wages as the latter were primarily represent.

Gibrat's Law, or the Law of the Proportionate Effect is formally stated as follows:

$$x_t - x_{t-1} = \epsilon_t x_{t-1} \tag{1}$$

where x is a firm's size at time t, and ϵ_t is a random parameter denoting a proportionate rate. For incremental time periods, logarithm of the (1) yields

$$\log x_t \cong \log x_0 + \epsilon_1 + \epsilon_2 + \cdots + \epsilon_t \tag{2}$$

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while the random term assumed to be independently distributed, randomized effects dominate the value in the limit. So, the distribution of log x_t becomes well approximated by normal distribution [2].

This historical questioning about the dynamics and representations of skew distributions have been largely debated in different lines of research, mostly focused on verifying or rejecting the Gibrat's Law. Being verified suggests that firm size distribution is non-stationary resulting in the lognormal size distribution [3–8]. Until the last decades, Gibrat's Law had been commonly regarded as non-existent when aggregated or small firm data were considered due to enormous efforts to reject its validity [9–14]. Lately, increasing popularity of the *complexity theory*, and *econophysics* have revolutionized this way of thinking [15–19]. Although it has earlier roots in the literature [5], these new interdisciplinary perspectives on firm dynamics take into account a modified Gibrat's Law [20,21] and label the upper tail of the firm size distribution as a power law [22].

Reviving the debates on the overall distribution and simple rules of the firm dynamics have gained significant attention e.g. [15,23–28] and many others. The modified Gibrat's Law incorporates a lower bound for the firm size in order to let the mechanism mathematically converge to a power law [21]. This lower bound can be formally stated as:

$$s_i(t+1) = \max\left[s_0, \gamma\,(t)\, s_i(t)\right] \tag{3}$$

where s_0 is the lower bound for the firm size usually equal to 1 and γ is a random growth rate [15].

As the Occam's razor principle leads, scientists traditionally treat the simplest tool as the superior if there is no evidence against it. However, the power of rejection or validation seems to be caught in the middle, lacking a common acceptance. This ambiguity could be overcome with a complete dataset and conclusively distant or close findings to the Law, using the claimants' newly adopted methods.

The original work by Robert Gibrat [1] proposes the Law to be valid on aggregated samples. He tested the *formulae* with employment data of 509,352 French firms "in 87 departments of pre-1914 France," along with food expenditure and income taxes. The two major drawbacks could be considered as taking a snapshot of the economy to label a regularity, and binning the data to overcome computational difficulties of the time. Benefiting from improved data coverage and computational power to put an end to a specific question of the validity seems inevitable.

When looking for a conclusion about the Gibrat's Law for the firm growth, the approaches in literature gather around dividing already limited samples of firms into subgroups and measuring the randomness of the growth. It could be a subgroup regarding industry, or age, it holds for some, while may not hold for others. On the other hand, Hall [29] complies with the original proposition and posits that a balanced panel data of firms is a must for this quest which is somehow overlooked in many papers.

The aim of this paper is to tackle one of the mostly debated, previously agreed, yet surprisingly revived discussions on the validity of the Gibrat's Law of the Proportionate Effect. While avoiding the search for the *perfect partitioning* of firms, in this study, an exhaustive data based testing of the Gibrat's Law is aimed with a balanced panel of nearly a million Turkish firms for twelve years by utilizing a newly used addition to the methodology. Benefiting from the earlier work on the same dataset [30], it has been possible to incorporate the distributional properties of the Turkish firms within an innovative framework. Using the mixed effects methodology, the Law of the Proportionate effect is conclusively rejected. The paper presented can be considered as a challenge to the recent literature which looks for over-simplifying assumptions to import innovative modeling from other disciplines.

2. Data

To study Gibrat's Law, there needs to be an unbiased sample of firms. Although there are various size measurements employed in the firm dynamics literature such as sales [31], assets [32], revenue [14], output and value added [33], it can be said that employment is the generic size measure of firms starting as early as Gibrat [1]. Numerous studies concerning multiple size definitions report the results being similar to each other [13–15].

The availability and accuracy of employment data makes it indispensable for a comprehensive study. Turkish Statistical Institute (TURKSTAT) holds a large and confidential dataset regarding Turkish firms of which contribute to the GDP for a given year, called Annual Business Registers Framework, between 2005–2016. It is supplied within micro data collections, accessible through written permission and only to be analyzed in TURKSTAT research centers. The data consists of 3,395,647 firms on average for 12 years and all businesses have unique identification codes. The IDs make tracking possible over the period, and a balanced panel of 953,177 firms is gathered to test Gibrat's Law.

2.1. Balanced panel of Turkish firms

Table 1 provides descriptive statistics of the surviving firms from 2005 to 2016 in Turkish economy. Mean firm size starts from 5 employees per firm in 2005, increasing 70% to 8.49 in 2016. Increase in employee count continued until 2008, then two years of shrinking occurred in 2009 and 2010. Another positive momentum was observed between 2011–2014, followed by a negative but slower decrease in size until 2016. Nevertheless, the tracked surviving firms in Turkey had enjoyed a substantial growth over the period in average terms.

The data has always been highly skewed, as seen in Table 1. Median size differs from 1 only in 2009, when the Global Financial Crisis hit the Turkish economy. Moreover, 95% of firms had less than or equal to 12 in 2005 and 22 in 2016.

Table 1
Summary of employment data for surviving firms, 2005–2016.
Source: TURKSTAT.

Year	Obs.	Median	Mean[a]	Std.[b]	b/a	95%	Skewness
2005	953,177	1	5.0	90	18	12	247
2006	953,177	1	5.1	96	19	12	351
2007	953,177	1	5.4	88	16	14	254
2008	953,177	1	6.9	186	27	18	217
2009	953,177	2	6.8	141	21	14	448
2010	953,177	1	6.5	94	14	18	176
2011	953,177	1	7.0	101	14	20	158
2012	953,177	1	7.8	102	13	21	146
2013	953,177	1	8.5	120	14	24	197
2014	953,177	1	8.9	132	15	22	192
2015	953,177	1	8.7	116	13	21	115
2016	953,177	1	8.5	115	14	22	121

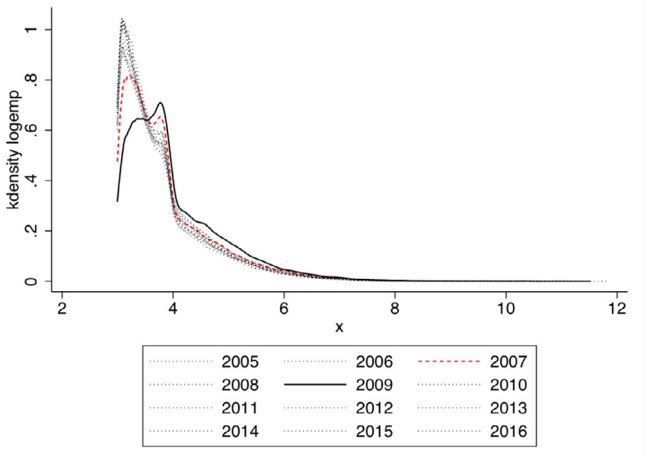


Fig. 1. Size distribution of Turkish firms between 2005–2016. x = log(size) and size≥20 to improve visualization.

2.2. Distribution of the firm sizes

Analysis of the firm data incorporates different approaches. If a specific growth law was considered to be valid, overall distribution of the firms were expected to follow certain types. As [1] suggests, firm sizes should follow a lognormal distribution if the Law of the Proportionate Effect holds in Turkey. On the other hand, firm sizes were to follow a power law in tail if the modified Gibrat's Law were to be true.

The recent popularity of the connection between modeling firm dynamics and expecting a specific firm size distribution can best be tackled on methodological grounds. In theory, proofs of growth models approach to solid distributions but labeling observed regularities of firm sizes are hard to measure. Serious efforts have been made to improve accuracy of measuring power laws [34,35]. Recent literature on the firm size distribution simply overlooked these concerns and propositions, except [30].

The tail distribution of Turkish firms are plotted in Fig. 1. It can be seen that the distribution is rather stable over time, except 2007 and 2009 (laying interesting questions for the researchers of the Global Financial Crisis). Regarding this dataset, [30] estimate the Turkish firm size distribution incorporating the model in [34]. It is reported that the tail of the distribution were not following a power law in any given year. It is also shown that the tail is well approximated by the lognormal distribution only in 5 out of 12 years, i.e. 2006, 2007, 2010, 2015 and 2016.

2.3. Growth of firms

Table 2 summarizes the growth rate statistics of tracked 953,177 firms over the period 2006–2016. Growth for 2005 is not available due to the obvious reasons. A fraction of surviving firms does not have any registered employees for some years which causes the observation count slightly differ in time. Mean growth rate of firms unsurprisingly close to zero in non-disturbance periods, while significantly closer than pre-crisis in recent years. Also variance of the growth rates has a stable nature despite increasing overall firm sizes during the period as presented in Table 2, except from 2008 to 2010. During the Global Financial Crisis, firm growth rates in Turkey changed drastically. There is an extraordinary high mean growth rate in 2009, yet another extraordinary high shrinking rate in the following year of 2010.

Table 2
Summary of growth data for surviving firms, 2005–2016.

Year	Obs.	Mean[a]	Std.[b]	b/a	Variance	Skewness	Kurtosis
2006	953,177	0.01	0.3	34	0.11	1.4	37
2007	953,177	0.06	0.2	4	0.05	2.7	27
2008	953,174	0.08	0.5	6	0.20	2.2	24
2009	953,174	0.35	0.5	1	0.25	−2.0	16
2010	952,889	−0.36	0.5	−1	0.27	1.6	12
2011	952,881	0.05	0.3	6	0.10	1.8	32
2012	952,903	0.15	0.4	3	0.17	0.5	13
2013	950,451	0.03	0.4	12	0.12	0.9	25
2014	948,557	0.01	0.4	41	0.17	−0.5	22
2015	950,076	0.03	0.4	12	0.12	−1.2	25
2016	943,919	0.01	0.3	32	0.10	−0.3	26

Table 3
Random intercept model output.

Variables	MLE	Std. Err.
logsize_{t-1}	0.9313**	0.0001
Year	−0.0038**	0.0001
Cons	7.7632**	0.0823
sd(_cons)	0.0001**	0
sd(residual)	0.4153**	0.0001
Wald p-val	[0.0001]	
LR p-val	[0.99]	

**$p \leq 0.001$.

3. Random intercept estimation

There are several main pillars of model selection with panel data. Considering current exhaustingly large dataset with not much of a time length, the fixed and random effect models can be candidates. The practicality of these models dealing with heterogeneity is a bonus. When specifying the fixed effects model, all time-invariant heterogeneity between entities are accounted for in order to single out the variance within unit. By this notion it is impossible to detect time-invariant answers because the model incorporates all time-independent characteristics of the entities [36]. So, fixed effects modeling is ruled out for testing the Gibrat's Law over a time period. Then mixed effects estimation is taken into consideration. It is largely put forward that Random Effects (and Mixed Effects) modeling is to be preferred due to its high flexibility and saves the researcher from the blockage on extracted information caused from Fixed Effects due to the extended controlling variables [37].

If we assume that unobserved firm specific heterogeneity is uncorrelated with the parameters in the model, which are the last period's size and time vector, we may estimate the random effects with Maximum Likelihood:

$$\log \text{size}_{it} = \beta_0 + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + u_{it} \tag{4}$$

where,

$$u_{it} = \gamma_{1i} + \epsilon_{it} \tag{5}$$

Here, time_t is a vector of fixed effects capturing time-dependent heterogeneity in firm growth, $\gamma_{1i} \sim N(0, \sigma_1)$ is time-invariant random effects parameter which accounts for the variation not captured by β_0, and $\epsilon_{it} \sim N(0, \sigma_\epsilon)$.

In this model, $\log \text{size}_{it-1}$ and time_t are assumed to be exogenous, bringing on $E(\gamma_{1i}|\log \text{size}_{it-1}, \text{time}_t) = 0$ and $E(\epsilon_{it}|\log \text{size}_{it-1}, \text{time}_t, \gamma_{1i}) = 0$. The model also can be stated as:

$$\log \text{size}_{it} = \beta_0 + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + \gamma_{1i} + \epsilon_{it} \tag{6}$$

or

$$\log \text{size}_{it} = \beta_{0i} + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + \epsilon_{it} \tag{7}$$

where $\beta_{0i} = \beta_0 + \gamma_{1i}$ incorporates the random intercept and leaves β_1 as the unbiased estimator of the effect of the firm size over firm growth. Gibrat's Law is to be confirmed if $\beta_1 = 1$, rejected otherwise.

Table 3 shows highly significant coefficients for all the parameters, leaving the Gibrat's law rejected with a β_1 estimation smaller than 1. But the random effect parameter γ_{1i} has a very small value. To improve variance capturing, an additional term of random slope might be added to the equation in the following section, allowing to benefit further from the unprecedented magnitude of the dataset.

Table 4
Random intercept and random slope model output.

Variables	MLE	Std. Err.
log $size_{t-1}$	0.6093**	0.0004
Year	0.0001*	0
Cons	−0.0697	0.0781
sd(log $size_{t-1}$)	0.2711**	0.0004
sd(_cons)	0.0868**	0.0005
sd(residual)	0.3786**	0.0001
Wald p-val	[0.0000]	
LR p-val	[0.0001]	

**$p \leq 0.001$.
*$p \leq 0.01$.

4. Random intercept and random slope estimation

After incorporating the random effect on the unobserved individual heterogeneity, one can also include a random effect regarding the observed heterogeneity as a slope, namely firm size. There would be an interesting feature of the developed model, if the estimated coefficient of log $size_{ti}$ turns out close to 1: the Gibrat's Law could be tested for individual firms similar to [14] with Swedish energy firms. In any case, the new model would become:

$$\log \text{size}_{it} = \beta_0 + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + \upsilon_{it} \tag{8}$$

where,

$$\upsilon_{it} = \gamma_{1i} + \gamma_{2i} \log \text{size}_{it-1} + \epsilon_{it} \tag{9}$$

where $\gamma_{1i} \sim N(0, \sigma_1)$ is again the random part missed in β_0, $\gamma_{2i} \sim N(0, \sigma_2)$ is the random slope on the last period's firm size, and $\epsilon_{it} \sim N(0, \sigma_\epsilon)$ is the error of the model. Rearranging yields:

$$\log \text{size}_{it} = \beta_0 + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + \gamma_{1i} + \gamma_{2i} \log \text{size}_{it-1} + \epsilon_{it} \tag{10}$$

Assuming the terms $\log \text{size}_{it-1}$ and time_t exogenous, $E(\gamma_{1i}|\log \text{size}_{it-1}, \text{time}_t) = 0$, $E(\gamma_{2i}|\log \text{size}_{it-1}, \text{time}_t) = 0$ and $E(\epsilon_{it}|\log \text{size}_{it-1}, \text{time}_t, \gamma_{1i}, \gamma_{2i}) = 0$. Thus a more general form can be specified as:

$$\log \text{size}_{it} = (\beta_0 + \gamma_{1i}) + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{time}_t + \epsilon_{it} \tag{11}$$

This model was estimated using Maximum Likelihood method. Table 4 presents the estimated parameters of the model incorporating both random intercept and random slope.

According to the estimates on Table 4, slope of the last period's employment is far lower from 1. There are only a handful of firms of which Gibrat's Law may hold individually, if any. Adding the firm specific random slope one can assert that Gibrat's Law is strongly rejected in Turkey.

5. Does the resulting distribution matter?

As noted earlier, modeling firm dynamics involves certain distributional propositions e.g. Pareto or lognormal. An accurate assessment on the Gibrat's Law should be built upon this notion. In order to improve the fundamentals of testing the Gibrat's Law, one needs to incorporate the resulting distributional effects on firm sizes as well, where mixed effects modeling can be exceptionally useful. It can be seen from Tables 3 and 4 that disregarding this information might return statistically significant but small time dependent fixed effects.

Benefiting from earlier work on the same dataset [30], one can build up a dummy variable of distribution that represents lognormally distributed firm size periods. Adding to the Eq. (11) yields Model 1 in Table 5:

$$\log \text{size}_{it} = (\beta_0 + \gamma_{1i}) + \beta_1 \log \text{size}_{it-1} + \beta_2 \text{distribution}_t + \beta_3 \text{time}_t + \epsilon_{it} \tag{12}$$

Intuitively, one can drop the time-dependent fixed effect time_t, which has a small on firm size, forming the Model 2. Furthermore, if one updates the data with respect to the rule in Eq. (3) then the Model 3 would be capable of testing the Modified Gibrat's Law with distributional information at hand.[1]

The estimation results of the proposed three new models are presented in Table 5. Model 1 posits that confirmed lognormal tail distribution of particular periods are associated with significantly smaller firm sizes. This feature does not change with the exclusion of time variable in Model 2 and setting a lower bound for the firm size in Model 3. It suggests that contractionary periods lead to lognormal distribution of firm sizes. All estimations highlight the importance of the tail distribution on firm sizes. On the other hand, results of the Model 3 indicates that proposed mathematical modifications to the Gibrat's Law has little effect in real terms and the modified Gibrat's Law is also rejected.

[1] Setting a lower bound for the firm size had impact on 15,647 data points out of 11,438,124, corresponding to only 0.1% of the firm data.

Table 5
Comparison of the proposed models.

	Model 1: Distributional Info. added	Model 2: No time-dependent heterogeneity	Model 3: Modified law with distributional Info.
Variable	MLE	MLE	MLE
log $size_{t-1}$	0.6627** (0.0003)	0.6566** (0.0003)	0.6631** (0.0003)
Distribution	−0.1509** (0.0002)	−0.1502** (0.0002)	−0.1511** (0.0002)
Year	−0.0011** (0.0001)		
Cons	2.4197** (0.0769)	0.2171** (0.0002)	0.2145** (0002)
sd(log $size_{t-1}$)	0.2207** (0.0004)	0.2218** (0.0004)	0.2187** (0.0004)
sd(_cons)	0.1017** (0.0005)	0.1077** (0.0004)	0.1018** (0.0004)
sd(residual)	0.3728** (0.0001)	0.3720** (0.0001)	0.3754** (0.0001)
Wald p-val LR p-val	[0.0000] [0.0001]	[0.0000] [0.0001]	[0.0000] [0.0001]

**$p \leq 0.001$, *$p \leq 0.01$.

6. Conclusions

The main contribution of the paper to the literature is beyond being a mere empirical exercise. Despite the consensus about the rejection in the economics literature, recent adoption of the Gibrat's Law in *econophysics* deserves a proper response on common grounds. To the best of our knowledge, the most comprehensive dataset in the literature to test the validity of the Gibrat's Law is employed in the present study. Without random effect parameter on the size from previous year, econometric analysis provide evidence on Gibrat's Law as almost holds with a coefficient close to one. However, when all surviving firms' data incorporated to the equation with firm specific variance component, coefficient significantly drops. Since it is a general benchmark for firm dynamics, this result puts importance on reconsidering the validity with complete data for developed countries.

It is shown in Section 5 that, as an extension to the previous efforts on validation, recent revival of the attention on the Gibrat's Law through interdisciplinary perspectives turns out to be misleading. The observed distribution is statistically significant to the firm sizes. In fact, properly labeled lognormal distribution of the tail has an important negative impact on the individual firm sizes. Estimation results also undermine the recent attention on power law distributed firms along with the retaining the modified Gibrat's Law.

In this study, it is shown that when a complete data set is considered, Gibrat's Law is rejected by a large gap. Departure from the Law is so strong that it should be expected to be likewise in different countries. Turkish firms do not grow in proportion to their size, i.e. growth is not independent of size. Economic environment in Turkey during the period of 2005–2016 seems to support smaller firms to grow faster, with a considerably high random effect parameter. The outcome suggests that the policymakers may benefit from targeting such size classes to promote economic development in Turkey.

The most important conclusion of the paper relates to the future of the field. Economists with interdisciplinary perspectives and other scientists with interest in economics enthusiastically try to revisit and improve the toolkit in economic research. However, each regularity like the Gibrat's Law has its specific fundamentals to be carefully regarded. Some of the breakthroughs building on applications of the newly available methods from other disciplines might require extra precision in order to be considered theoretically valid. These concerns have made it inevitable to present a complete and convincing recap of the Law of the Proportionate Effect, employing the new perspectives in economics.

While rejecting Gibrat's Law conclusively, further research opportunities provided with a focus on heterogeneity of macroeconomic effects on individual firms offer prospect, as seen above, shall be fruitful in terms of combining the macro and micro theory of the firm. Growth distribution over the years yielded an unexpected claim for turbulent times. It could be further examined that Turkish firms gets affected from economic downturn collectively, while benefits from booms distributed sparsely. Although the literature reports similar results as mentioned in Section 2, the rejection of the Law could be re-checked when or if the data for assets or revenues of all firms in an economy gets to be available in the near future. Finally, sectoral or regional dimensions of the data could be combined in a more complex framework to spot different regularities regarding the growth distribution of firms.

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