

Practice Questions

- ① Find the domain of the following function and show them in the coordinate plane.

a) $f(x,y) = \sqrt{\frac{1-x^2-y^2}{xy}}$

b) $f(x,y) = \sqrt{x^2-y} + \ln(y-1)$

- ② Examine the existence of the limit of function

$$f(x,y) = \frac{y \sin \pi x}{x+y-2} \quad \text{at } (1,1).$$

③ $\lim_{(x,y) \rightarrow (0,1)} \frac{\sin(xy)}{x} = ?$

④ $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{1-\sqrt{1+x^2+y^2}}$

- ⑤ Show that the following functions don't have limit at $(0,0)$.

a) $f(x,y) = \frac{x-y}{x+y}$

b) $f(x,y) = \frac{x^4}{x^4+y^2}$

⑥ $f(x,y) = \begin{cases} x^2 \sin \frac{1}{y} & , y \neq 0 \\ 0 & , y = 0 \end{cases}$, Investigate the continuity of the function f at $(0,0)$.

⑦ $f(x,y) = \begin{cases} \frac{\sin(xy^2)}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$, Show that the function f is continuous everywhere.

⑧ Let $f(x,y) = x^2y$. Find the partial derivative f_x and f_y by using the definition of derivative.

⑨ Find the values of $f_x(0,0)$ and $f_y(0,0)$

a) $f(x,y) = \sqrt[3]{x^4+y^2}$ b) $f(x,y) = \sin \sqrt{x^2+y^4}$

⑩ Let $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$.

a) Examine the continuity of f at $(0,0)$.

b) Examine the existence of $f_x(0,0)$ and $f_y(0,0)$.

⑪ Let $z = x^2 + 3xy + y^2$. If $x = \sin r + \cos s$ and $y = \sin r - \cos s$, $\partial z / \partial r = ?$

⑫ Let $w = f\left(\frac{y-x}{xy}, \frac{z-y}{yz}\right)$. Show that the following equation holds

$$x^2 \frac{\partial w}{\partial x} + y^2 \frac{\partial w}{\partial y} + z^2 \frac{\partial w}{\partial z} = 0.$$

⑬ Find the approximate value of $\sqrt{(2.06)^2 + 5(0.97)^4}$.

(14) Determine directional derivative of $f(x,y) = xy^2$ at the point $P(1,3)$ in the direction of the line that connects $P(1,3)$ and $Q(4,5)$ points

(15) Determine directional derivative of $z = x \ln y$ at the point $(1,2)$ in the direction given by the angle $\theta = \frac{\pi}{6}$.

(16) Find the local extremum points of $f(x,y) = 3xy - x^3 - y^3$