

Chapter 6

Some Continuous Probability Distributions

Probability & Statistics *for Engineers & Scientists*

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Section 6.1

Continuous Uniform Distribution

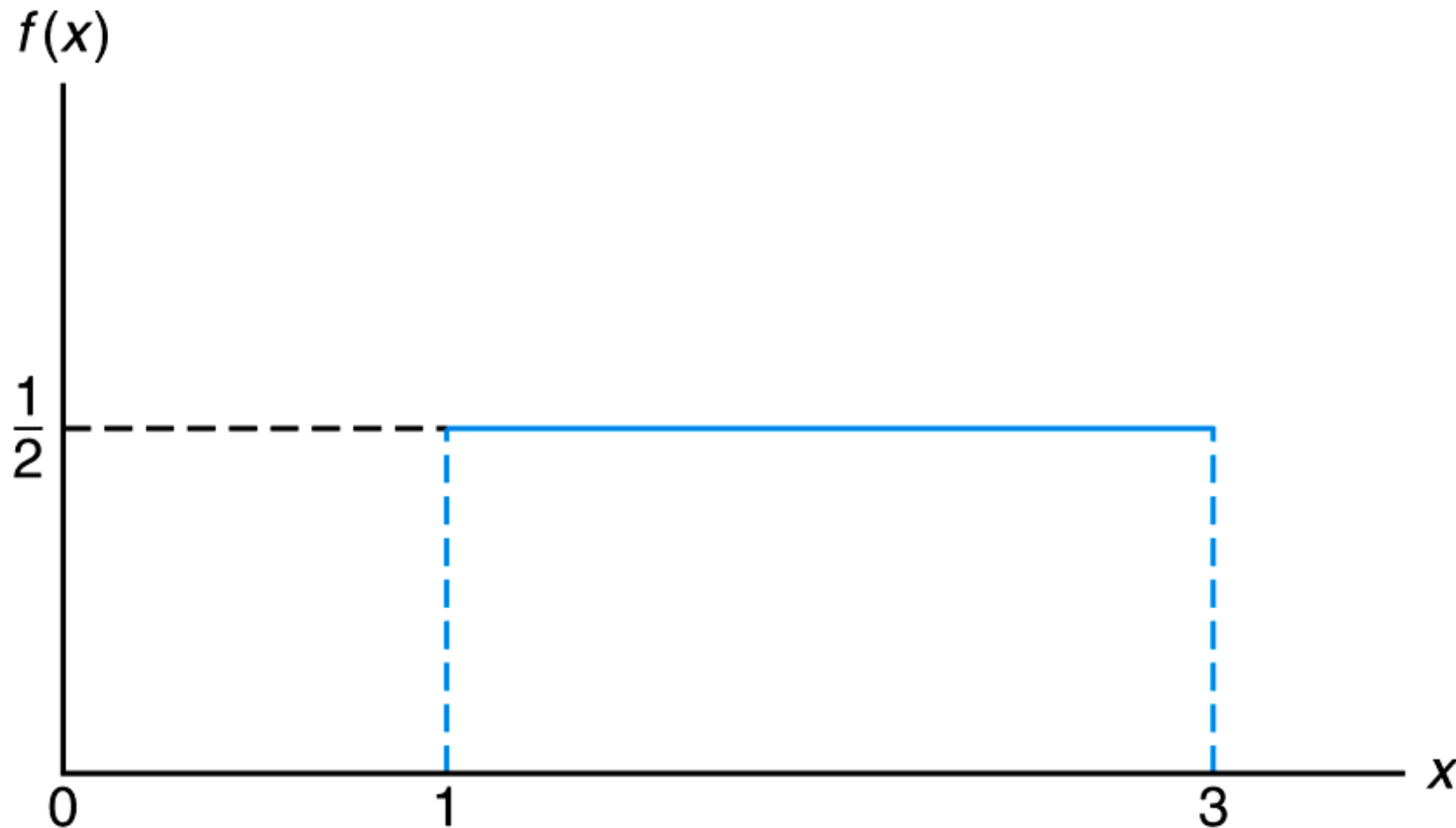
Probability & Statistics *for Engineers & Scientists*

NINTH EDITION

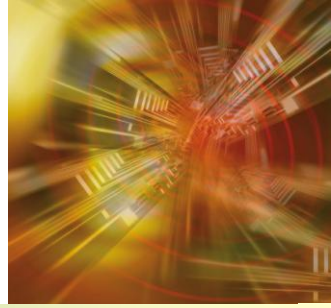


WALPOLE | MYERS | MYERS | YE

Figure 6.1 The density function for a random variable on the interval $[1,3]$



Theorem 6.1



The mean and variance of the uniform distribution are

$$\mu = \frac{A + B}{2} \text{ and } \sigma^2 = \frac{(B - A)^2}{12}.$$

Section 6.2

Normal Distribution

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Figure 6.2 The normal curve

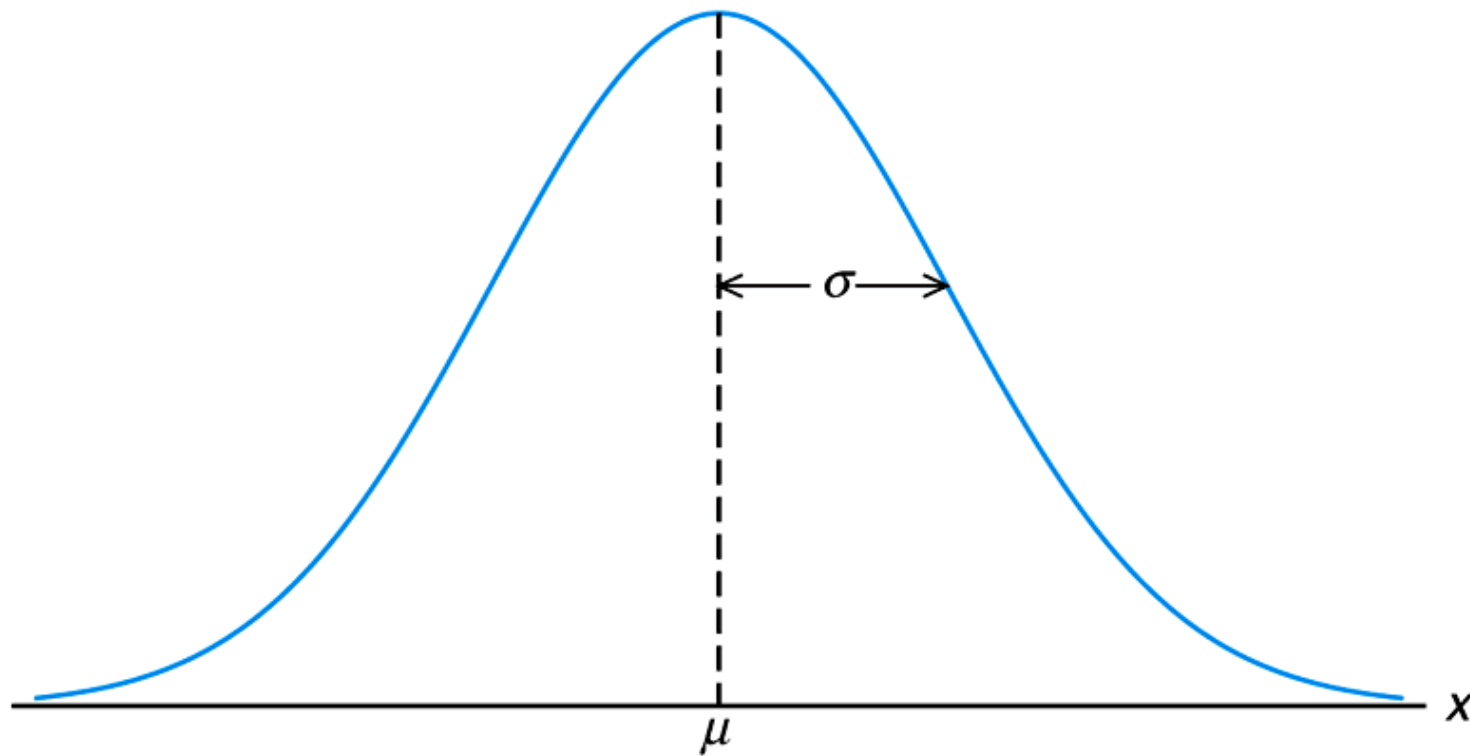


Figure 6.3 Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \mu_2$

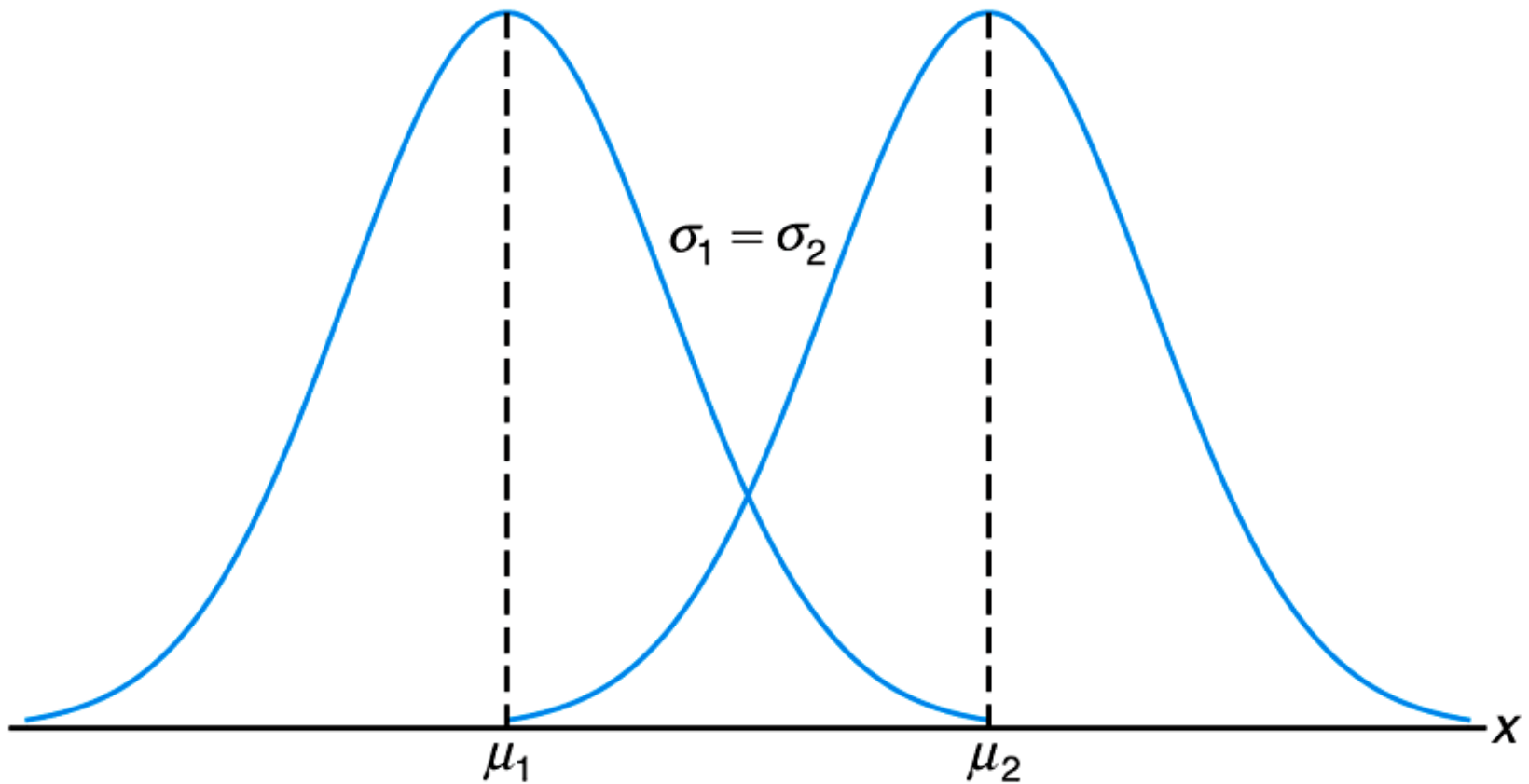


Figure 6.4 Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$

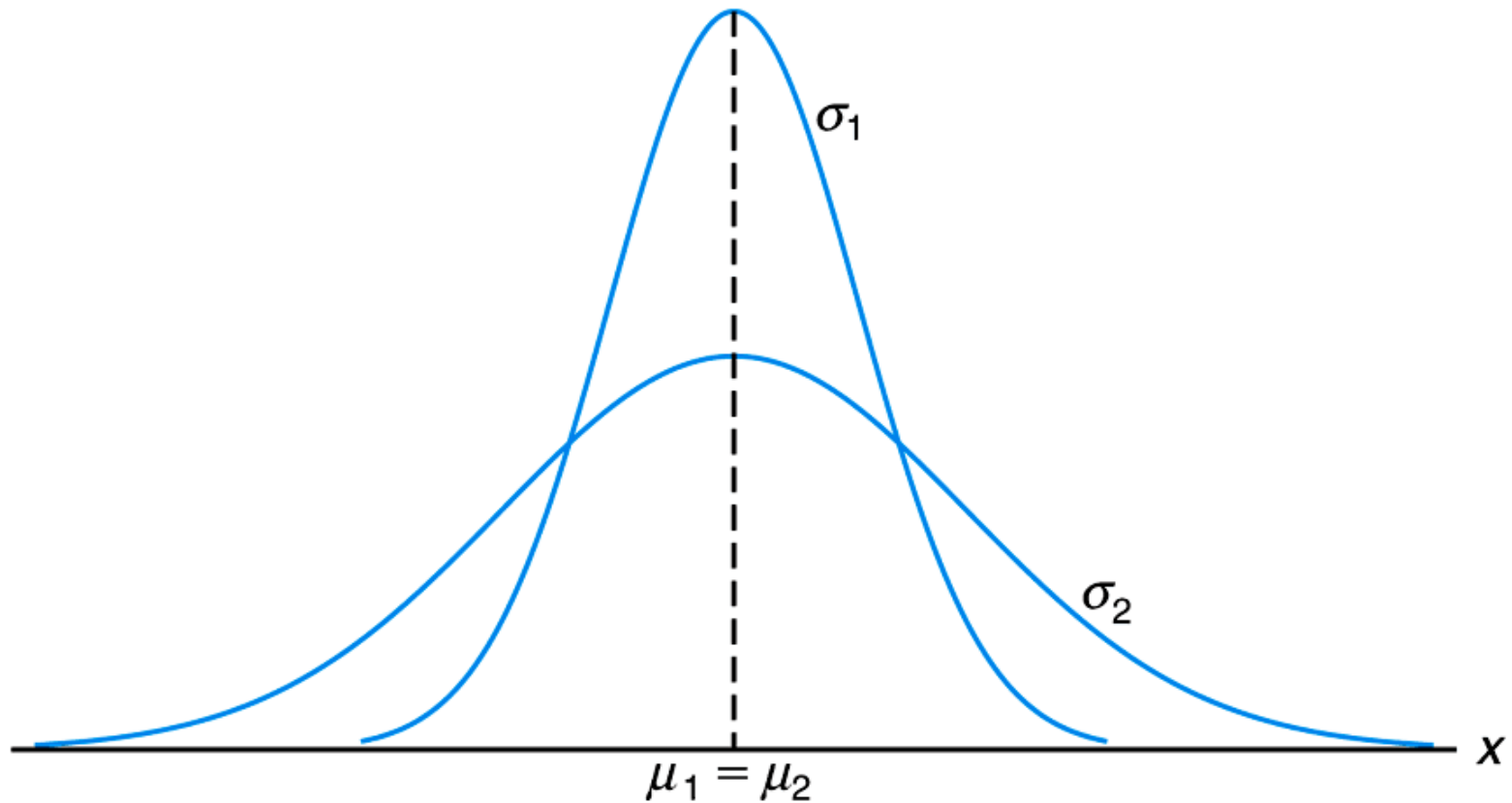
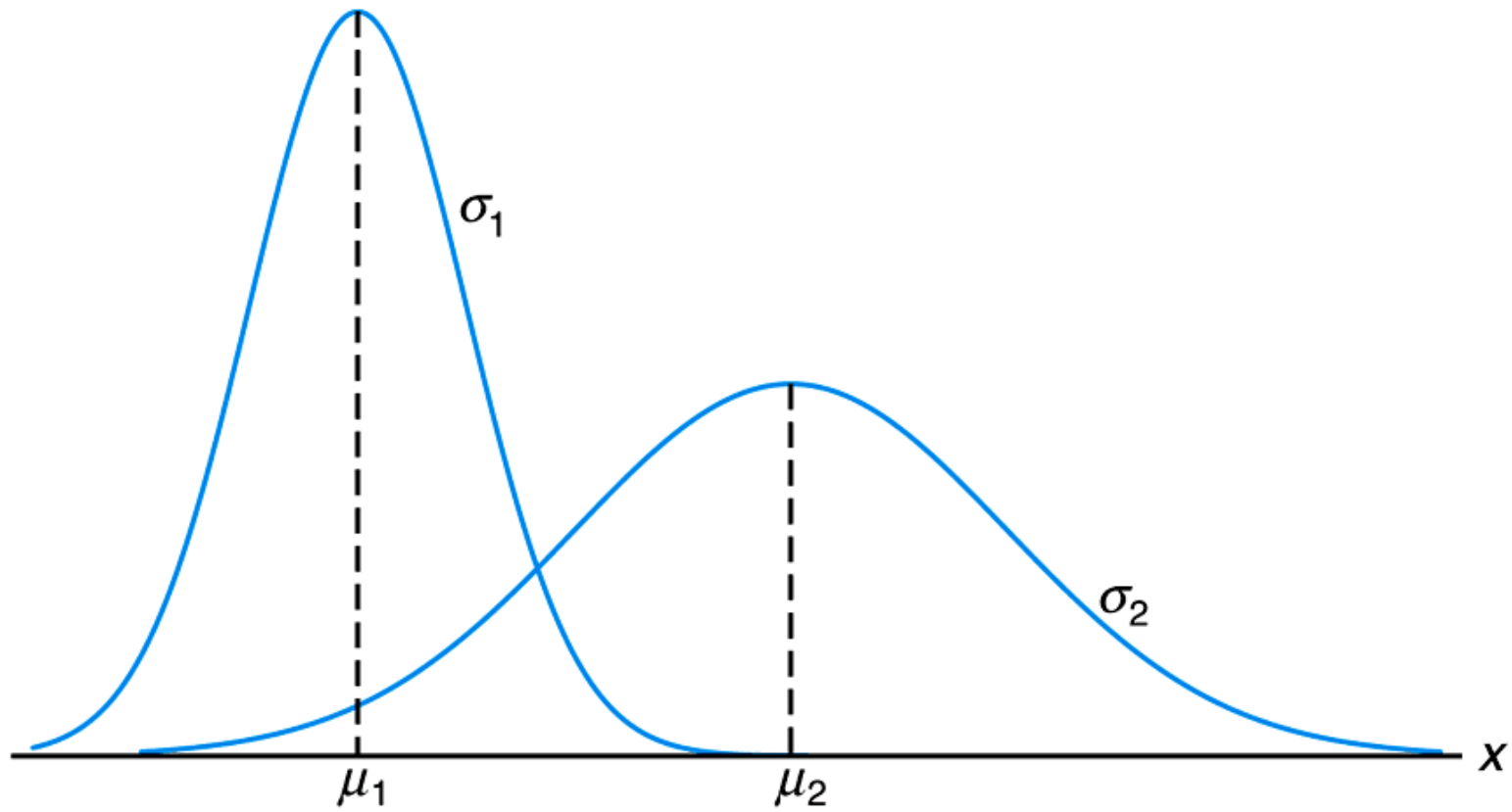


Figure 6.5 Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \mu_2$



Theorem 6.2



The mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2 , respectively. Hence, the standard deviation is σ .

Section 6.3

Areas under the Normal Curve

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Figure 6.6 $P(x_1 < X < x_2) = \text{area of the shaded region}$

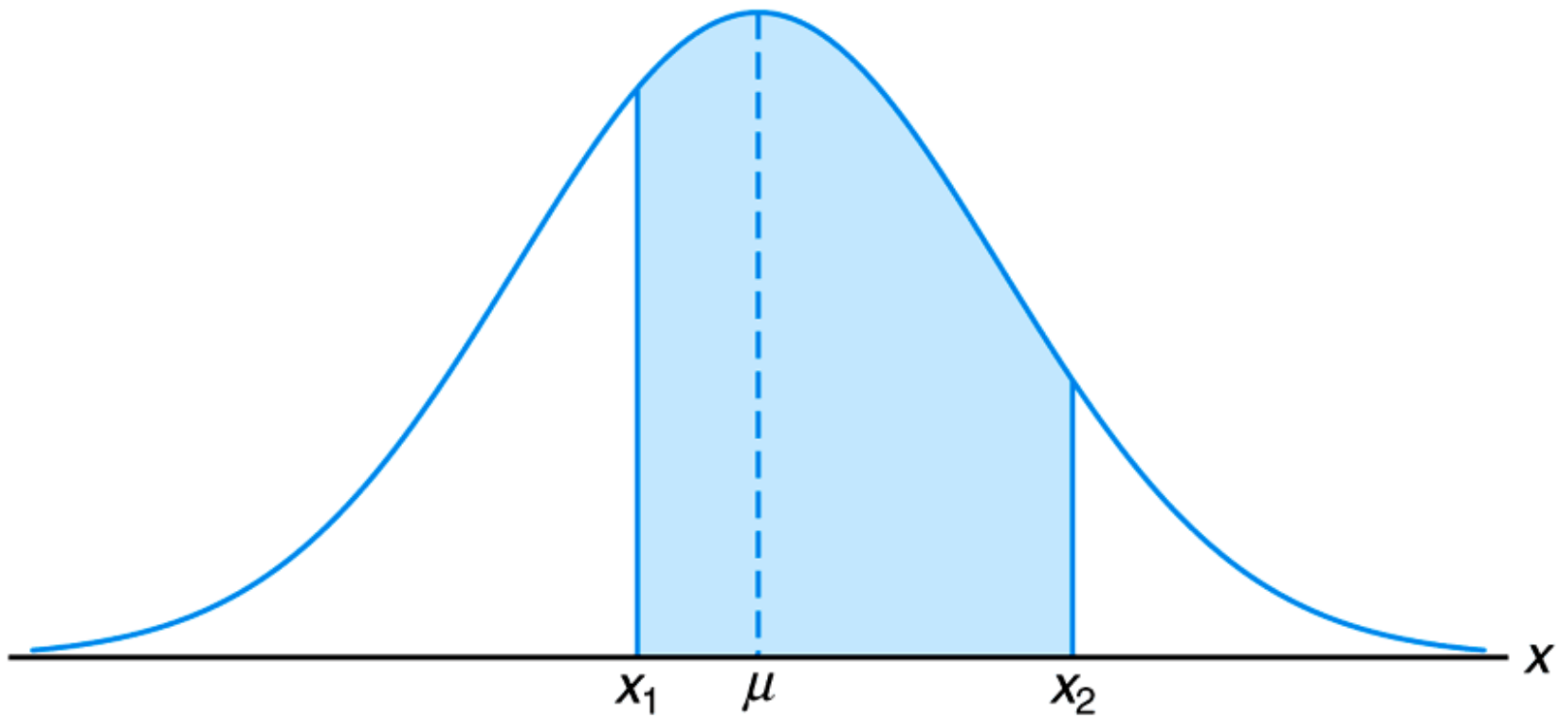
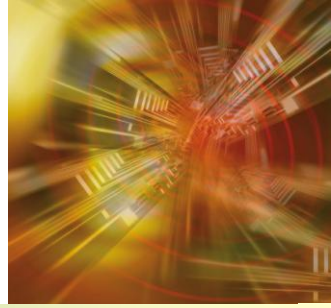
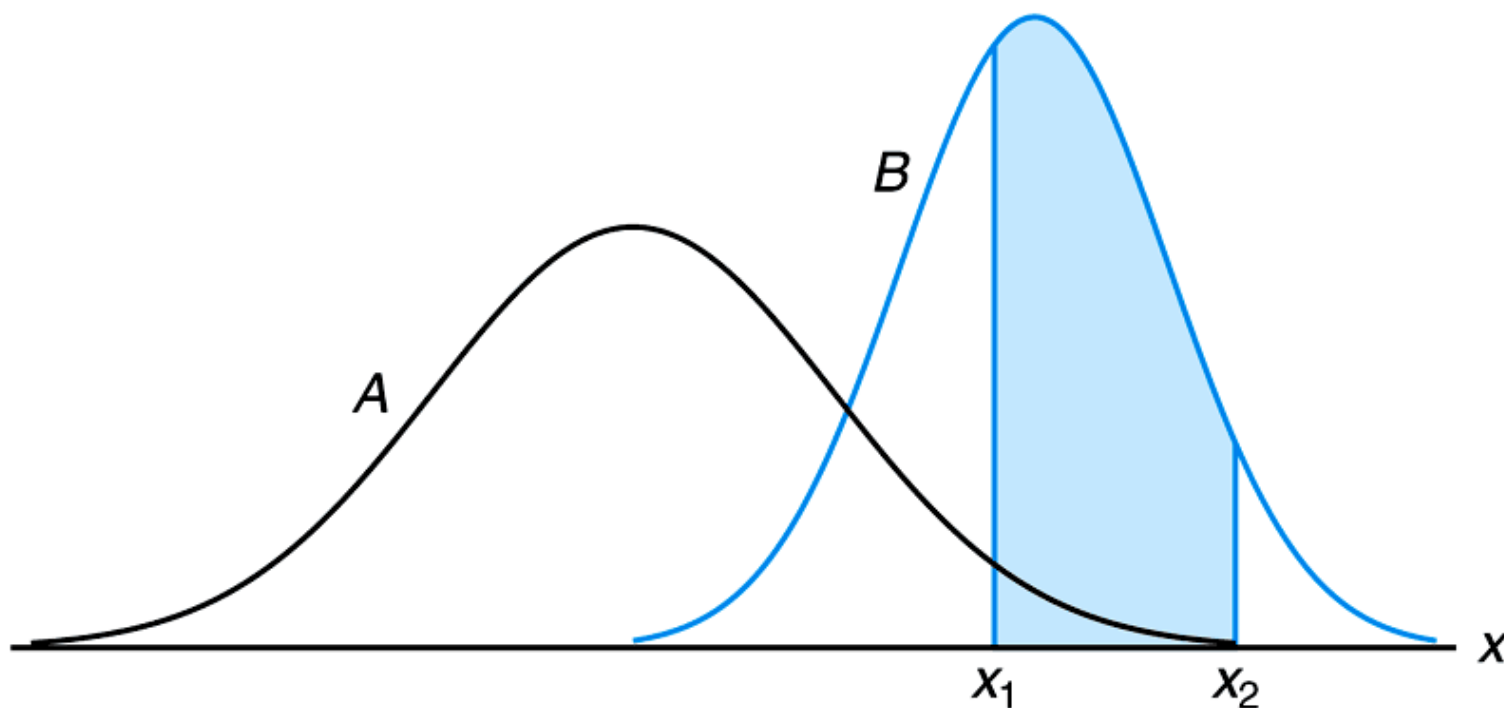
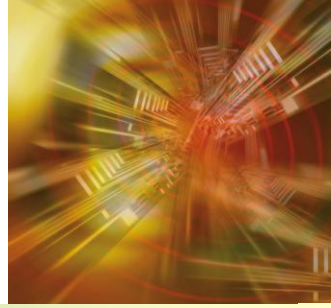


Figure 6.7 $P(x_1 < X < x_2)$ for different normal curves



Definition 6.1



The distribution of a normal random variable with mean 0 and variance 1 is called a **standard normal distribution**.

Figure 6.8 The original and transformed normal distributions

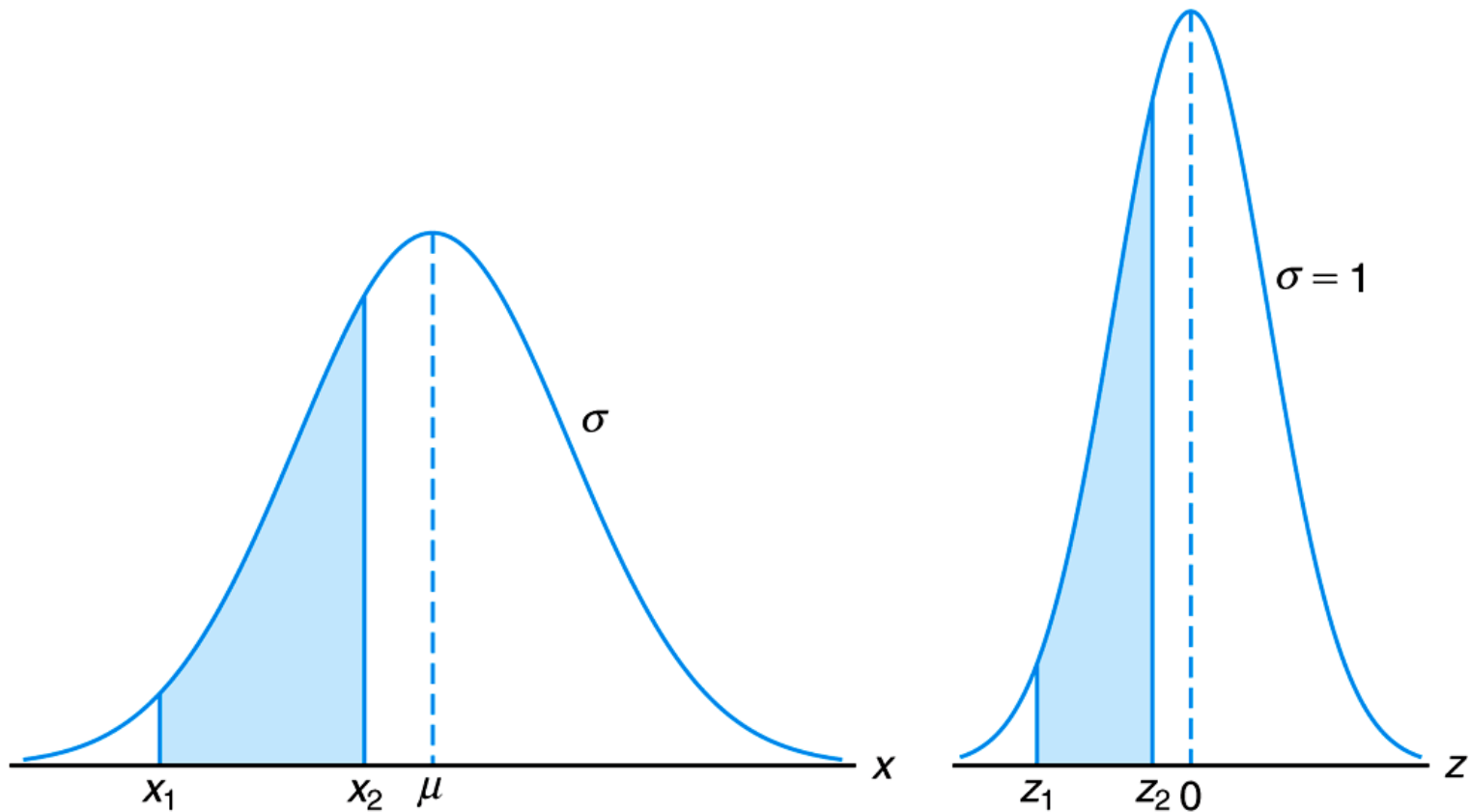


Figure 6.9 Areas for Example 6.2

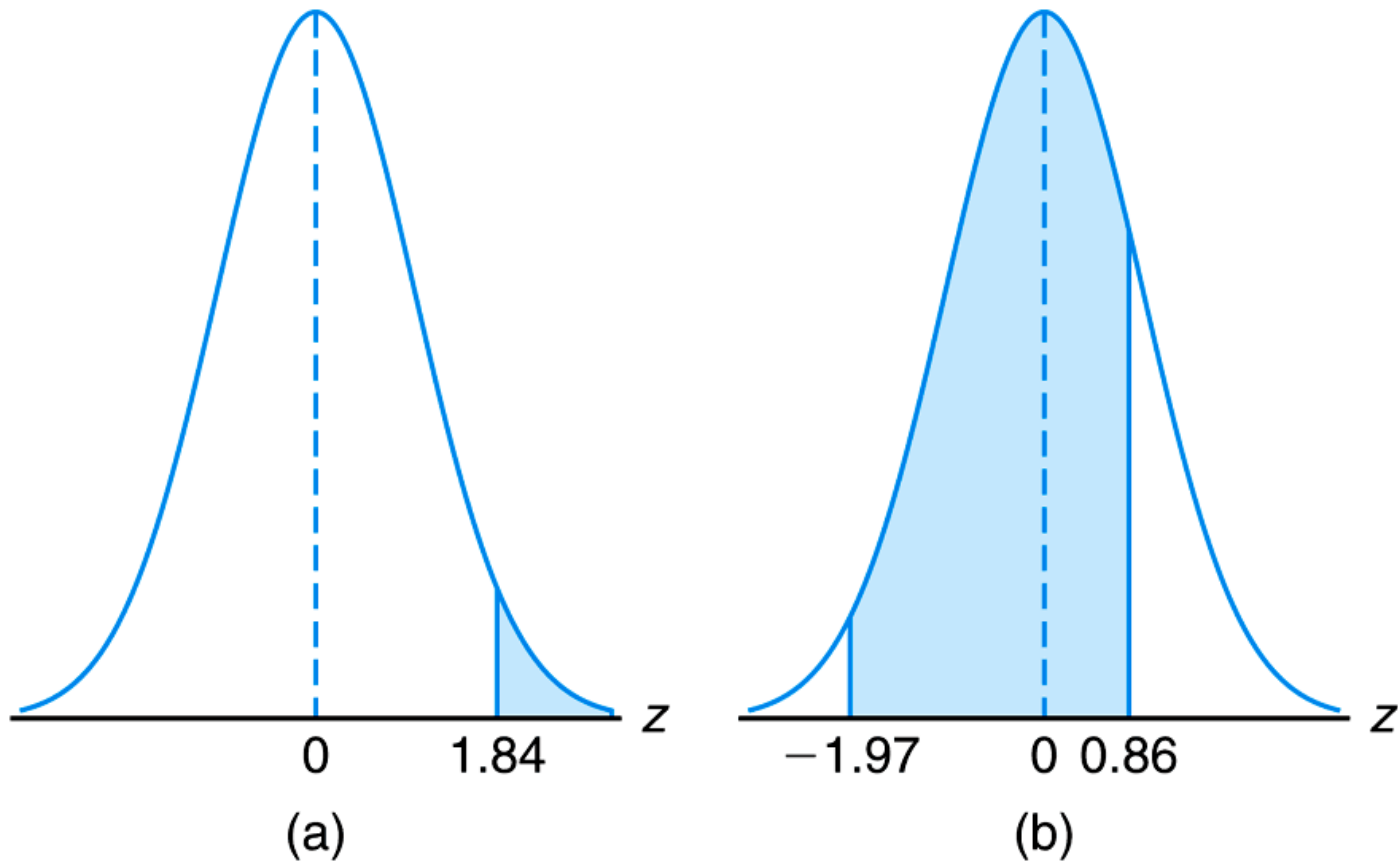


Figure 6.10 Areas for Example 6.3

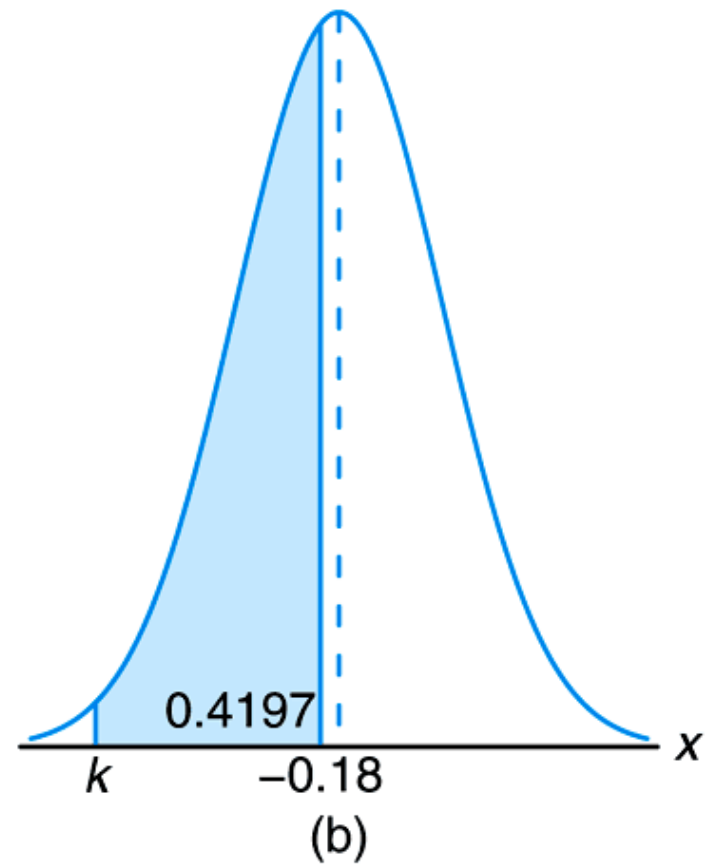
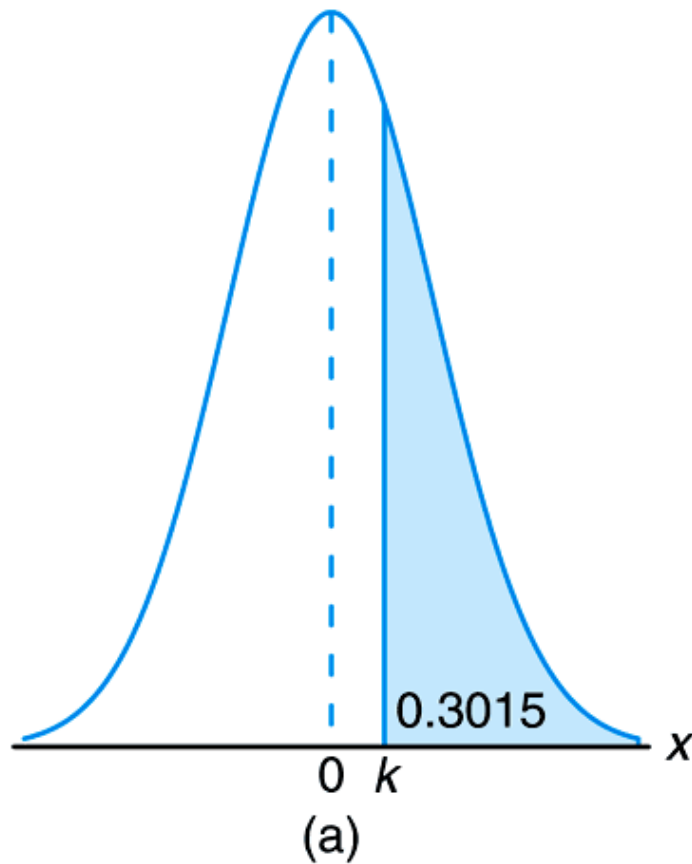


Figure 6.11 Area for Example 6.4

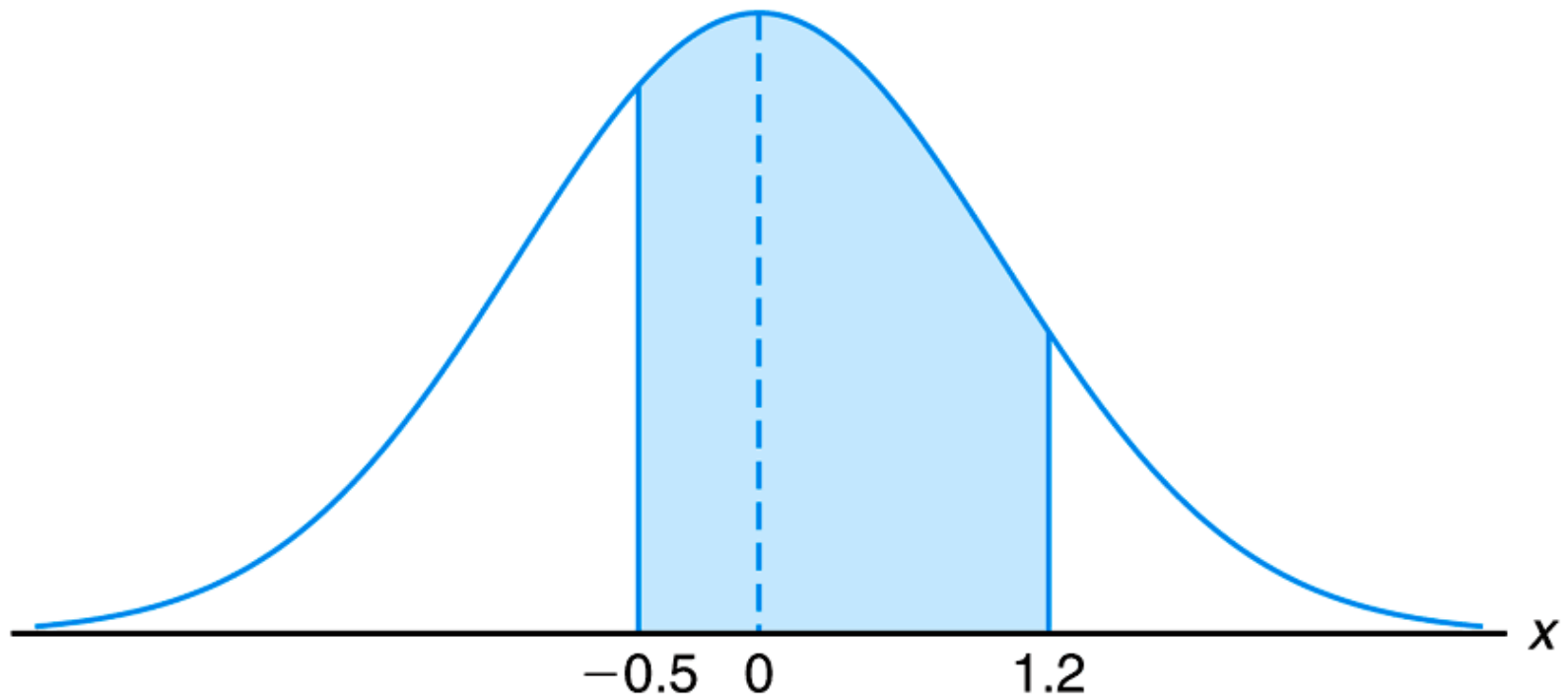


Figure 6.12 Area for Example 6.5

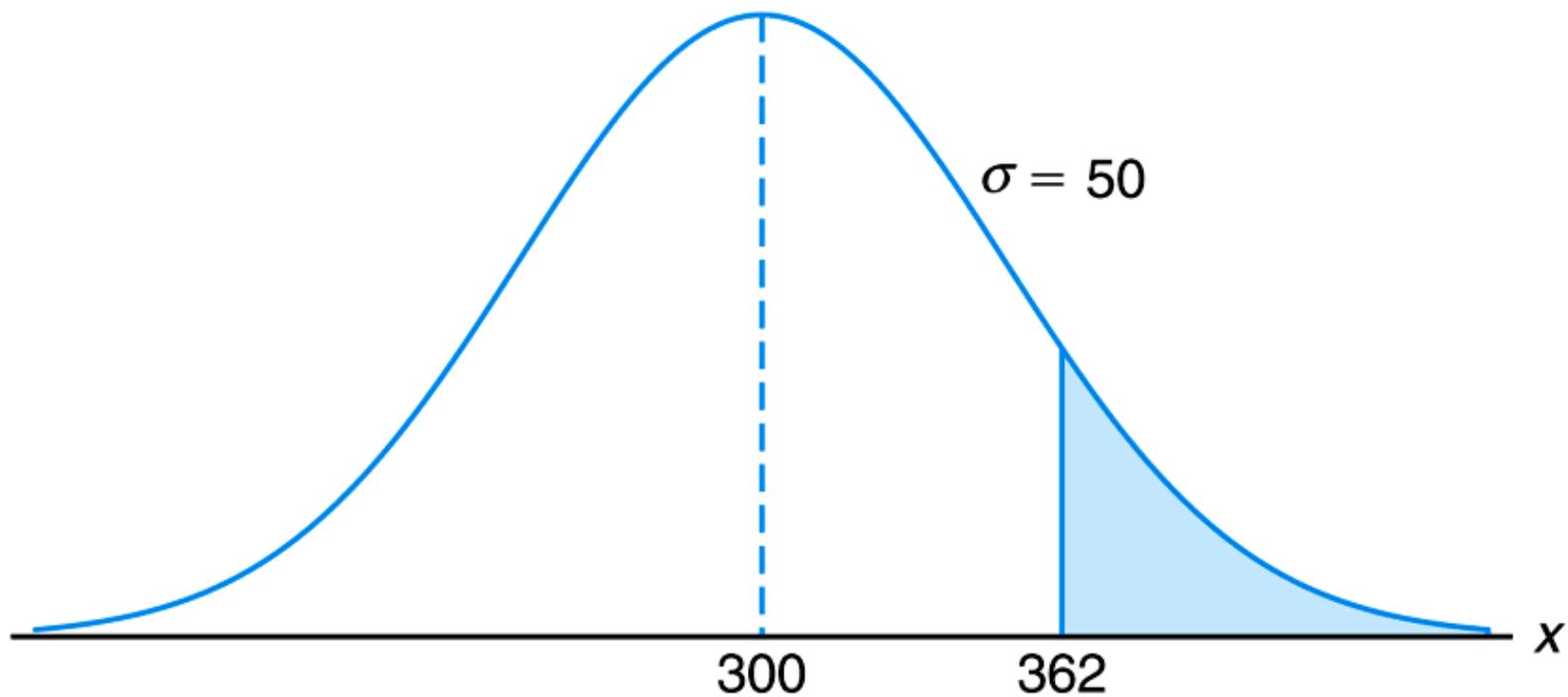
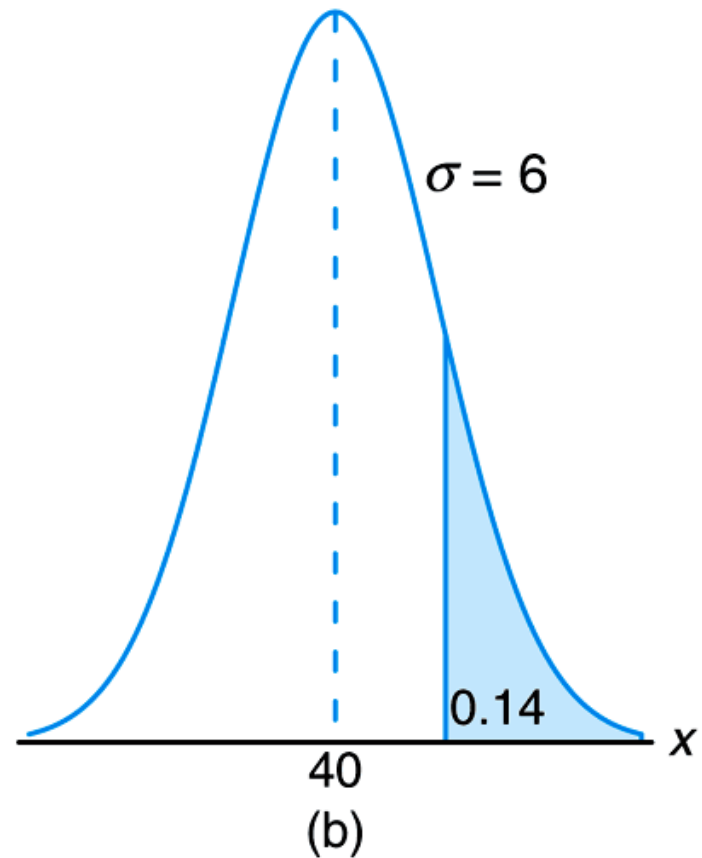
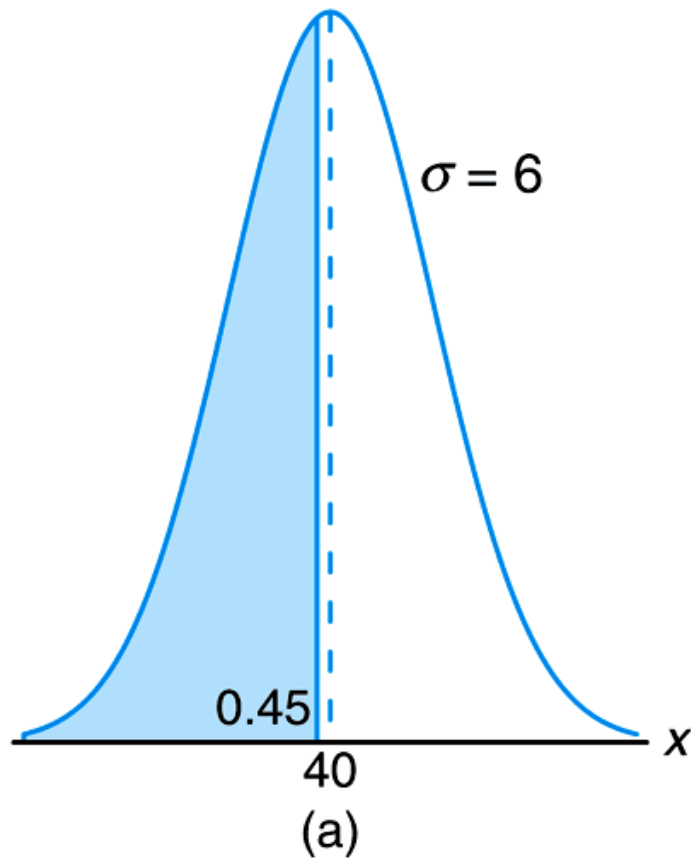


Figure 6.13 Areas for Example 6.6



Section 6.4

Applications of the Normal Distribution

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Figure 6.14 Area for Example 6.7

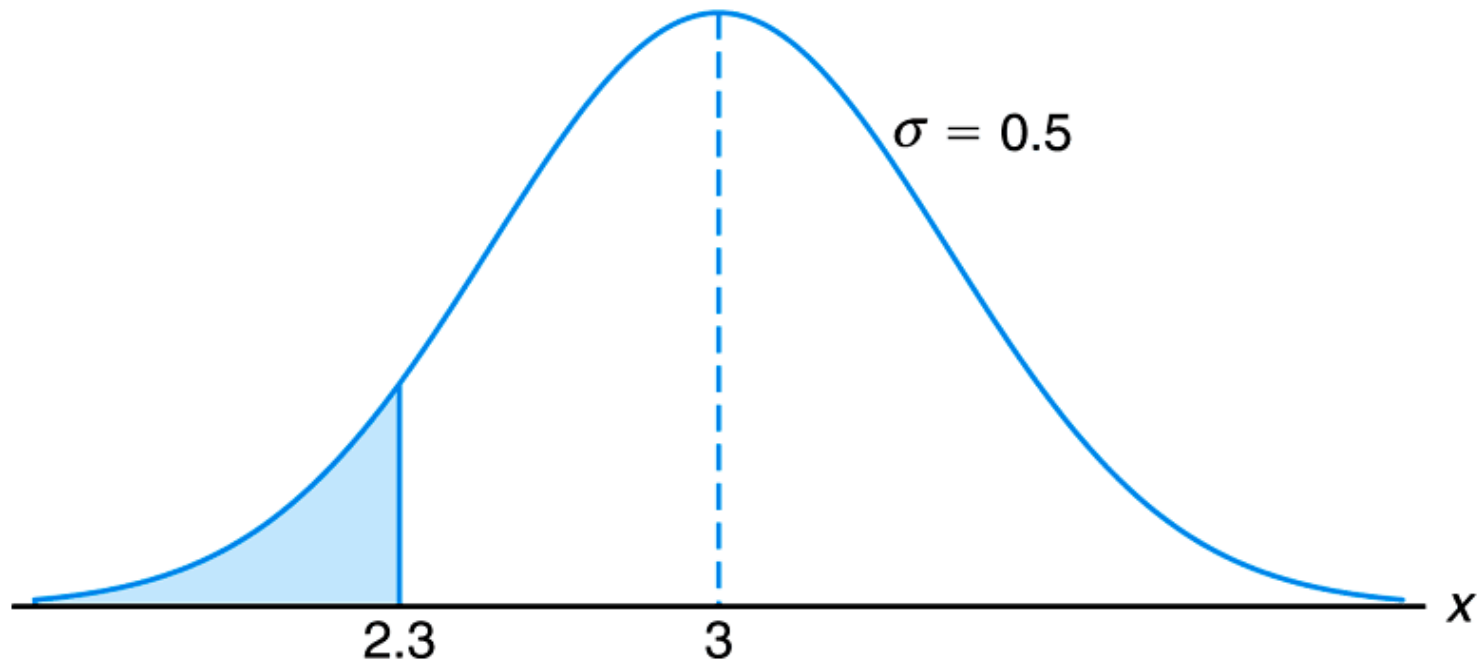


Figure 6.15 Area for Example 6.8

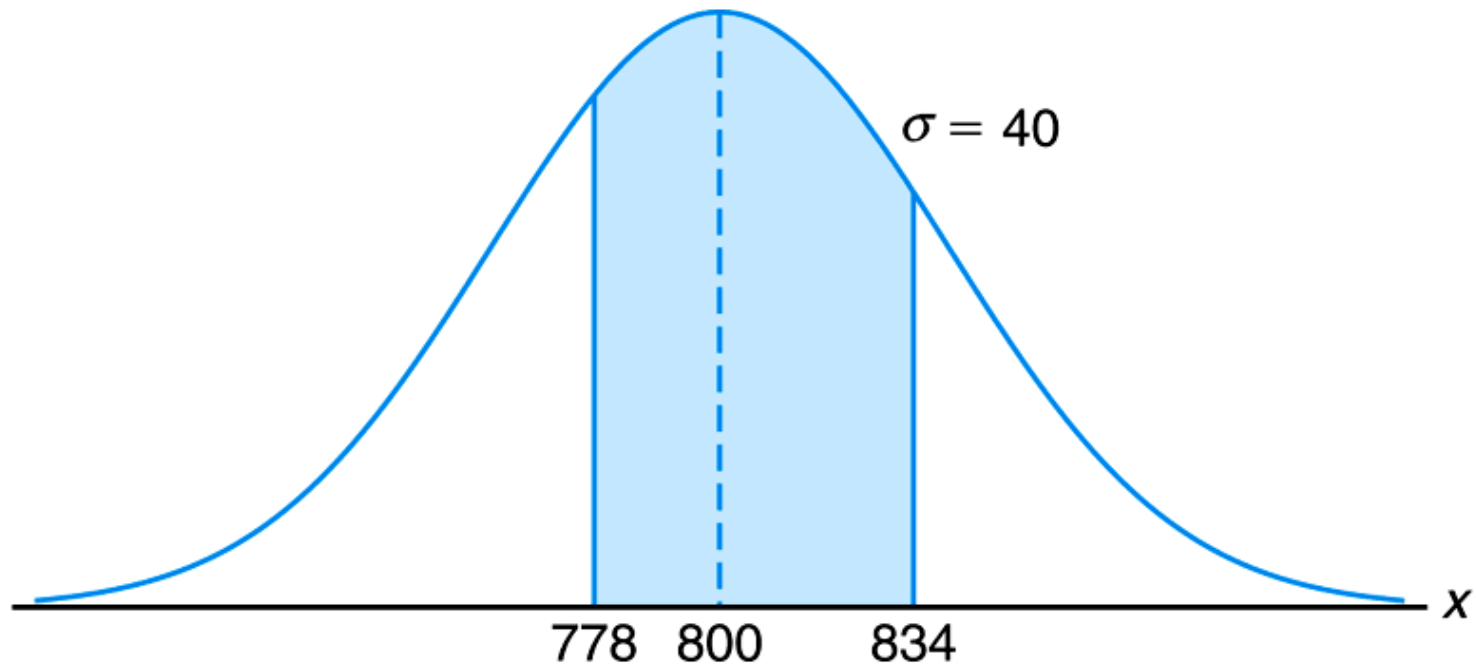


Figure 6.16 Area for Example 6.9

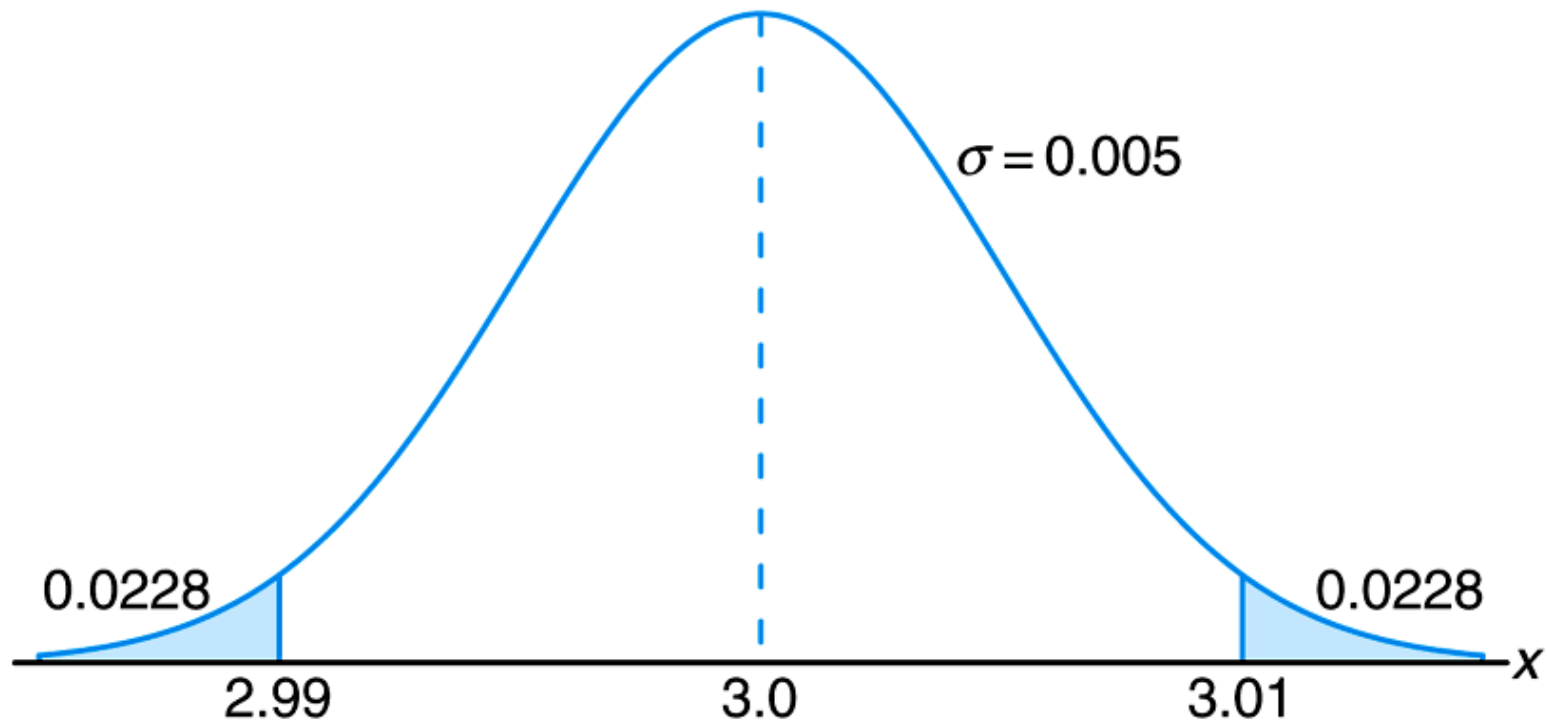


Figure 6.17 Specifications for Example 6.10

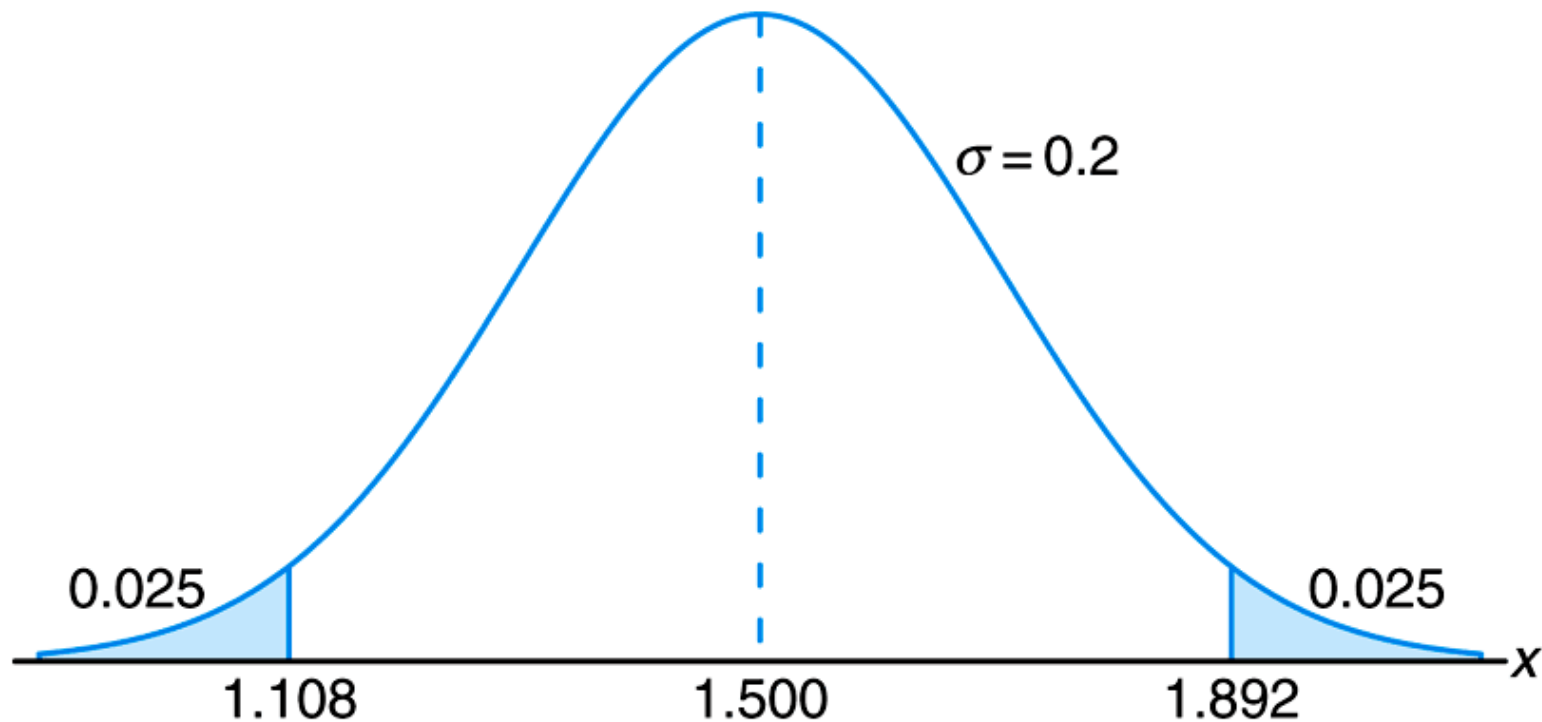


Figure 6.18 Area for Example 6.11

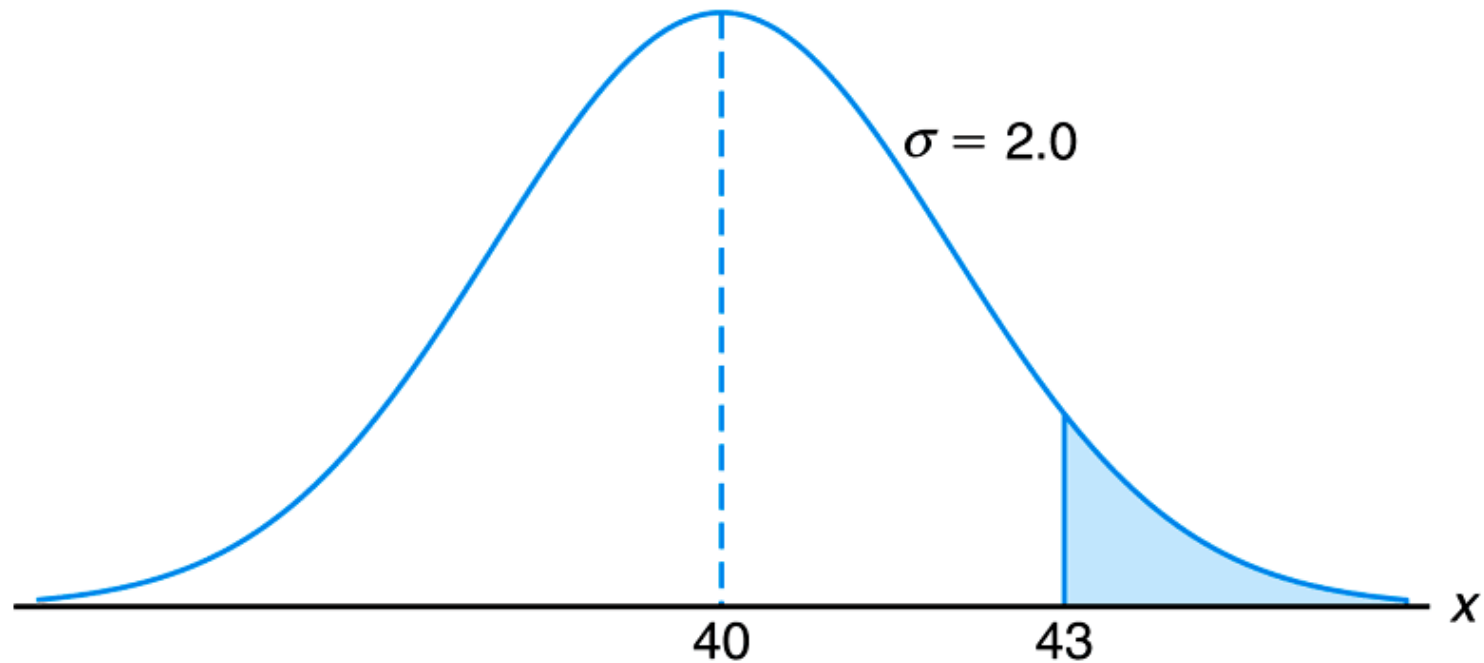


Figure 6.19 Area for Example 6.12

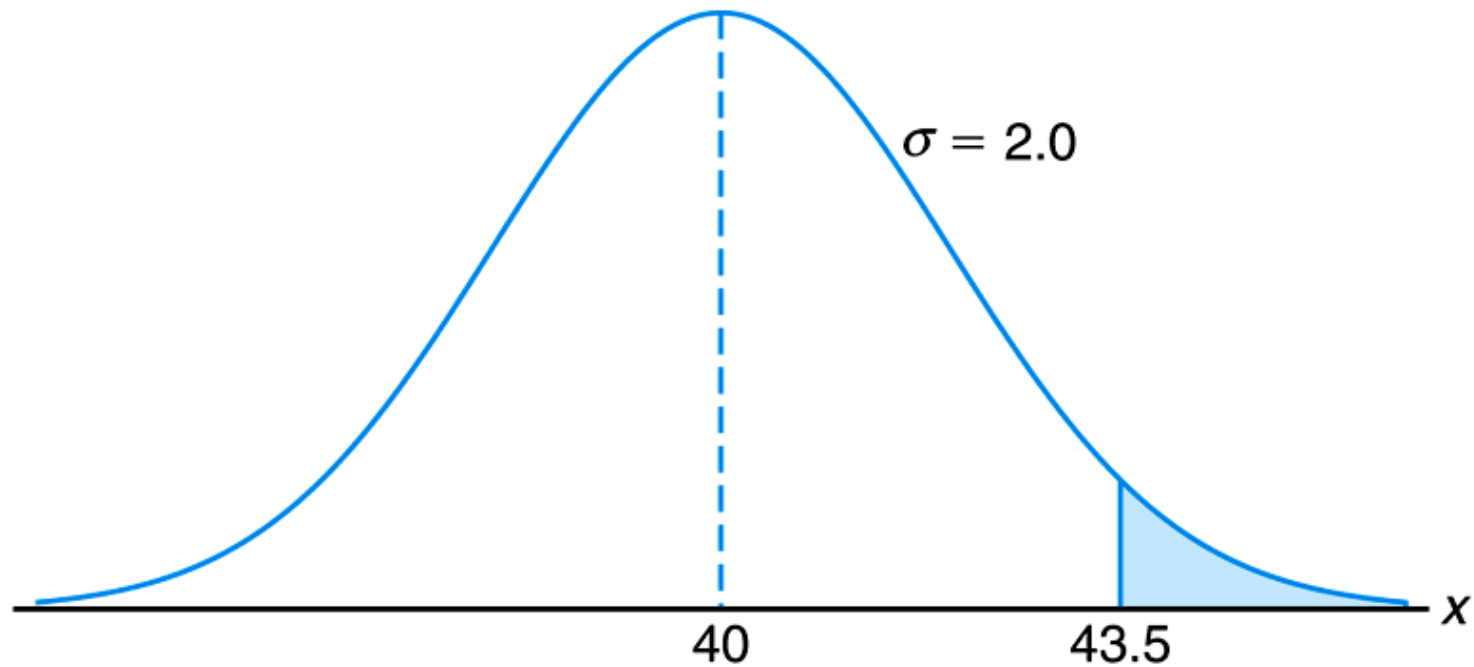


Figure 6.20 Area for Example 6.13

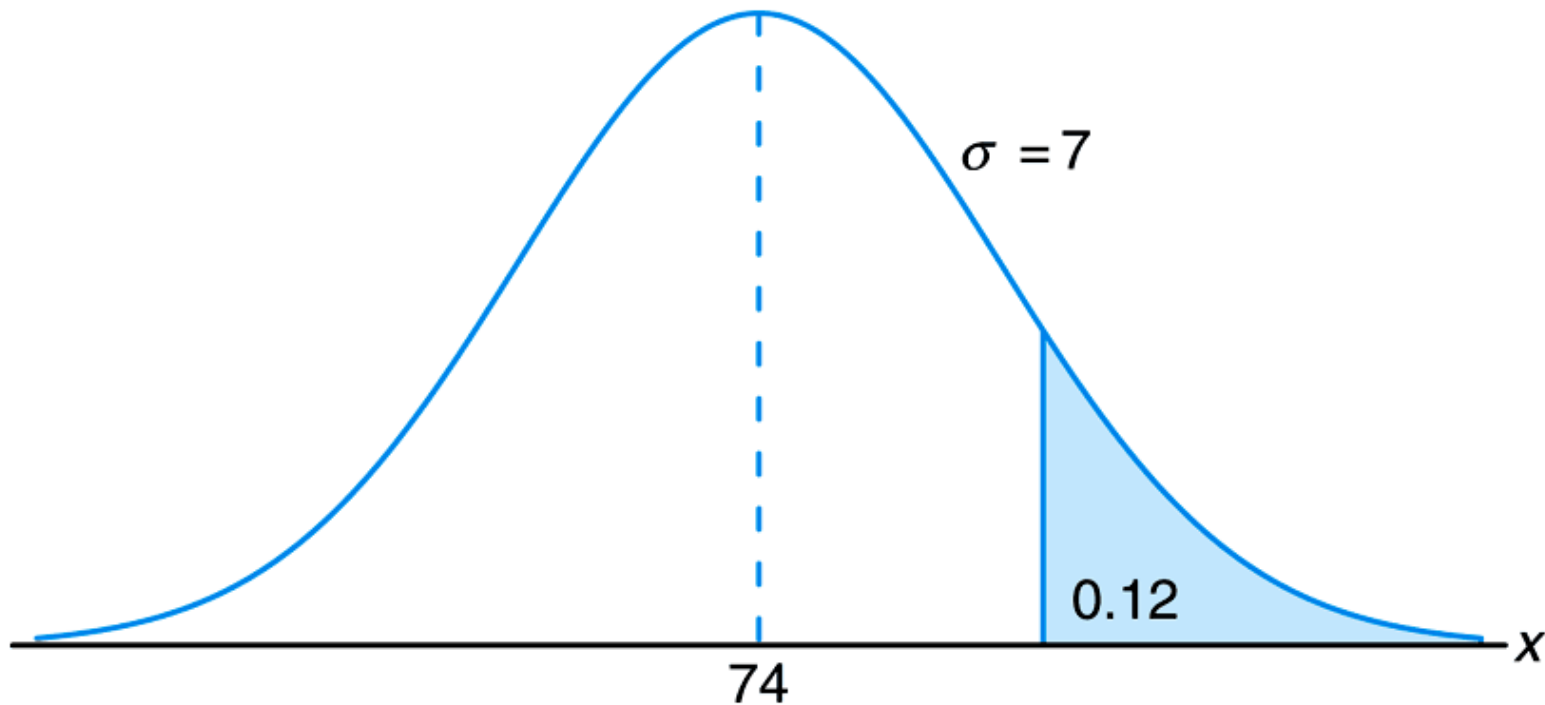
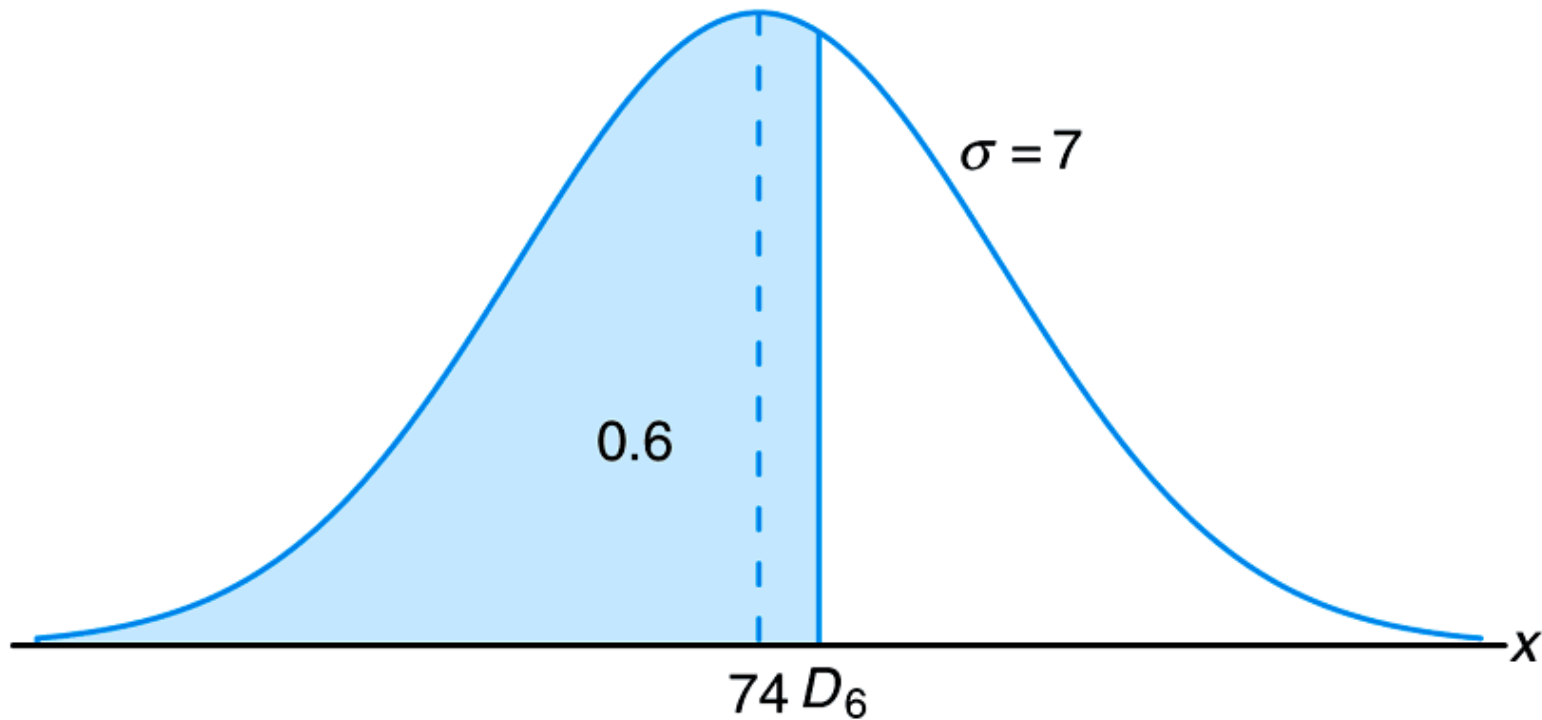


Figure 6.21 Area for Example 6.14



Section 6.5

Normal Approximation to the Binomial

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Theorem 6.3



If X is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

$$Z = \frac{X - np}{\sqrt{npq}},$$

as $n \rightarrow \infty$, is the standard normal distribution $n(z; 0, 1)$.

Figure 6.22 Normal approximation of $b(x; 15, 0.4)$

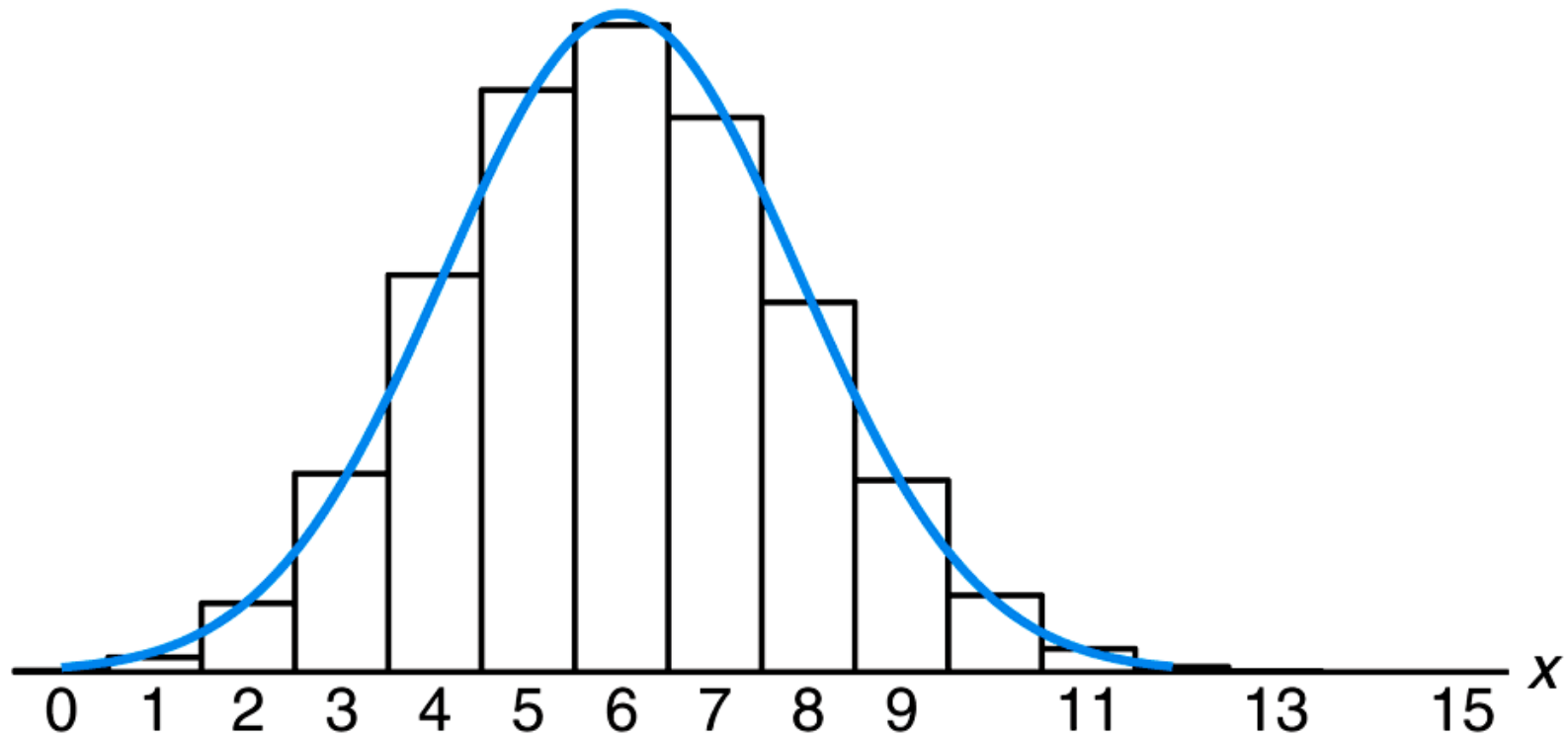


Figure 6.23 Normal approximation of $b(x; 15, 0.4)$ and $\sum_{x=7}^9 b(x; 15, 0.4)$

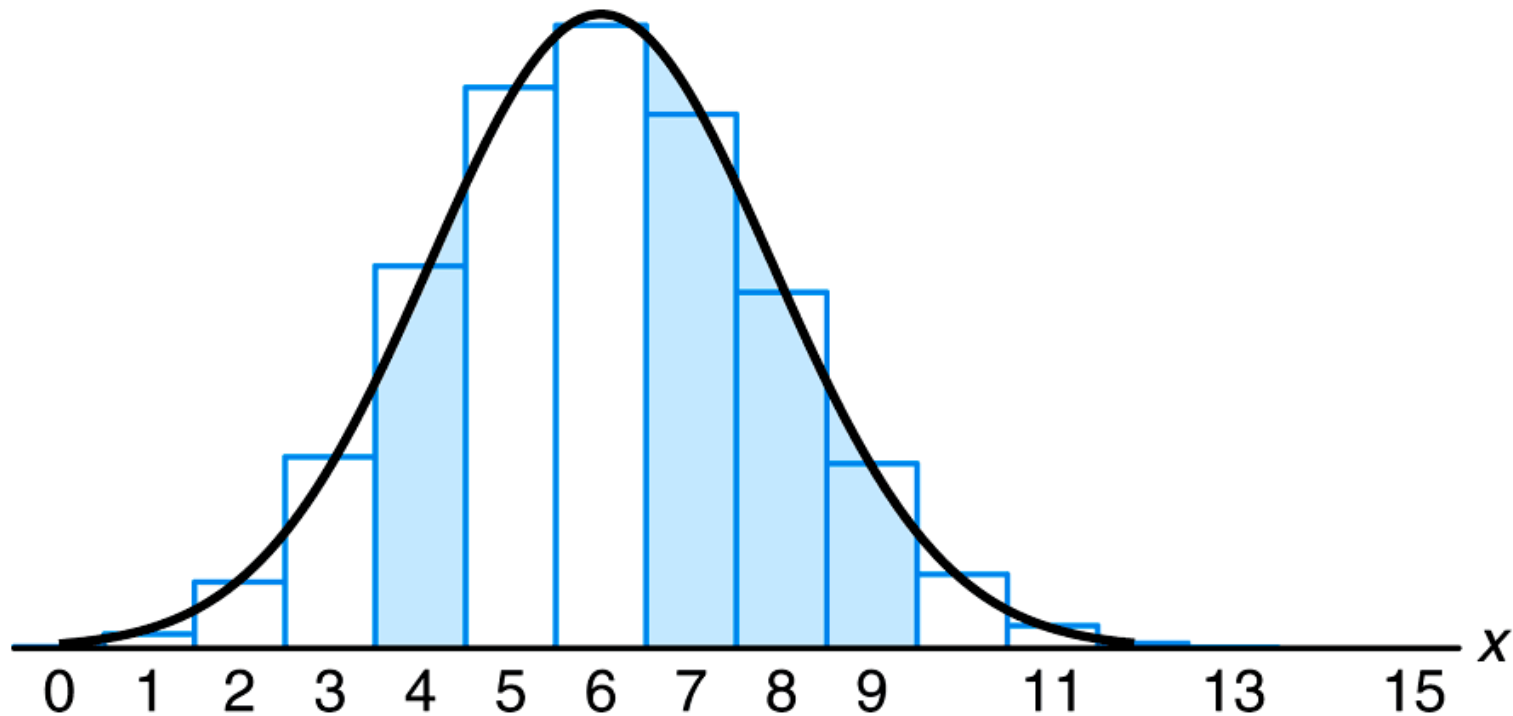


Figure 6.24 Histogram for $b(x; 6, 0.2)$

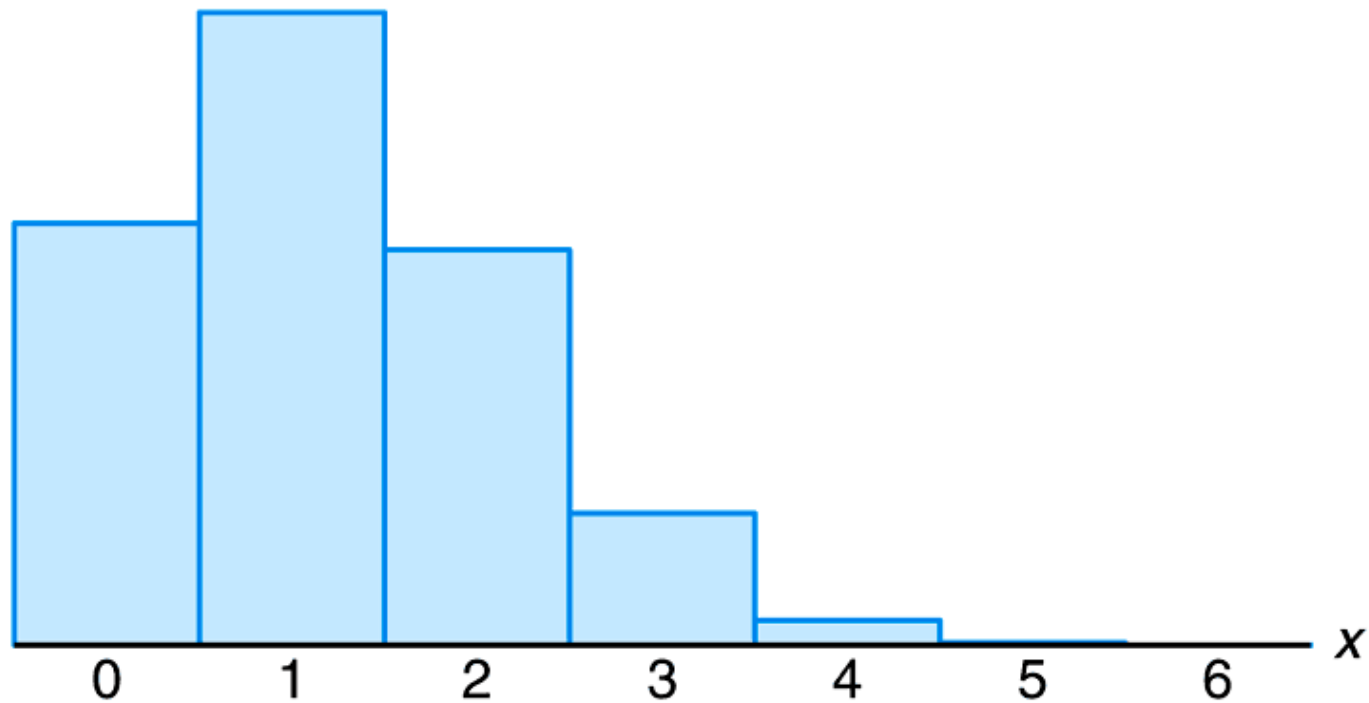


Figure 6.25 Histogram for $b(x; 15, 0.2)$

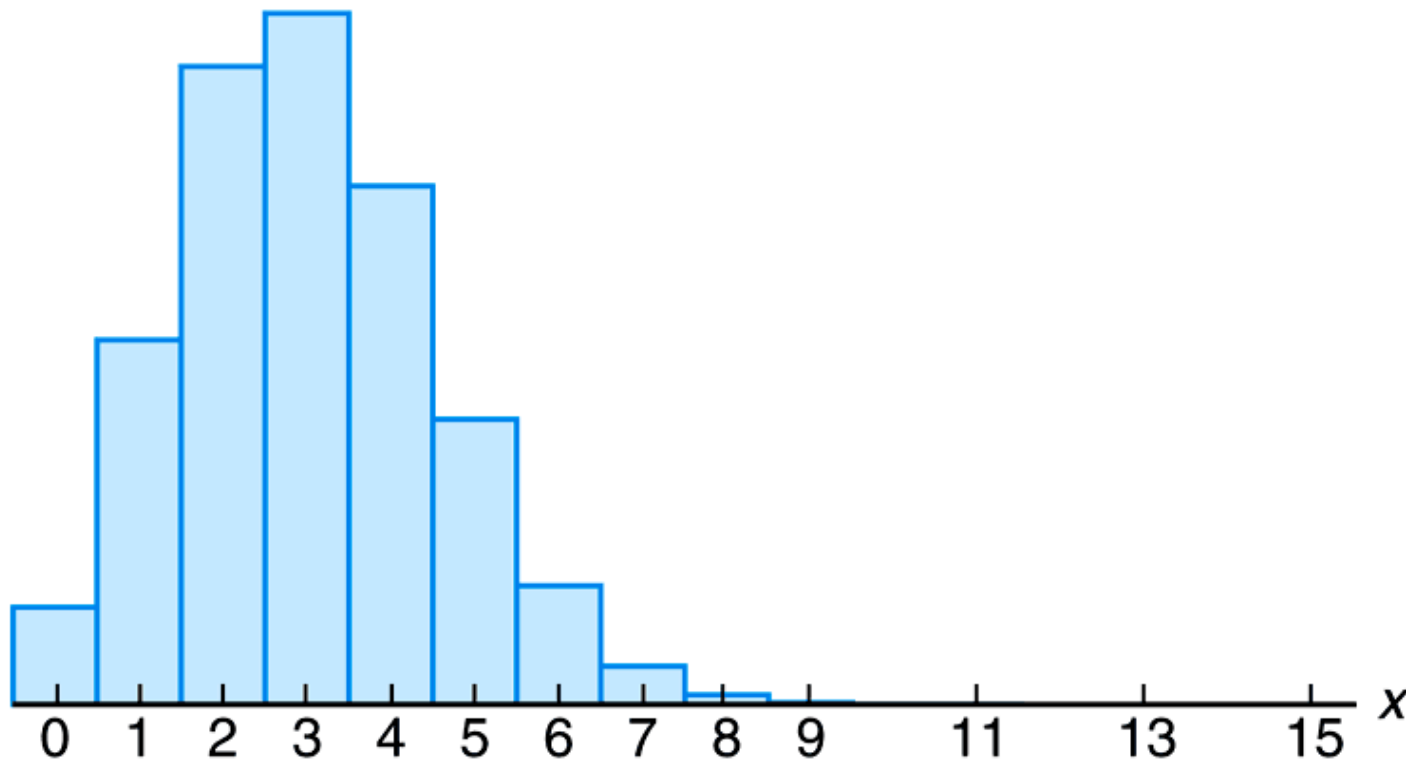


Table 6.1 Normal Approximation and True Cumulative Binomial Probabilities



r	$p = 0.05, n = 10$		$p = 0.10, n = 10$		$p = 0.50, n = 10$	
	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	0.5987	0.5000	0.3487	0.2981	0.0010	0.0022
1	0.9139	0.9265	0.7361	0.7019	0.0107	0.0136
2	0.9885	0.9981	0.9298	0.9429	0.0547	0.0571
3	0.9990	1.0000	0.9872	0.9959	0.1719	0.1711
4	1.0000	1.0000	0.9984	0.9999	0.3770	0.3745
5			1.0000	1.0000	0.6230	0.6255
6					0.8281	0.8289
7					0.9453	0.9429
8					0.9893	0.9864
9					0.9990	0.9978
10					1.0000	0.9997

r	$p = 0.05$					
	$n = 20$		$n = 50$		$n = 100$	
	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	0.3585	0.3015	0.0769	0.0968	0.0059	0.0197
1	0.7358	0.6985	0.2794	0.2578	0.0371	0.0537
2	0.9245	0.9382	0.5405	0.5000	0.1183	0.1251
3	0.9841	0.9948	0.7604	0.7422	0.2578	0.2451
4	0.9974	0.9998	0.8964	0.9032	0.4360	0.4090
5	0.9997	1.0000	0.9622	0.9744	0.6160	0.5910
6	1.0000	1.0000	0.9882	0.9953	0.7660	0.7549
7			0.9968	0.9994	0.8720	0.8749
8			0.9992	0.9999	0.9369	0.9463
9			0.9998	1.0000	0.9718	0.9803
10			1.0000	1.0000	0.9885	0.9941

Figure 6.26 Area for Example 6.15

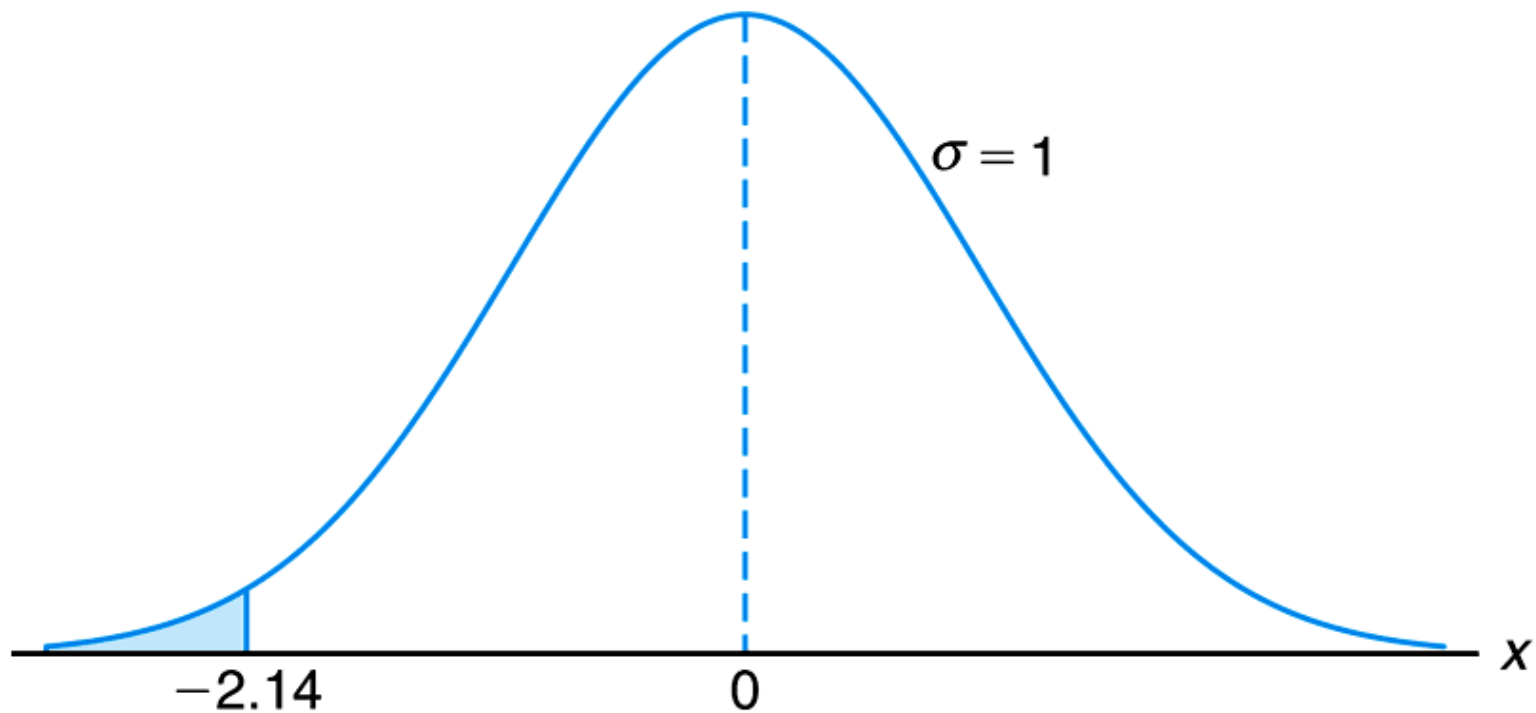
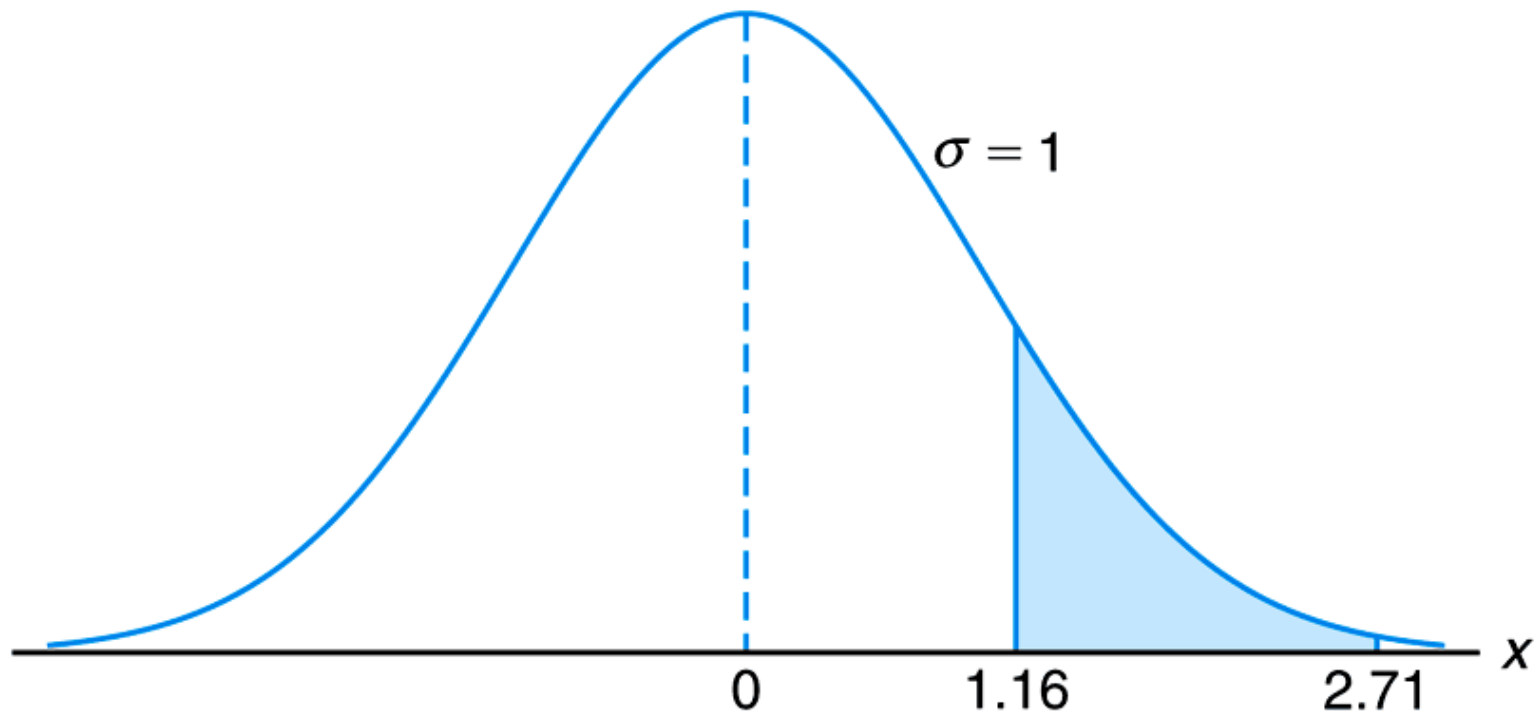


Figure 6.27 Area for Example 6.15



Section 6.6

Gamma and Exponential Distributions

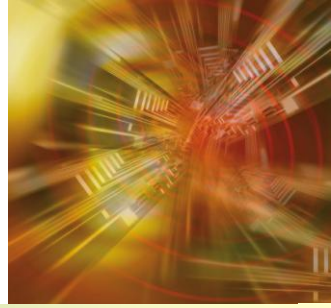
Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

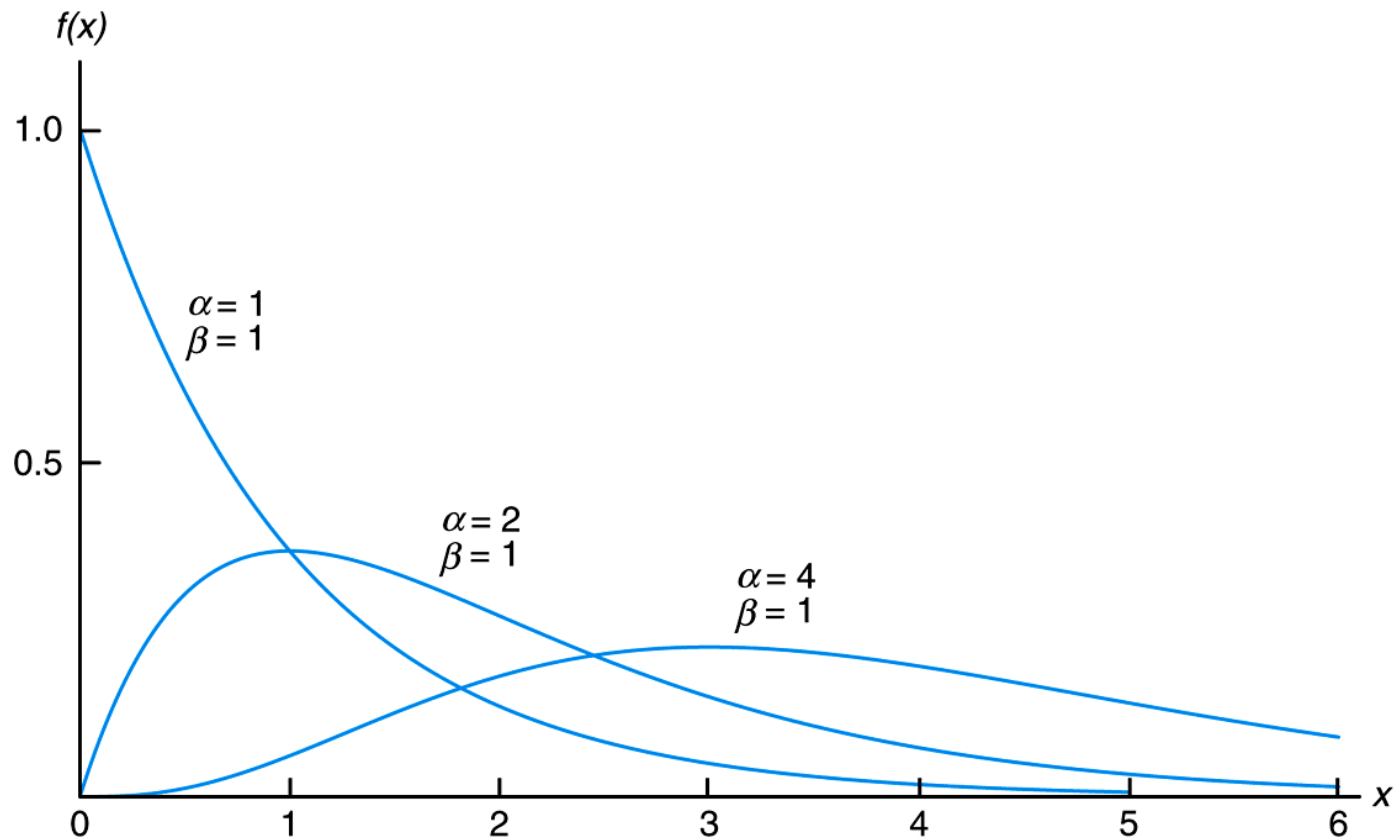
Definition 6.2



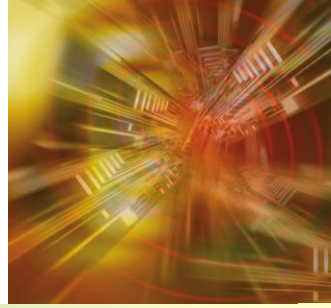
The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx, \quad \text{for } \alpha > 0.$$

Figure 6.28 Gamma distributions



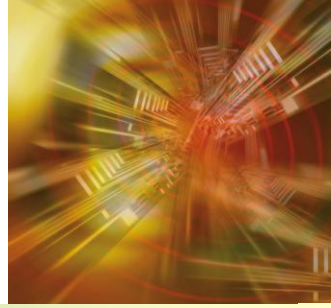
Theorem 6.4



The mean and variance of the gamma distribution are

$$\mu = \alpha\beta \text{ and } \sigma^2 = \alpha\beta^2.$$

Corollary 6.1



The mean and variance of the exponential distribution are

$$\mu = \beta \text{ and } \sigma^2 = \beta^2.$$

Section 6.7

Chi-Squared Distributions

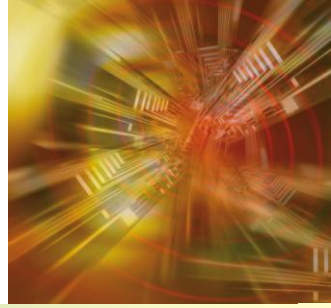
Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Theorem 6.5



The mean and variance of the chi-squared distribution are

$$\mu = v \text{ and } \sigma^2 = 2v.$$

Section 6.8

Beta Distribution

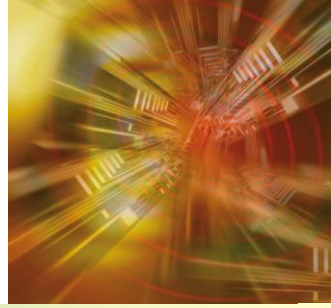
Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

Definition 6.3



A **beta function** is defined by

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \text{ for } \alpha, \beta > 0,$$

where $\Gamma(\alpha)$ is the gamma function.

Theorem 6.6



The mean and variance of a beta distribution with parameters α and β are

$$\mu = \frac{\alpha}{\alpha + \beta} \text{ and } \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)},$$

respectively.

Section 6.9

Lognormal Distribution

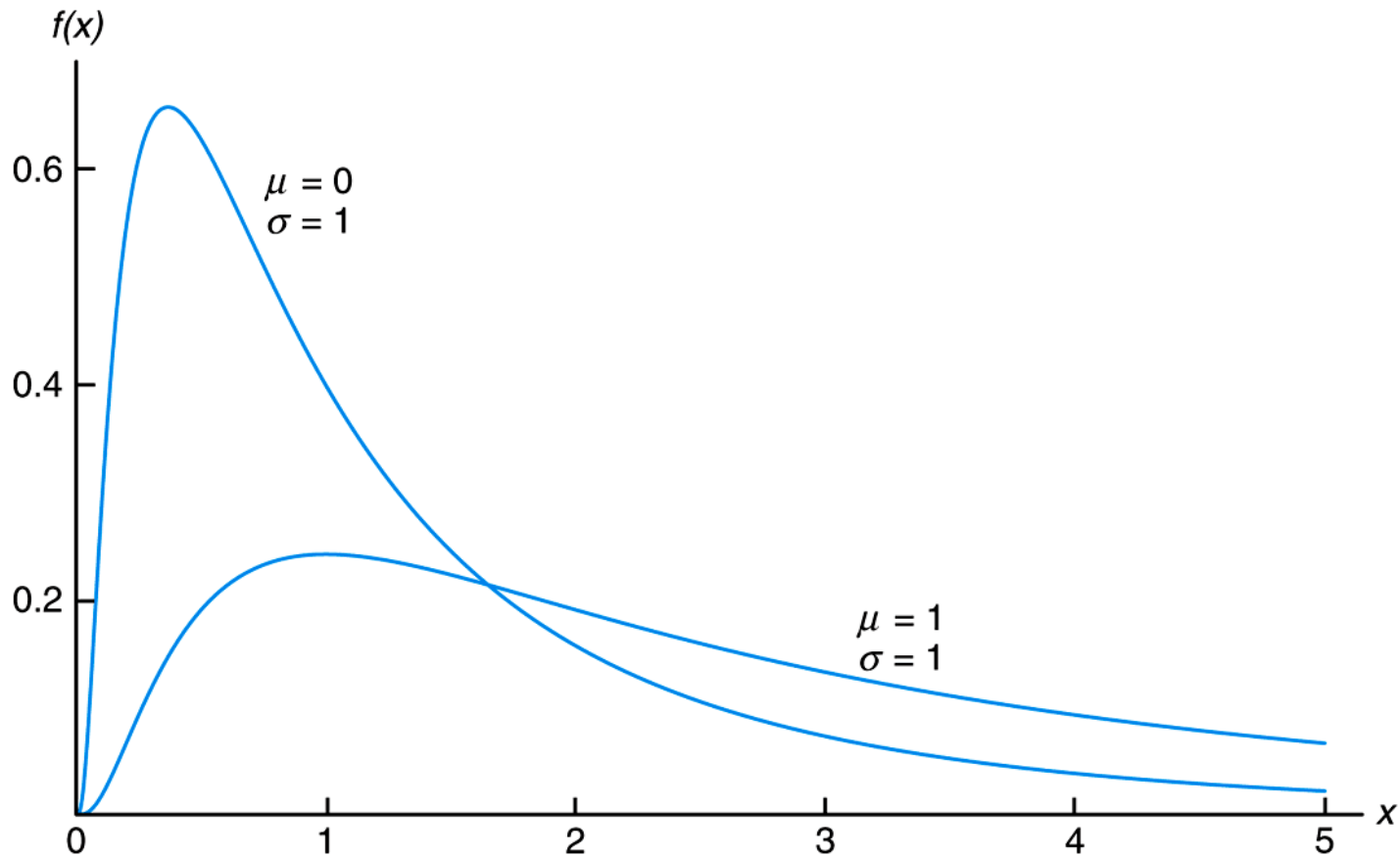
Probability & Statistics
for Engineers & Scientists

NINTH EDITION

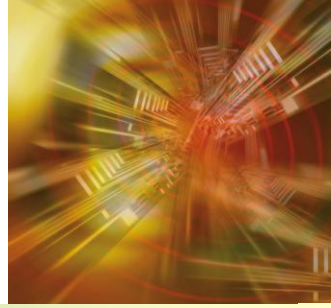


WALPOLE | MYERS | MYERS | YE

Figure 6.29 Lognormal distributions



Theorem 6.7



The mean and variance of the lognormal distribution are

$$\mu = e^{\mu + \sigma^2/2} \text{ and } \sigma^2 = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1).$$

Section 6.10

Weibull Distribution (Optional)

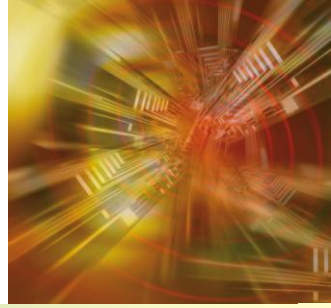
Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

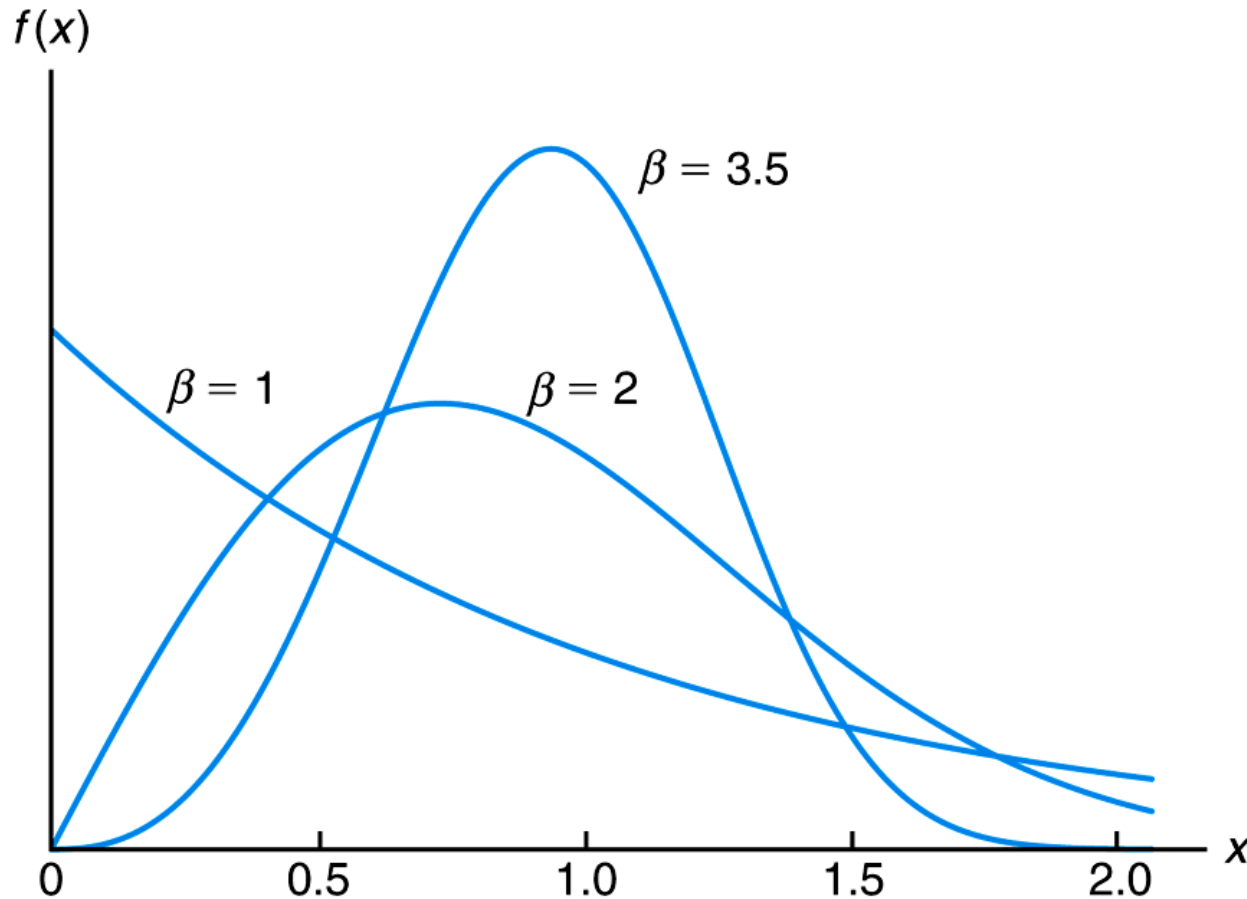
Theorem 6.8



The mean and variance of the Weibull distribution are

$$\mu = \alpha^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \text{ and } \sigma^2 = \alpha^{-2/\beta} \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right) \right]^2 \right\}.$$

Figure 6.30 Weibull distributions ($\alpha = 1$)



Section 6.11

Potential
Misconceptions
and Hazards;
Relationship to
Material in Other
Chapters

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE