Q3.(35 pts)

For the following series , determine whether it converges or diverges. Show your justifications and state the name of the convergence test you use.

$$\sum_{n=1}^{\infty} \frac{2n^2+3^n}{-3+2(5^n)}$$

First,

$$\lim_{n\to\infty} \frac{\frac{2n^2+3^n}{-3+2(5^n)}}{\frac{3^n}{5^n}} = \lim_{n\to\infty} \frac{2\frac{n^2}{3^n}+1}{-3\frac{1}{5^n}+2} \longrightarrow \frac{1}{2} < \infty$$

where

$$\lim_{n\to\infty} \frac{n^2}{3^n} = \lim_{n\to\infty} \frac{2n}{3^n \cdot \ln 3} = \lim_{n\to\infty} \frac{2}{3^n \ln 3 \cdot \ln 3} = 0$$

Because $\sum_{n=1}^{\infty} \frac{3^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ is convergent,

the given series is convergent by the limit comparison test.

Q3.(35 pts)

For the following series , determine whether it converges or diverges. Show your justifications and state the name of the convergence test you use.

$$\sum_{n=1}^{\infty} \frac{2n^2+3^n}{-3+2(5^n)}$$

First,

$$\lim_{n\to\infty} \frac{\frac{2n^2+3^n}{-3+2(5^n)}}{\frac{3^n}{5^n}} = \lim_{n\to\infty} \frac{2\frac{n^2}{3^n}+1}{-3\frac{1}{5^n}+2} \longrightarrow \frac{1}{2} < \infty$$

/10 pts

where

$$\lim_{n\to\infty} \frac{n^2}{3^n} = \lim_{n\to\infty} \frac{2n}{3^n \cdot \ln 3} = \lim_{n\to\infty} \frac{2}{3^n \cdot \ln 3 \cdot \ln 3} = 0$$

/10 pts

Because $\sum_{n=1}^{\infty} \frac{3^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ is convergent,

/10 pts

the given series is convergent by the limit comparison test.

/5 pts