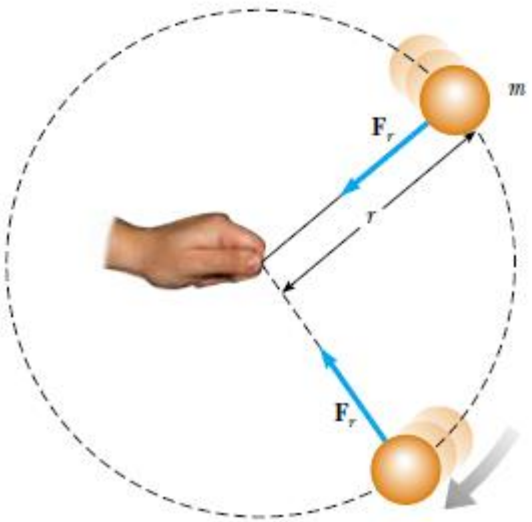
Circular Motion and Other Applications of Newton's Laws

Newton's Second Law Applied to Uniform Circular Motion

A particle moving with uniform speed *v* in a circular path of radius *r* experiences an acceleration that has a magnitude

$$a_c = \frac{v^2}{r}$$

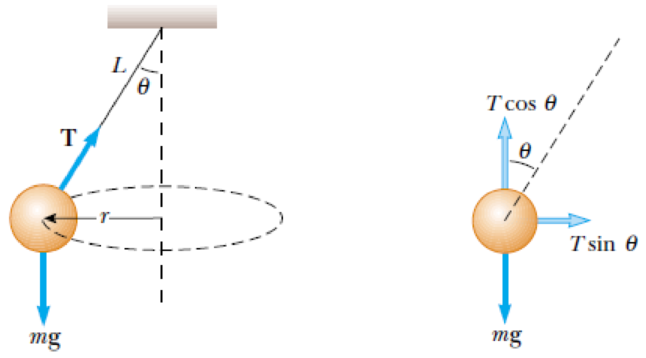


an object of mass *m* is moving ona horizontal circular path with a constant speed of *v*. a_c is *always* perpendicular to *v*.

If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

$$\sum F = ma_c = m\frac{v^2}{r}$$

Ex/ A small object of mass *m* is suspended from a string of length *L*. The object revolves with constant speed *v* in a horizontal circle of radius *r*, as shown in Figure. Find an expression for *v*.



$$\Sigma F_y = ma_y = 0 \qquad T\cos\theta = mg$$

Radial force; $\sum F = T\sin\theta = ma_c = \dfrac{mv^2}{r}$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta} \qquad r = L\sin\theta$$

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Ex/ A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in the Figure. If the radius of the curve is 35 m and the coefficient of static friction between the tires and dry pavement is 0.5, find the maximum speed the car can have and still make the turn successfully.

$$n = mg \qquad f_s = m\frac{v^2}{r}$$

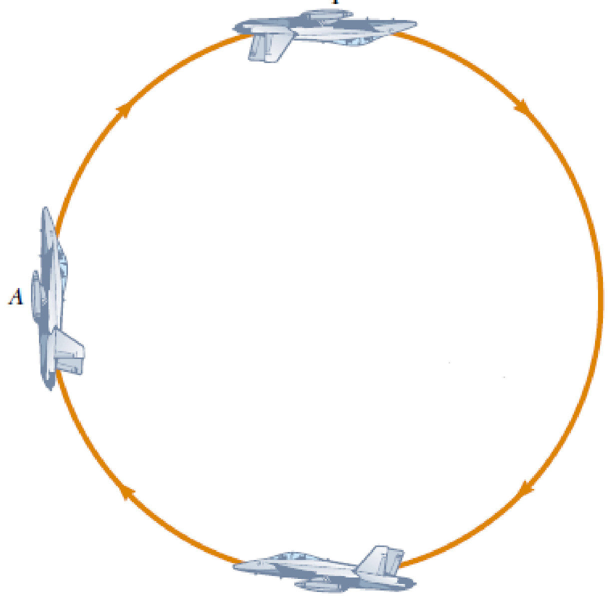
$$f_{s,\max} = \mu_s mg$$

$$v_{\max} = \sqrt{\frac{f_{s,\max}\,r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

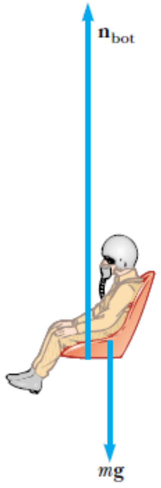
$$= \sqrt{(0.500)(9.80\ \mathrm{m/s^2})(35.0\ \mathrm{m})}$$

$$= \boxed{13.1\ \mathrm{m/s}}$$

Ex/ A pilot of mass *m* in a jet aircraft executes a loop-the-loop, as shown in the Figure. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot **(a)** at the bottom of the loop and **(b)** at the top of the loop. Express your answers in terms of the weight of the pilot *mg*.



a)

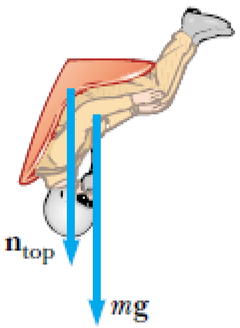


$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{bot} = mg\left(1 + \frac{(225\ \text{m/s})^2}{(2.70 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)}\right) = \boxed{2.91\,mg}$$

b)



$$\sum F = n_{top} + mg = m\frac{v^2}{r}$$

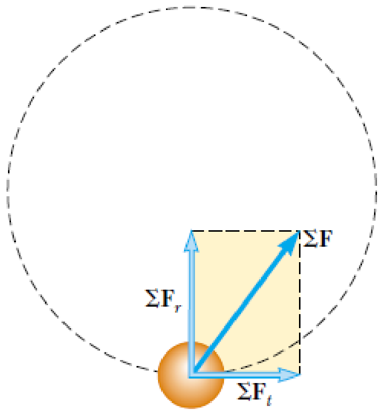
$$n_{top} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

$$n_{top} = mg\left(\frac{(225\ \text{m/s})^2}{(2.70 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)} - 1\right)$$

$$= \boxed{0.913\,mg}$$

Nonuniform Circular Motion

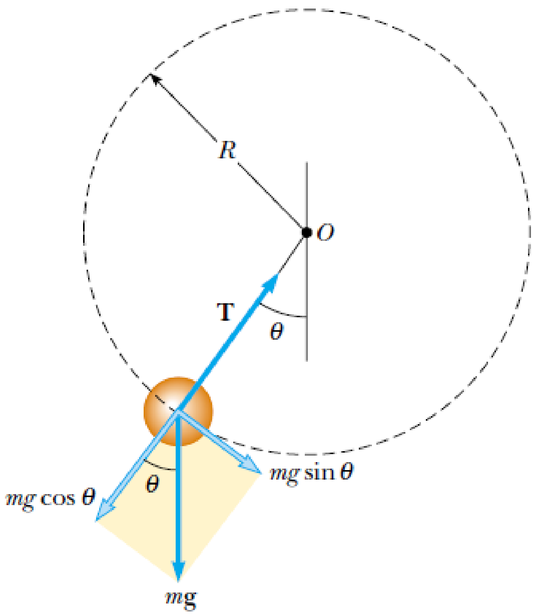
Speed of the particle can be varible on a circular path. In this case motion will be non-uniform and there will be a tangential force in order to change the magnitude of the velocity.



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_r + \Sigma \mathbf{F}_t$$

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t,$$

Ex/ A small sphere of mass *m* is attached to the end of a cord of length *R* and set into motion in a *vertical* circle about a fixed point *O*, as illustrated in the Figure. Determine the tension in the cord at any instant when the speed of the sphere is *v* and the cord makes an angle θ with the vertical.



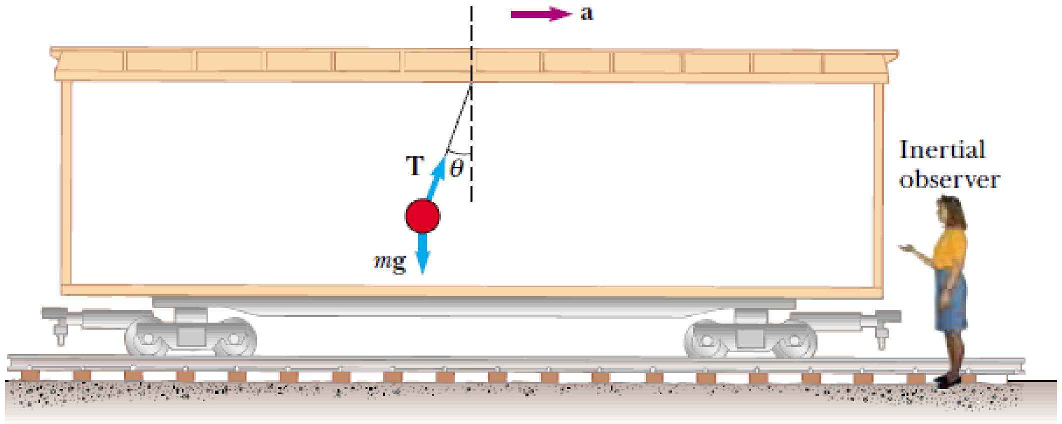
$$\sum F_t = mg \sin\theta = ma_t$$

$$a_t = g\sin\theta$$

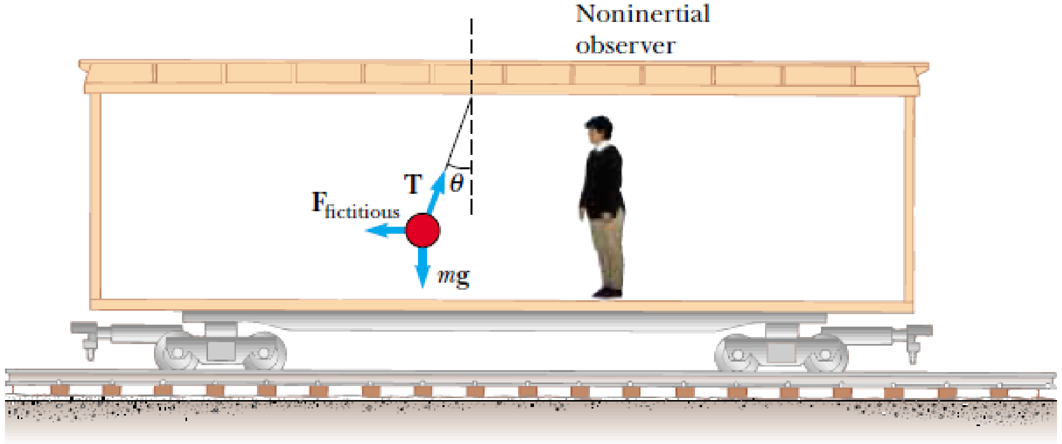
$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = \boxed{m\left(\frac{v^2}{R} + g\cos\theta\right)}$$

Motion in Accelerated Frames



(a)



(b)

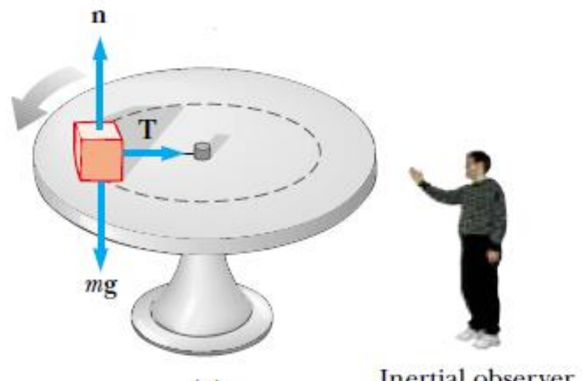
A small sphere of mass *m* is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in Figure (a). The noninertial observer in Figure (b) claims that a force, which we know to be ***fictitious***, must act in order to cause the observed deviation of the cord from the vertical.

Inertial observer
$$\begin{cases} (1) \quad \sum F_x = T\sin\theta = ma \\ (2) \quad \sum F_y = T\cos\theta - mg = 0 \end{cases}$$

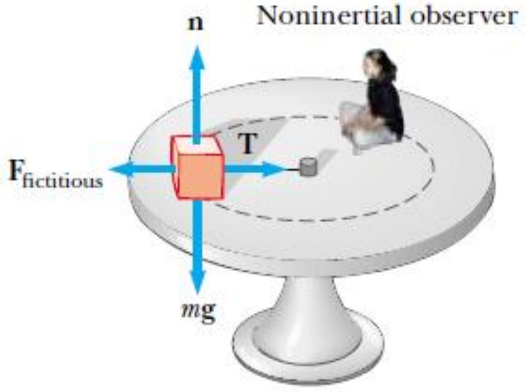
Noninertial observer
$$\begin{cases} \sum F'_x = T\sin\theta - F_{\text{fictitious}} = 0 \\ \sum F'_y = T\cos\theta - mg = 0 \end{cases}$$

these expressions are equivalent to (1) and (2) if $F_{\text{fictitious}} = ma$, where *a* is the acceleration according to the inertial observer. If we were to make this substitution in the equation for F'_x above, the noninertial observer obtains the same mathematical results as the inertial observer. However, the physical interpretation of the deflection of the cord differs in the two frames of reference.

Ex/ Suppose a block of mass m lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown in Figure. How would each of the observers write Newton's second law for the block?



(a)



(b)

According to an inertial observer (Fig. a), if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r, where v is its linear speed. The inertial observer concludes that this centripetal acceleration is provided by the force T exerted by the string and writes Newton's second law as $T = mv^2/r$.

According to a noninertial observer attached to the turntable (Fig b), the block is at rest and its acceleration is zero. Therefore, she must introduce a fictitious outward force of magnitude mv^2/r to balance the inward force exerted by the string. According to her, the net force on the block is zero, and she writes Newton's second law as $T - mv^2/r = 0$.

And she concludes again $T = mv^2/r$