A Short Review of Fuzzy Linear Equation Systems

S

Hale Gonce Kocken
Yildiz Technical University, Turkey

Inci Albayrak
Yildiz Technical University, Turkey

INTRODUCTION

Systems of linear equations play a major role in various areas such as operational research, physics, statistics, engineering and social sciences. In many applications, the parameters of linear equation systems are not always exactly known and stable. This imprecision may follow from the lack of exact information, changeable economic conditions, etc.. A frequently used way of expressing the imprecision is to use the fuzzy numbers rather than the crisp numbers. It enables us to consider tolerances for parameters of linear equation systems (the entries of the coefficient matrix and right hand side vector) in a more natural and direct way. Therefore, fuzzy linear equation systems seem to be more realistic and reliable than the crisp case.

This study has attempted to provide a short review of notable papers on fuzzy linear equation systems with a basic structural classification. Considering the vast literature, we only aim to present the most cited and leading papers for each classification. Our short review will give a general framework about the progression of the subject and its solution approaches.

BACKGROUND

Wide range of real world applications in many areas including financial engineering, scientific management and engineering technology are using linear equation systems for modeling and solving their respective problems. In many situations, the estimation of the system parameters is imprecise because of the lack of exact information, environmental and changeable economic conditions, etc. The uncertainty of parameters involved in the process of actual mathematical modeling is often represented by fuzzy numbers, so it is important to develop mathematical models that would appropriately treat fuzzy linear equation systems.

One of the most important applications of linear equation systems to electrical engineering is to analyze electronic circuits that cannot be described using the rules for resistors in series or parallel. The goals are to calculate the current flowing in each branch of the circuit and to calculate the voltage at each node of the circuit, which are known Loop Current and Nodal Voltage Analysis, respectively. Owing to environmental conditions, tolerance in the elements and noise, these analyses can be modeled in the form of fuzzy linear equation systems.

Another important application of linear equation systems is input-output analysis developed by V. Leontief. Input-output analysis is an extremely effective tool used in more than 70 countries over the world for manufacturing processes optimization, economy condition improvement and intersectors costs allocation analysis. Considering dynamic nature of economics, it can be easily stated that extending this analysis to fuzzy version of linear equation system is significant.

There is a vast literature on the investigation of solutions for fuzzy linear equation systems. Early works in the literature are on to linear equation systems whose coefficient matrix is crisp and the right hand vector is fuzzy, that is known as Fuzzy Linear Equation System (FLS). The crispness of the coefficient matrix makes the modeling of real life problems restricted. Linear systems, whose all the parameters i.e. both coefficient matrix and right hand vector are fuzzy, are named Fully Fuzzy Linear Equation System (FFLS). The main intend of FFLS is to widen the scope of FLS in scientific applications by removing the crispness assumption on the entries of coefficient matrix.

DOI: 10.4018/978-1-4666-5888-2.ch687

Copyright © 2015, IGI Global. Copying or distributing in print or electronic forms without written permission of IGI Global is prohibited.

Besides FLS and FFLS, there exist the dual forms of both systems in the literature.

Generally, both FLS and FFLS are handled under two main headings: square (n x n) and nonsquare (m x n) forms. Most of the works in the literature deal with square form. Fuzzy elements of these systems can be taken as triangular, trapezoidal or generalized fuzzy numbers in general or parametric form. While triangular fuzzy numbers are widely used in earlier works, trapezoidal fuzzy numbers are neglected for a long time. Besides, there exist lots of works using the parametric and level cut representation of fuzzy numbers. Another classification for FFLS can be made also depending on whether FFLS has sign restrictions on its parameters. Having sign restrictions for FFLS means that all parameters of FFLS are assumed as positive. Since the parameters are assumed as positive in the most of the papers, further work is needed for FFLS with arbitrary (no restrictions on sign) fuzzy numbers.

Solution methods for FLS and FFLS can be gathered into two groups: direct and iterative methods. Direct methods which are also known as computational or classical methods in the literature are generally used for square systems. Some of the well known direct methods can be given as: the Gauss elimination method, Cramer's rule, QR Decomposition method, LU Decomposition method, Quasi-Newton method. The iterative methods, which are mathematical procedures that generate a sequence of improving approximate solutions for the related problem, are used for both square and nonsquare linear equation systems. Some of the well known iterative methods can be given as: Gauss Seidel, Jacobi, Adomian Decomposition, Homotopy perturbation.

This paper is organized with a basic structural classification. Primary classification is made considering whether the system is square or not. Secondary classification is based on the fuzziness of the coefficient matrix that is FLS or FFLS. Final classification is constructed with respect to the solution methods, i.e. direct or iterative. Considering the limitation of the chapter, each classification involves the most cited and notable papers consist of scientific refereed journals. The objective of our review is to provide structural methodology on FLS and to reemphasize a number of gaps in the literature.

SQUARE SYSTEMS

Fuzzy Linear Equation Systems

Direct Methods

Friedman, Ming and Kandel (1998) firstly introduces a general model for solving a n x n FLS, $Ax = \tilde{b}$ where A is a crisp matrix and $\tilde{b}$ is an arbitrary fuzzy number vector. Using embedding approach, the original system is replaced by a 2n x 2n crisp linear equation system and then they give the conditions for the existence of a unique fuzzy solution to a n x n FLS.

Besides Abbasbandy and Alavi (2005) proposes an efficient method replacing the original system by two n x n crisp linear equation systems, Abbasbandy, Ezzati and Jafarian (2006) discusses the LU decomposition method in case the coefficient matrix is symmetric positive definite.

Starting from the work by Friedman, Ming and Kandel (1998), Ma, Friedman and Kandel (2000) analyses the solution of dual fuzzy systems of the form $A_1\tilde{x} = A_2\tilde{x} + \tilde{b}$. They remark that this system is not equivalent to the system $\left(A_1 - A_2\right)\tilde{x} = \tilde{b}$, since there is no element v such that u + v = 0, for an arbitrary fuzzy number u.

Allahviranloo and Salahshour (2011) proposes a simple and practical method for the system $A\tilde{x} = \tilde{b}$, where $\tilde{x}$ and $\tilde{b}$ are fuzzy triangular vectors with non-zero spreads and A is nonsingular. They derive three types of solutions: maximal ($\tilde{x}^L$) and minimal ($\tilde{x}^U$) symmetric solutions and a fuzzy vector solution ($\tilde{x}^\lambda$). The last type solution corresponds the convex combination of the maximal and minimal spreads, and enables the decision maker to analyse the system in desire positions by attaining some values to λ .

Ezzati (2011) gives a new method for solving square FLS by using the embedding method given in Cong-Xin and Ming (1991) and replaces the original n x n FLS by two n x n crisp linear equation systems. And that study also investigates the perturbation analysis in two n x n crisp linear equation systems instead of a 2n x 2n crisp linear equation system as the authors of Wang, Chen and Wei (2009) have done.

7 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the product's webpage:
www.igi-global.com/chapter/a-short-review-of-fuzzy-linear-equation-systems/112396?camid=4v1

This title is available in InfoSci-Books, InfoSci-Computer Science and Information Technology, Science, Engineering, and Information Technology, InfoSci-Select, InfoSci-Select, InfoSci-Select, InfoSci-Select, InfoSci-Select, InfoSci-Select. Recommend this product to your librarian:
www.igi-global.com/e-resources/library-recommendation/?id=1

Related Content

Classification of Sentiment of Reviews using Supervised Machine Learning Techniques
Abinash Tripathy and Santanu Kumar Rath (2017). *International Journal of Rough Sets and Data Analysis (pp. 56-74).*
www.igi-global.com/article/classification-of-sentiment-of-reviews-using-supervised-machine-learning-techniques/169174?camid=4v1a

A Rough Set Theory Approach for Rule Generation and Validation Using RSES
Hemant Rana and Manohar Lal (2016). *International Journal of Rough Sets and Data Analysis (pp. 55-70).*
www.igi-global.com/article/a-rough-set-theory-approach-for-rule-generation-and-validation-using-rses/144706?camid=4v1a

Missing Part of Halal Supply Chain Management
Ratih Hendayani and Yudi Fernando (2018). *Encyclopedia of Information Science and Technology, Fourth Edition (pp. 5456-5464).*
www.igi-global.com/chapter/missing-part-of-halal-supply-chain-management/184248?camid=4v1a

Institutional Repository
Om Prakash Saini and Malkeet Singh Gill (2015). *Encyclopedia of Information Science and Technology, Third Edition (pp. 6697-6704).*
www.igi-global.com/chapter/institutional-repository/113132?camid=4v1a