**Örnek:** $A = \begin{bmatrix} -1 & 2 & -4 \\ 0 & 4 & 2 \\ 0 & -1 & 1 \end{bmatrix}$ matrisinin özdeğer ayrışımını yapınız. Bu sonuçtan yararlanarak $A^4 = ?$ hesaplayınız.

**Çözüm:** $(A-\lambda I)X = 0$

$$\left.\begin{aligned} (-1-\lambda)x_1 + 2x_2 - 4x_3 &= 0 \\ (4-\lambda)x_2 + 2x_3 &= 0 \\ -x_2 + (1-\lambda)x_3 &= 0 \end{aligned}\right\}$$

$|A-\lambda I| = 0 \quad \begin{vmatrix} -1-\lambda & 2 & -4 \\ 0 & 4-\lambda & 2 \\ 0 & -1 & 1-\lambda \end{vmatrix} = 0$

$(-1-\lambda)[\lambda^2 - 5\lambda + 6] = 0 \Rightarrow \lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 3$

$\lambda_1 = -1$ için $\left.\begin{aligned} 2x_2 - 4x_3 &= 0 \\ 5x_2 + 2x_3 &= 0 \\ -x_2 + x_3 &= 0 \end{aligned}\right\}$ $x_2 = 0$, $x_3 = 0$ olur. $x_1 = s$ keyfi $(s \neq 0)$

$s = 1$ olsun. $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_2 = 2$ için $\left.\begin{aligned} -3x_1 + 2x_2 - 4x_3 &= 0 \\ 2x_2 + 2x_3 &= 0 \\ -x_2 - x_3 &= 0 \end{aligned}\right\}$

$x_2 = 1$ keyfi, $x_3 = -1$, $x_1 = 2$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\lambda_3 = 3$ için $\left.\begin{aligned} -4x_1 + 2x_2 - 4x_3 &= 0 \\ x_2 + 2x_3 &= 0 \\ -x_2 - 2x_3 &= 0 \end{aligned}\right\}$ $x_3 = 1$ keyfi, $x_2 = -2$, $x_1 = -2$

$$X_3 = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

$$R = [X_1\ X_2\ X_3]$$

$$R = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[R,I]=\left[\begin{array}{ccc|ccc}1&2&-2&1&0&0\\0&1&-2&0&1&0\\0&-1&1&0&0&1\end{array}\right]\sim\left[\begin{array}{ccc|ccc}1&0&2&1&-2&0\\0&1&-2&0&1&0\\0&0&-1&0&1&1\end{array}\right]$$

$$\sim\Big[\underbrace{\begin{array}{ccc}1&0&0\\0&1&0\\0&0&1\end{array}}_{I}\Big|\underbrace{\begin{array}{ccc}1&0&2\\0&-1&-2\\0&-1&-1\end{array}}_{R^{-1}}\Big]\ ,\quad R^{-1}=\begin{bmatrix}1&0&2\\0&-1&-2\\0&-1&-1\end{bmatrix}$$

$$R^{-1}AR=B=\text{köş}(\lambda_1,\lambda_2,\lambda_3)\ ,\quad B=\begin{bmatrix}-1&0&0\\0&2&0\\0&0&3\end{bmatrix}$$

$$A=RBR^{-1}\Longrightarrow A^4=RB^4R^{-1}$$

$$A^4=\begin{bmatrix}1&2&-2\\0&1&-2\\0&-1&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&16&0\\0&0&81\end{bmatrix}\begin{bmatrix}1&0&2\\0&-1&-2\\0&-1&-1\end{bmatrix}$$

$$A^4=\begin{bmatrix}1&32&-162\\0&16&-162\\0&-16&81\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&-1&-2\\0&-1&-1\end{bmatrix}=\begin{bmatrix}1&130&100\\0&146&130\\0&-65&-49\end{bmatrix}$$

$$A^4=\begin{bmatrix}1&130&100\\0&146&130\\0&-65&-49\end{bmatrix}$$