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Non-Cutting Moving Toolpath Optimization with Elitist Non-Dominated Sorting Genetic Algorithm-II

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Abstract: Path planning (PP) is fundamental in the decision-making and control processes of computer numerical control (CNC) machines, playing a critical role in smart manufacturing research. Apart from improving optimization in PP, enhancing efficiency while decreasing CNC machine cycle time is important in manufacturing. Many methods have been offered in the literature to improve the cycle time for obtaining optimal trajectories in toolpath optimization, but these methods are mostly considered for improvements in path length or machining time in optimal PP. This study demonstrates a method for creating a smoothing path. It aims to minimize both cycle time and toolpath length, while demonstrating that the non-dominated sorting genetic algorithm (NSGA-II) is efficient in addressing the multi-objective PP problems within static situations. Pareto optimality for performance comparisons with multi-objective genetic algorithms (MOGAs) is presented in order to highlight the positive features of the non-dominant solving generated by the NSGA-II. According to the comprehensive analysis results, the optimization of the path carried out with the NSGA-II emphasizes its shorter and smoother attributes, with the optimal trajectory achieving approximately 30% and 7% reductions in path length and machining cycle time, respectively.

Keywords: computer numerical control; smart process planning; multi-objective optimization; evolutionary algorithm

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Citation: Demir, G.; Acar Vural, R. Non-Cutting Moving Toolpath Optimization with Elitist Non-Dominated Sorting Genetic Algorithm-II. *Appl. Sci.* **2024**, *14*, 4471. https://doi.org/10.3390/app14114471

Academic Editor: Mark J. Jackson

Received: 23 April 2024
Revised: 15 May 2024
Accepted: 20 May 2024
Published: 23 May 2024

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1. Introduction

CNC machine tools are generally preferred for the manufacturing of very complex industrial products in automotive, aerospace, defense, and molding industries to improve efficiency and mass productivity [1]. Computer-aided design (CAD) is responsible for the design phase. On the other hand, computer-aided manufacturing (CAM) manages the manufacturing and production processes and the computer-controlled machinery, automating various aspects of the manufacturing process. These CAD/CAM tools are complementary in the manufacturing process, each serving its distinct purpose [2].

The current CAM technology usually relies on geometric computations for toolpath generation, which often leads to the deviation of the generated toolpaths from the optimal perspective of manufacturing engineering. This software is utilized to program toolpaths and generate the necessary G-code to operate CNC machines. Production engineers can choose one of the standard toolpath methods (zig-zag, concentric, radial, etc.) available in machining sector [3]. Nevertheless, conventional toolpaths, which are developed based solely on geometric calculations without considering the mechanics of the processes, may not be optimal for machining all types of parts. Consequently, when compared to optimized paths, the cutting magnitudes become larger, increase the risk of tool damage, and can cause surface problems. It is crucial to optimize the toolpath in machining to achieve extended tool life and reduce the possibility of tool breakage. Additionally, optimizing the toolpath helps limit tool deflection and minimize surface problems on the machined part [4].

The complex geometries and parts can be manufactured thanks to the advances in CAM systems. Any manual control in CAM software can cause the generation of both productive and non-productive toolpaths based on workpiece geometry [5]. This produced toolpath is subsequently transformed into G-code through CAM software to be executed by CNC machines [6]. Optimized software is crucial in manufacturing as it enhances automation, resulting in time savings, error reduction, and improved precision and productivity throughout the manufacturing process [7].

Machine tools need various and complex control methods for their airtime, spindle speed, feed rate, depth of cut, path length, target materials, cutting processes, and so on. This can cause several issues such as the surface quality, tool lifetime, machining time, etc. [8]. These control processes require a considerable amount of energy, and a tool's power consumption changes dynamically during the process [9,10]. A critical topic in the literature is the prediction of the energy usage of CNC machine tools for optimal power consumption. This issue can be effectively utilized for monitoring, analyzing, and improving their usage according to their purpose [11].

Path planning is another active research area in the literature. Researchers working on the PP matter have been focused on geometric constraints, distance, cycle time, cutting conditions, part materials, jerk-limited feed rates, and collision avoidance [12]. Studies in this field have typically been conducted with a focus on single-factor improvements. With the active and precise application of intelligent technologies such as evolutionary algorithms (EA) and artificial neural networks (ANN) in manufacturing, the overall usability of the system is intended to be improved, leading to more favorable outcomes such as performance optimization and other positive attributes [13,14].

Numerous studies have explored the optimization of toolpaths in EAs, primarily using methods such as genetic algorithm (GA), particle swarm optimization (PSO), and artificial immune systems (AISs), etc. [13]. For instance, Cheng et al. [15] suggested employing GAs to determine optimal path and location planning for workpieces, aiming to reduce the time needed for robots to process these items. Additional research introduced an algorithm that minimized the amount of non-cutting time in the milling process via optimally dividing the toolpaths [4]. Consequently, GAs are recommended for optimizing the cutting parameters during machining processes [16,17].

The traveling salesman problem (TSP), used with the sequential ordering problem (SOP) to establish generalized precedence constraints, can also be found in the literature. The validation of toolpath optimization through the use of the TSP confirmed for the multi-cavity machining of intricate parts [18]. In the drilling process, it is demonstrated that many studies have utilized the TSP model to find an optimal path solution [19]. This leads to the minimization of path length during movement, which has been shown to be an effective strategy for sequencing issues in manufacturing [20]. However, numerous studies in the field of drilling toolpath optimization have mostly focused on simple shapes and hole arrangements [21].

Another study utilized AISs to enhance the surface smoothness of milling operations through determining optimal parameters and focusing on various factors such as the cutting depth, feed rate, and cutting speed [22]. Numerous studies have been performed in attempts to apply ant colony optimization (ACO). For instance, Ghaiebi et al. [23] aimed to minimize non-cutting airtime during hole-making processes, applying the ant colony algorithm. In addition, Wu et al. [24] adopted a modified ACO to achieve optimal machining parameters that help to reduce production costs.

Drilling operations modeled as a TSP have been addressed through the use of PSO on CNC machines. In [25], the PSO model was implemented with only a few control variables, providing a user-friendly and applicable model that contributed to reducing manufacturing costs. In another study, the applicability of PSO in defining the optimal process parameters for turning the machining is investigated. This analysis aimed to detect the best feed rate and tooling speed that would enhance the machining accuracy while simultaneously reducing the machining time and costs [26]. In the context of CNC end

milling optimization, the use of the PSO method is recommended to improve surface roughness [27].

Furthermore, PSO is utilized to choose the optimal parameters for reducing both the machining cycle time and the production costs [28]. In another study, a comparison of unit production cost reductions is performed between the PSO, GA, and SA methods [29]. Additionally, fully informed and advanced PSOs are introduced to the optimization of the toolpath planning of a five-axis milling machine process. The findings indicated that fully informed PSO is more effective in reducing the errors when compared to the PSO method [30].

Researchers have proposed various methodologies to improve the cycle time and energy consumption in order to achieve optimal trajectories for a high-speed CNC system design. Altintas and Erkorkmaz [31] presented a method for time-optimal feed rate scheduling, specifically tailored for the quantic spline toolpath. Mori et al. [32] introduced a time-optimal path planning technique that considers the second-order dynamics of feed drives during the traversal of contour curves. Dong et al. [33] suggested an algorithm to optimize the time allocation for jerk-limited feed rates along the parametric curves. Erkorkmaz and Heng [34] proposed an approach aimed at minimizing feed rate fluctuations and modulating the feed continuously through optimizing NURBS toolpaths.

Generating toolpaths for complex geometries requires additional caution to prevent collisions between the tool and workpiece, especially in high-speed multi-axis machines [35]. Therefore, it is crucial to explore the methods to optimize the toolpaths analytically [36]. In contrast to single-purpose improvements, studies concerning multi-objective optimization that simultaneously consider both the minimum toolpath length and the collision avoidance are limited and need to be explored for efficient and high-speed CNC systems. Zahra and Ali [37] focused on a new optimization model that considers these constraints for drilling processes only.

The NSGA-II is recognized for its robust optimization capabilities and successfully handling of various optimization problems [38–40]. The NSGA-II is a robust multi-objective evolutionary algorithm approach that has been tested across various real and simulated problems [41,42]. Research comparing the path smoothness between the NSGA-II and GA is provided in [43]. In a different study, Castillo et al. [44] analyzed the PP problem on a path, focusing on two different objectives, which were difficulty and path length. The authors applied a user-defined weight to path segments relying on the NSGA-II. NSGA-II and GA optimization methods are performed on the path clearance and path length. The NSGA-II model has been utilized to address the challenges presented in the models [45].

The contribution of this paper is to propose an evolutionary algorithm approach for a multi-objective optimal PP which minimizes the toolpath length and improves path smoothness for CNC part machining processes, while considering both simulation and experimental results. This study explores the advantages of efficiently using the NSGA-II to solve the PP problem of CNC machines. The developed algorithm is able to minimize path length and decrease the maximum turning angle of the non-cutting moves generated by the CNC machine. In this paper, PP trajectories are obtained in both simulation and experimental environments. The experimental results show that the non-dominated solutions obtained through the NSGA-II exhibit favorable parameters, revealing a shorter and smoother path option. The NSGA-II offers significant advantages over single-objective optimization methods due to its robust multi-objective problem-solving capabilities. Furthermore, this study distinguishes itself from others in the literature via implementing PP optimization processes on actual CNC machining data. MOGAs are also performed to evaluate the efficiency and suitability of the non-dominated sorting nature of the NSGA-II for this problem. Since the aim is to optimize paths for multiple objectives, it is preferred to find a solution in the framework of Pareto optimality between the objectives. However, the MOGA heavily relies on selecting shared functions, which can create significant selection pressure and ultimately result in premature convergence [46,47].

After introduction, details of the equations of the PP optimization problem in CNC systems from a design perspective are presented in Section 2. Section 3 describes the multi-objective PP optimization using the NSGA-II model. Comparative Pareto graphs, with the MOGA and the analysis of the effect of the NSGA-II on the travel distance and machining cycle time on the workpieces, are presented in both the simulation and experimental environments in Section 4. Lastly, the benefits and remarks of the offered method are presented and discussed in this paper.

2. Path Optimization Problem Description

The path planning process involves determining an optimized route from a starting point to an ending point, ensuring the avoidance of obstacles and maintaining smoothness, safety, and efficiency. The reduction of the non-cutting moves for a tool requires the connection of the toolpath regions in the shortest distance.

2.1. Traveling Salesman Problem

The toolpath optimization problem can be expressed using the TSP framework, where each toolpath is considered a city coordinate that requires a visit. In describing the toolpath, the decision variable k is employed, where $k_{ij} = 1$ indicates that the salesman (cutting tool) moves from city (machining) i to city j to the ending toolpath. Conversely, $k_{ij} = 0$ signifies that the cutting tool does not move from city i to city j within the total toolpath. The TSP problem, which is a minimization problem, can be formulated as follows [48]:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} k_{ij} \tag{1}$$

where c_{ij} represents the Euclidean distance from city i to j. To guarantee that each city j is visited a single time during the tour presented in (2), Equation (3) guarantees that the cutting tool departs from each individual city only once.

$$\sum_{i=1}^{n} k_{ij} = 1; \ \forall_j \tag{2}$$

$$\sum_{j=1}^{n} k_{ij} = 1; \ \forall_i \tag{3}$$

Finding the most efficient solution for TSP problems is not feasible, and heuristic algorithms are employed to obtain solutions that closely approximate the optimal solution within a relatively short running time.

2.2. Path Length

In machining, the cutting tool needs to travel a number of cities, n, in the shortest path. The path length between any two points in the x and y coordinates can be expressed using the Euclidean. Equation (4) calculates the distance for a path via connecting a sequence of points in a two-dimensional space, which is typically used to determine the shortest path that a tool must travel in applications like machining. Each point p_i in the sequence is represented as a coordinate (x_i, y_i). $D(p_i, p_{i+1})$ represents the total distance traveled from the starting point p_i to the endpoint p_{i+1}. x_i, and y_i are the Cartesian coordinates of the ith point. x_{i+1} and y_{i+1} are the Cartesian coordinates of the point following p_i.

Therefore, for two sequential points ($p_i = (x_i, y_i)$, $p_{i+1} = (x_{i+1}, y_{i+1})$), the path length connecting these points can be calculated. The total traveling distance can be obtained through summing all of the ordered path lengths as expressed by (5).

$$D(p_i, p_{i+1}) = \sum_{i=1}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{4}$$

$$L(p) = D(p_i, p_{i+1}) \tag{5}$$

It is shown in Figure 1 that the path length is minimized by the shortest operator. This operator removes extra orange lines to obtain the shortest path. The updated lines in the new toolpath are illustrated in Figure 1b.

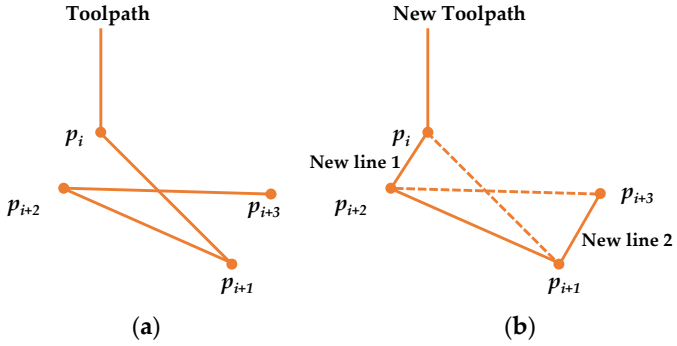


Figure 1. Toolpath planning: (**a**) The original toolpath, (**b**) the newly generated toolpath considering the path length optimization.

PP also includes smoothness optimization. This is essential to validate it thoroughly, as any failure in the tool could result in significant damages, putting the rest of the mission at risk [49]. Path smoothness is a critical consideration in designing algorithms for PP to guarantee the safe navigation of robots or CNCs. It involves evaluating and optimizing the path to prioritize smoothness trajectories while achieving the desired objectives, such as minimizing travel time or energy consumption.

2.3. Path Smoothness

The smoothness of a path states how curved the path is. *A* smoother path results in lower energy consumption for the motion trajectory. The smoothness of a curve is described by three consecutive points, p_i, p_{i+1}, and p_{i+2}, using the angle between the two lines formed by these points. The smoothness of the path is quantified via calculating the average turning angle, which can be computed using (6), where the angle between two successive lines defines a corner. This equation uses the cosine rule to determine the angle. The cosine of the angle is computed by taking the dot product of the vectors $p_{i+1} - p_i$ and $p_{i+2} - p_{i+1}$, and then dividing this by the product of the magnitudes of these vectors. This relates the geometric layout of the points to their angular relationship.

$$A[p_i, p_{i+1}, p_{i+2}] = \pi - \cos^{-1}\left(\frac{(x_{i+1} - x_i)(x_{i+2} - x_{i+1}) + (y_{i+1} - y_i)(y_{i+2} - y_{i+1})}{D(p_i, p_{i+1}).D(p_{i+1}, p_{i+2})}\right) \tag{6}$$

In this expression, $p_i = (x_i, y_i)$ represents the ith point in the curve, and $D(p_i, p_{i+1})$ and $D(p_{i+1}, p_{i+2})$ denote the distance between points p_i and p_{i+1} and p_{i+1} and p_{i+2}, respectively. In addition, $\cos^{-1}$ is the inverse cosine function. The overall smoothness of the sequence is then calculated as the average of these angles:

$$S(p) = \frac{1}{n}\sum_{i=0}^{n-1}\{A[p_i, p_{i+1}, p_{i+2}]\} \tag{7}$$

The smoother the curve, the closer the angle will be to 180° (straight line). Here, the smoothness is improved by minimizing the difference in the arithmetic mean between the angles and 180° for the n number of traveling points, which is given as $S(p)$. Smaller average angles imply sharper turns and less smooth paths, while larger average angles indicate smoother transitions between points. This measure can be used to optimize paths for applications requiring minimal sharp turns, which is beneficial in machining.

A smoother path can be achieved through the path smoothness objective function. In this function, the largest angle of rotation between two sequential lines on the path is reduced. Two sequential lines are removed to create a new line, thereby reducing the maximum angle on the path. Figure 2 shows the concept of path smoothness in toolpath planning. On the left-hand side, the original toolpath planning is illustrated, while, on the right-hand side, two consecutive lines, where the largest angle of rotation is positioned, are removed to create a new line, represented by dashed lines. It demonstrates the newly generated toolpath.

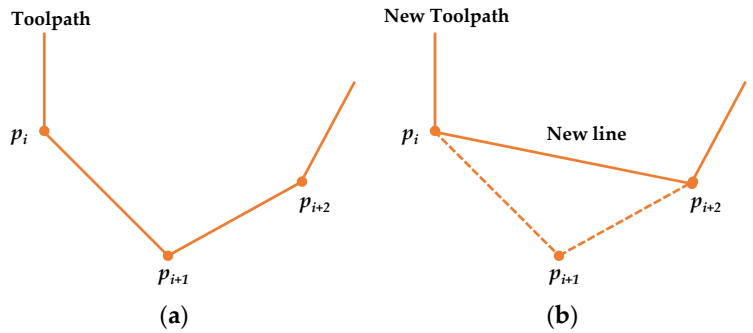


Figure 2. Toolpath planning: (**a**) the original toolpath, (**b**) the newly generated toolpath considering the path smoothness optimization.

2.4. Multi-Objective Functions and Weight Factors

Typically, multiple objectives may present conflicting goals, meaning that improving one objective may require sacrificing one or more other objectives. This scenario represents a multi-objective optimization problem, where there is not a single optimal solution, but rather a collection of solutions between various objectives. Furthermore, the solutions are non-dominated, signifying that, if one or more objectives of a solution are superior to another, it is necessary that there are at least one or more inferior objectives. Recently, sophisticated evolutionary algorithms were proposed to overcome PP problems.

The CNC PP process involves numerous variables. This paper focuses on optimizing the toolpath length and smoothness as they play a crucial role in achieving the desired machining results. The thoughtful selection of these two parameters significantly impacts both the energy usage and the machining performance, making them pivotal optimization variables.

The PP has defined an optimization problem involving a sequence point, denoted as p. These multi-objective functions should be minimized. Currently, PP formulation can be expressed through the following:

$$p = [p_0, p_1, p_2, \ldots, p_n, p_{n+1}] \tag{8}$$

The length of the sequence, denoted as $L(p)$, is minimized as given in (9). This typically involves minimizing the total distance or path length between consecutive points in the sequence. The smoothness of the sequence is improved by minimizing $S(p)$, as shown in (9). This is calculated as the average of angles formed by three consecutive points in the sequence, as described in the previous response.

The overall optimization problem is computed as follows:

$$F(p) = Min(\omega_1.L(p) + \omega_2.S(p)) \tag{9}$$

Here, ω_1 and ω_2 are the weighting factors that determine the relative significance of the objective functions as compared to the others. The specific values of these weights would depend on the specific goals and priorities of the optimization problem. The weighted sum approach consolidates all of the multi-objective functions into a single scalar composite objective function by employing weighted summation [50,51].

In this study, to solve the multi-objective model using the NSGA-II, different target weight factors will result in different solutions during the two objective weight determination processes. The weighting factors are determined through considering the relationship between optimization objectives and their variables. The weighting factor values for the relevant objective functions are shown in Table 1. These preferences rely on the specific needs and goals of the optimization problem.

Table 1. ω weighting factors of each objective function.

ω Weighting Factors	Related Objective	Value
ω_1	Path Length	0.9
ω_2	Path Smoothness	0.1

In the optimization process, weighting factors are crucial in determining the importance of each objective relative to others. In this case, ω_1 and ω_2 are the weighting factors assigned to path length and path smoothness, respectively. The value of ω_1 is 0.9, indicating that the path length is significantly prioritized in the optimization process. This proposes a strong emphasis on minimizing the total distance of the toolpath, which can lead to reduced machining time and potentially lower tool wear on the machinery. On the other hand, ω_2 has a value of 0.1, showing that the path smoothness is less prioritized but still considered. Smoother paths can reduce the occurrence of the sharp turns or irregular movements, enhancing the quality of the surface form and prolonging the life of the cutting tool. Balancing these factors allows for an optimization that not only focuses on efficiency, but also maintains an acceptable level of quality in the final output.

2.5. Study Conditions and Implementation of the Model

The conditions of this study concerning toolpath planning focus on optimizing both the path length and path smoothness. The main objectives are to minimize the total travel distance of the non-cutting tool and to ensure a smooth path to reduce energy consumption and avoid sharp turns that could lead to tool wear or damage.

This study assumes the following conditions and steps:

- The workspace is a two-dimensional plane where each point is represented by Cartesian coordinates (x_i, y_i).
- Determine the sequence of points (p_i) that the cutting tool must travel through.
- The cutting tool needs to travel through a set of predefined points in a specific sequence.
- The Euclidean distance calculation is used to measure the path length between each pair of consecutive points, as defined in (4).
- Find the sum of these distances to get the total path, as defined in (5).
- The total path length should be minimized to reduce the machining time and energy consumption without compromising the path smoothness.
- Three consecutive points, (p_i, p_{i+1}, and p_{i+2}) are used to calculate the angle θ between the segments using the cosine rule, as defined in (6). Path smoothness is evaluated based on the angles between the successive segments formed by these points.
- Compute the average angle for the entire path to quantify smoothness. A smoother path can be obtained via (7) to obtain the angles closer to 180°.
- Introduce weighting factors ω_1 and ω_2 for path length and path smoothness, respectively, to balance their importance in the optimization process.
- The objective function can be expressed using (9).
- The values of ω_1 and ω_2 can be adjusted based on the specific requirements of the application. For instance, if minimizing the travel distance is more critical, a higher value can be assigned to ω_1. Conversely, if a smoother path is prioritized, ω_2 can be increased.
- Adjust the sequence of points and recompute the path to minimize the objective function.

- Iterate the process to find an optimal balance between the shortest path and the smoothest path based on the chosen weighting factors.
- Apply the optimized toolpath to the CNC machine.
- Test the path on a sample workpiece to ensure it meets the desired criteria of minimal travel distance and smoothness.

By following these steps, the model helps in achieving an optimal toolpath that reduces machining time, energy consumption, and tool wear, thereby enhancing the efficiency and quality of the machining process. The weighting factors provide flexibility in balancing the trade-off between path length and smoothness according to specific application needs.

3. NSGA-II-Based PP Optimization Model

In this study, a meta-heuristic-based planner is designed to generate a trajectory comprising advanced and rotated primitives. Initially, the PP file is globally optimized for length and smoothness using the NSGA-II in the first step. The fitness function employed in optimization prioritizes the generation of paths characterized by shortness and smoothness.

In this section, the NSGA-II is presented as an optimization tool, and subsequently, the optimized procedures guided by the NSGA-II technique are described to generate an initial feasible path for the second part of the PP. The goal of this stage is to establish a short and feasible path using the NSGA-II algorithm. This path is systematically created by identifying a sequence of nodes derived from a set of geometrically random configurations within the scenario's free space. The selection of nodes for the feasible path is determined through a new generation process employing the NSGA-II.

Regarding the application of the NSGA-II, the optimization goal in theory is determined by two main factors when the manufacturing conditions are determined: path length and path smoothness. The correct selection of these two elements significantly impacts energy consumption and machining quality, making them the primary optimization variables.

The NSGA-II has demonstrated its capability to preserve various solutions and achieve optimization more effectively than other multi-objective optimization models [52]. The NSGA-II is a well-structured optimization model and a recognized algorithm with its popularity and maturity. The NSGA-II stands out as a commonly used multi-objective genetic algorithm, offering advantages such as reduced complexity when compared to the well-classification genetic algorithm, along with a fast execution speed and an effective solution process [53–56].

The NSGA-II algorithm operates the following steps:

1. An initial population, $P(0)$, (POP) of size, S, is arbitrarily created. Then, the non-dominated sorting with a child population, $Q(0)$, is attained, initializing $g = 0$.
2. The two generations, $P(g)$ and $Q(g)$, are associated to generate a new population $R(g)$.
3. The crowding distance (CD) of the individuals is determined while the non-dominated sorting of the population, $R(g)$, is carried out. The most suitable individual is then chosen based on the order value and CD to create a new parent set, $P(g + 1)$.
4. The new parent set $P(g + 1)$ is designated and mutated to generate a new children population, $Q(g + 1)$.
5. The algorithm checks if the stopping criteria has been met. If not, g is incremented by step 1, and the process returns to step 2.

The initial steps, from 1 to 2, involve the initial procedures, which include the generation and evaluation of the initial population $P(0)$. Additionally, a new population set, $R(g)$, is created to retain the non-dominated solutions discovered during the algorithm execution. Step 4 is the crucial stage of the algorithm, representing the main iteration that continues until the stopping criteria is met.

During the iteration, two new populations, namely $P(g)$ and $Q(g)$, are established. $P(g)$ is utilized to keep designated individuals from the population, $P(0)$, while $Q(g)$ keeps the offspring individuals. The selection step is utilized to determine the population from $P(0)$, and the chosen individuals are stored in $P(g)$. These individuals in $P(g)$ are paired, and offspring individuals are created via the crossover method, subsequently being incorpo-

rated into child population $Q(g)$. Various methods are executed for each individual within $P(g)$, resulting in the generation of new individuals stored in $Q(g)$. After applying genetical steps to the individuals in $P(g)$, the children population $Q(g)$ is calculated, and possible individuals are added to $R(g)$. Then, the individuals in the $R(g)$ population sets are updated, retaining non-dominated solutions while excluding others.

Following this, the elitist NSGA-II approach is implemented. To categorize the individual sets based on feasibility into sets of feasible and infeasible solutions, the $P(0)$ and $Q(g + 1)$ populations are combined. Non-dominated sorting and CD are applied to these sets, and individuals are selected to create a new $P(0)$. Infeasible individuals are not completely discarded in this stage. Instead, superior individuals are retained. The size of the $P(0)$ remains unchanged, and the counter of the loop in the algorithm is incremented by one.

Steps 4 and 5 are repeated until the stopping criteria is met, thereby completing the entire algorithm. At this point, the set $Q(g + 1)$ is determined as the output. Additionally, a detailed introduction of operators is provided. These evolutionary operators, designed to optimize the path length and smoothness beyond traditional methods, are presented with the flowchart of these operators and processes in Figure 3. These operators generate an offspring individual, which is stored in the child population $Q(g)$.

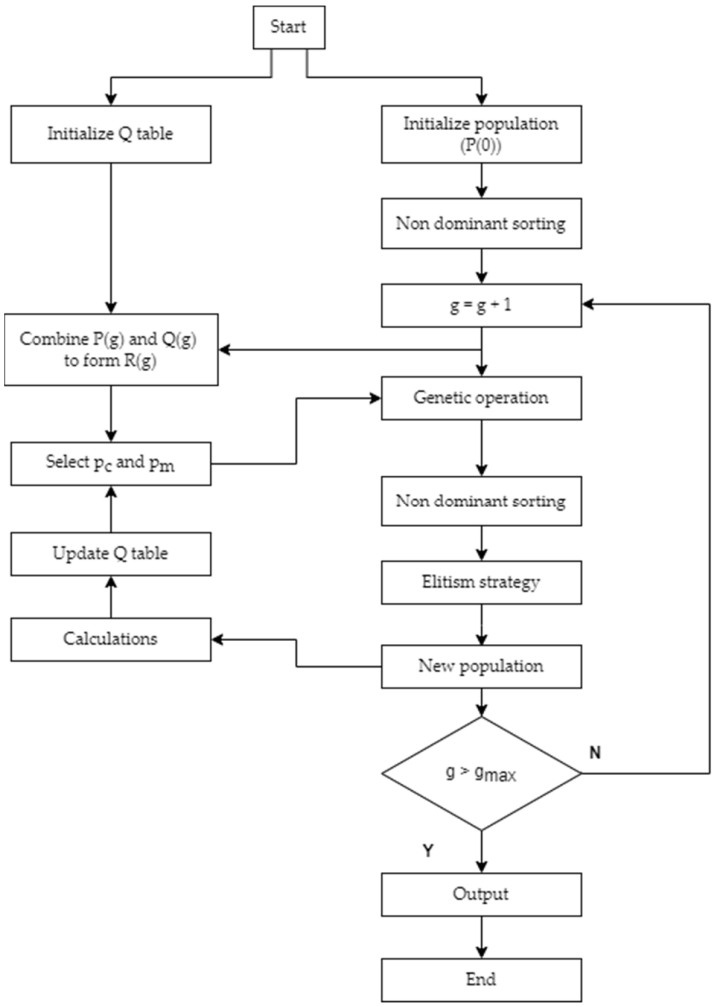


Figure 3. Flowchart of finding the shortest path through the application of the NSGA-II.

4. Simulation Results and Experimental Validations

The post-processing functionality within the study directory facilitates the generation of various outputs from the offline PP process, thus allowing for a comprehensive examination of the performance of the NSGA-II-based optimal path planner.

Figure 4 illustrates the approximate Pareto optimality. The red dot represents the optimal reference point. Orange triangles and blue dots denote the solutions attained by the optimized NSGA-II and MOGA, respectively. In non-dominated results, the corner points represent the solution nearest to the optimal reference point. According to the comparative simulation results, the NSGA-II exhibits a feature as good as the MOGA does. Furthermore, the non-dominated results provided by the NSGA-II are more intense. Additionally, it can be observed from the figure that the best possible results can be presented by the nearest points to the optimal reference point.

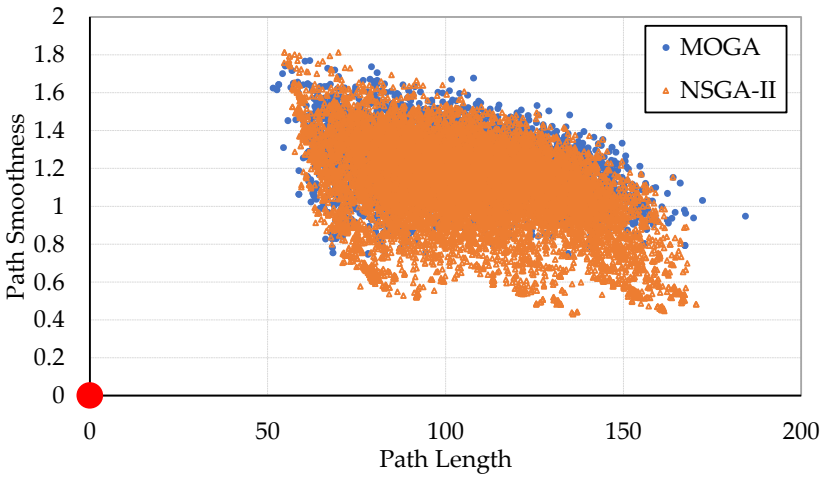


Figure 4. Presentation of the Pareto optimal solution formed by the NSGA-II and MOGA algorithms.

In this study, the NSGA-II is utilized for toolpath optimization. The NSGA-II integrates the CD metric to ensure diversity in the solution space, thus achieving more efficient cutting paths. The outcomes of the NSGA-II are discussed to better understand its results. CD, a strategy of the NSGA-II [41,57], is employed at the Pareto front to ensure a diversity of solutions. This is achieved through assessing the distribution of solutions near each population, which serves as an indicator of diversity within the Pareto optimal solutions. CD is calculated via summing the normalized distances between each individual and its two nearest neighbors across all objectives [38]. The solution that registers the highest crowding distance is considered the least crowded. This method helps in the optimization of cutting paths for workpieces with complex geometries via balancing the development of solutions with regard to both quality and efficiency. The usage of CD ensures that the solution population covers a broad spectrum of solutions, thereby increasing both the speed and accuracy of the cutting paths. This approach leads to cost and time savings in the production processes while maximizing the quality of the workpiece.

4.1. Analysis of the Impact of PP on the Distance between Cuts in CNC Processes

The effectiveness of optimizing PP using the NSGA-II algorithm to reduce the distance between cuts in CNC machining processes is validated with experimental case studies. Throughout our experimentations, consistent parameter settings are maintained for the NSGA-II, as outlined in Table 2.

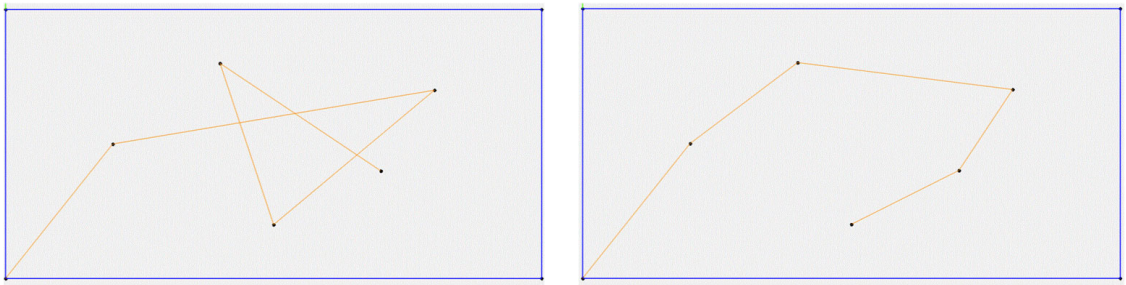
NSGA-II parameter values are given in Table 2. N refers to the number of individual solutions maintained in a population at any given time during the execution of the algorithm. G indicates the total number of cycles that the algorithm performs to evolve the population from one generation to the next. Each generation represents an iteration of the optimization process. n_c influences the spread of offspring around the parent solutions. It determines how tightly the offspring are clustered around the parents. p_c is the probability that crossover will occur during the reproduction process. n_m, similar to the crossover distribution index, controls the spread of mutations applied to an offspring. It affects how mutations are distributed across the chromosome of a solution. p_m defines the probability

of a mutation occurring in an offspring. Mutations introduce random variations into the genetic material, which helps to maintain genetic diversity and potentially discover new and better solutions.

Table 2. NSGA-II parameter values.

Property	Symbol	Value
Number of populations	N	300
Generation size	G	500
Crossover index	n_c	5
Crossover rate	p_c	0.9
Mutation index	n_m	20
Mutation rate	p_m	0.1

Figure 5 illustrates a rectangular workpiece with multiple holes. It can be seen from the figure that the toolpath optimization for holes drilled into a rectangular part is conducted using the NSGA-II algorithm, resulting in a shortened and smoothed path. The algorithm updated the path through shortening the distance to the necessary hole coordinates, tending to draw a smooth path by avoiding the sharp corners as much as possible. Significant improvements are achieved at the end of the optimization. The original toolpath length of 272.4 mm is reduced to 188.7 mm, representing a reduction of approximately 30%. This demonstrates the effectiveness of the NSGA-II in enhancing the efficiency of the machining processes through optimizing the movement and trajectory of the toolpaths.



(**a**) (**b**)

Figure 5. A rectangular workpiece with multiple holes; (**a**) the original toolpath, (**b**) the optimized toolpath.

This approach ensures reliable the comparative analysis and robust evaluation of the proposed PP methodology. Simulations are carried out using a computer with Intel i7 13th Gen Processor 20 Cores, resulting in an execution time of 8.72 s in the Python 3.10.12 simulator.

Workpiece 1 and 2 utilize both non-cutting and machining moves while being processed on CNC machines. The machining path is depicted with blue lines, while the non-cutting moves are represented with orange lines. The original PP for Workpiece 1 is shown in Figure 6. It can be seen from the figure that the PP is abundant in non-cutting moves.



Figure 6. The original PP for Workpiece 1. The machining path is shown with blue lines, while the non-cutting moves are rep-resented with orange lines.

Figure 7 illustrates the optimized PP for Workpiece 1. It can be seen from the figure that a significant improvement of 29.29% has been achieved in the non-cutting distances. The distance is reduced from 5678.4 mm to 4015.2 mm. This results in 1663.2 mm of travel distance being saved. The experimental results of the non-cutting distance for Workpiece 1 are presented in Table 3.



Figure 7. The optimized PP for Workpiece 1. The machining path is shown with blue lines, while the non-cutting moves are represented with orange lines.

Table 3. Experimental results of the non-cutting distance for Workpiece 1.

Versions of Workpiece 1	Non-Cutting Distance
Original	5678.4 mm
Optimized	4015.2 mm

Moreover, a new and different workpiece (Workpiece 2) is used to see the effectiveness of the performed approach (Figure 8). It can be seen from Figure 9 that the optimized PP for Workpiece 2 has reduced the non-cutting distances from 4703.6 mm to 3203.2 mm. This results in 1500.5 mm of travel distance being saved. As shown in Table 4, a significant improvement of 31.9% in the non-cutting distance is achieved.

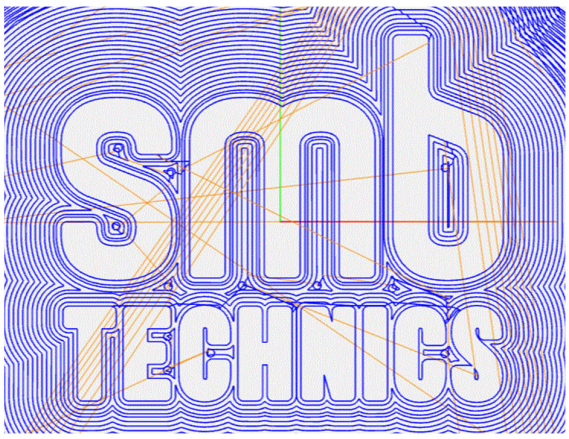


Figure 8. The original PP for Workpiece 2. The machining path is shown with blue lines, while the non-cutting moves are represented with orange lines.

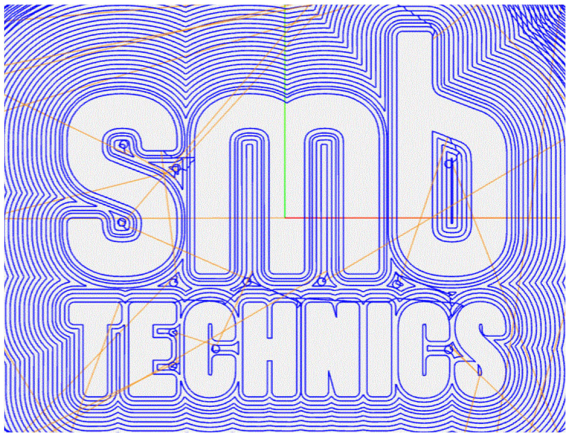


Figure 9. The optimized PP for Workpiece 2. The machining path is shown with blue lines, while the non-cutting moves are represented with orange lines.

Table 4. Experimental results of the non-cutting distance for Workpiece 2.

Versions of Workpiece 2	Non-Cutting Distance
Original	4703.6 mm
Optimized	3203.2 mm

This optimization process not only enhances the efficiency of CNC machining operations, but also contributes to reducing machining time and material wastage. The findings highlight the potential of leveraging PP optimization techniques to enhance manufacturing processes and ultimately improve productivity in CNC machining applications.

4.2. Analysis of the Impact of PP on the Machining Cycle Time in CNC Processes

4.2.1. Experimental Study Conditions

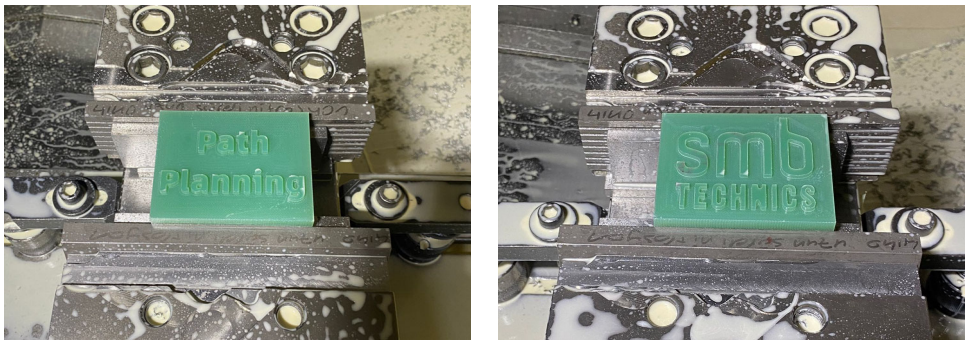
The effectiveness of optimizing PP using the NSGA-II algorithm to reduce the non-cutting distances in CNC machining processes has been demonstrated in the previous subsection. Here, both optimized and original versions of Workpiece 1 and Workpiece 2 are manufactured on a Trident TR60A model CNC machine (Taiwan). The technical parameters of the Trident TR60A CNC are provided in Table 5.

Table 5. Parameters of the CNC machine used in the study.

Items	Specifications	Value
Travel	x-travel	600 mm
	y-travel	400 mm
	z-travel	500 mm
Table	Table size	700 × 420 mm
Spindle	Speed	8000 rpm
	Max. Tapping Speed	3000 rpm
Feed Rate	Rapid Speed (G00)	48/48/48 m/min
	Cutting Feed Rate (G01)	10,000 mm/min
Motor	x-axis	1.5 kW
	y-axis	1.5 kW
	z-axis	3 kW
	Spindle	7.5 kW

4.2.2. Experimental Results

The pictures of workpieces during the manufacturing on the CNC machine are illustrated in Figure 10. Their machining cycle times during the manufacturing are compared in Table 6. It can be observed that, for Workpiece 1, the cycle time is reduced from 1692 s to 1573 s, saving approximately 119 s. This represents a significant improvement of approximately 7% in the machine cycle time, as provided in Figure 11.



(**a**) (**b**)

Figure 10. The pictures of workpieces during the manufacturing on the CNC machine: (**a**) Workpiece 1 and (**b**) Workpiece 2.

Table 6. Experimental results of the machining cycle time for Workpiece 1 and Workpiece 2.

Workpiece Versions	Cycle Time
Original Workpiece 1	1692 s
Optimized Workpiece 1	1573 s
Original Workpiece 2	1518 s
Optimized Workpiece 2	1416 s

Additionally, the same procedures were applied to Workpiece 2, resulting in a similar reduction in the machine cycle time. The decrement for the optimized Workpiece 2 is 102 s. This represents a significant improvement of approximately 6.72% in the machine cycle time, as shown in Figure 12.

As demonstrated in the experimental results of this study, the initial toolpaths contained unnecessary turns and extended distances. To make these paths more efficient, the NSGA-II with an elitist strategy has been examined. The NSGA-II optimizes the trajectory through reducing travel distances and simplifying the path. Consequently, as shown in the second optimized set of figures, a smoother and more direct route is achieved. The optimization process involves defining the path as a sequence of points that should be

connected in the most efficient way possible, aiming to minimize the overall travel distance while complying with the operational constraints. The objective function used for this optimization included factors such as the Euclidean distance between consecutive points and the angular deviation at each point, which facilitated a smoother transition between line segments. The results are quantified through comparing the total path lengths before and after optimization and through observing a significant reduction in angular changes. This ensures that the tool moves efficiently between points by reducing the path complexity.

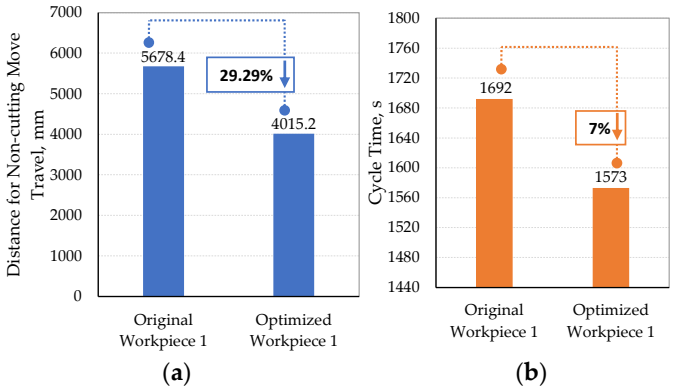


Figure 11. Comparison between original Workpiece 1 and optimized Workpiece 1 considering the (**a**) non-cutting distance and (**b**) cycle time.

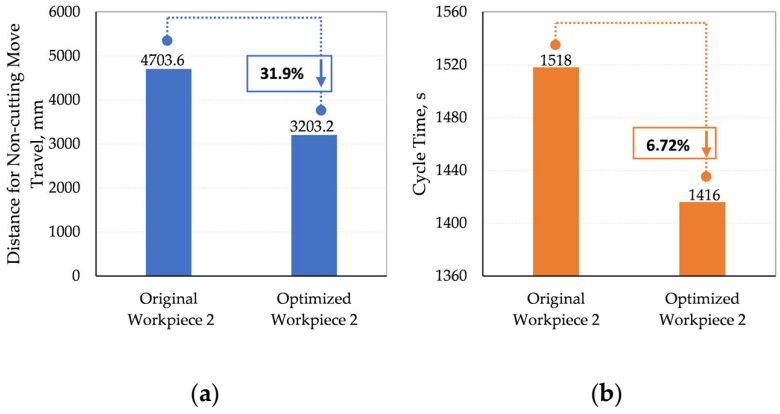


Figure 12. Comparison between original Workpiece 2 and Optimized Workpiece 2 considering the (**a**) non-cutting distance and (**b**) cycle time.

5. Conclusions

This study explores an evolutionary algorithm method for a multi-objective optimal PP which minimizes the toolpath length and improves the path smoothness for CNC part machining processes, while considering statically known environments. Results from the study demonstrate the excellent characteristics of the non-dominated solutions created by the NSGA-II with elitism in the PP. The developed algorithm is able to reduce the path length and the maximum turning angle during the non-cutting movements of a CNC machine. In the NSGA-II framework, genetic operators are offered to expedite the progress of individuals, facilitating the generation of paths that are shorter and smoother. The algorithm's parameters are systematically investigated to enhance its performance. PP trajectories have been analyzed through simulations and experimental settings. The experimental outcomes show that the non-dominated solutions created by the NSGA-II are characterized by advantageous parameters, offering options for paths that are both shorter and smoother.

Furthermore, the objectives, namely the path length and smoothness, are compared in the Pareto optimality of the NSGA-II and MOGA. The NSGA-II algorithm demonstrated

compatible results with the MOGA, indicating that both types of genetic algorithms are effective for the toolpath planning problem. However, the technique deliberated here entails an examination involving the individuals. While the MOGA approach can reduce computational costs, the solutions generally converge toward a specific point during the examination step, potentially ending up as a loss of solution diversity.

Simulation results have demonstrated that the NSGA-II, verified with a MOGA based on Pareto optimality, successfully optimizes the toolpath planning for two different workpieces, while minimizing both the path length and cycle time. According to the experimental results, the NSGA-II exhibits consistent performance across different terrain representations and achieved good performance for the multi-objective PP problem. Analyses conducted using G-code manipulation and PP optimization revealed significant improvements in the machining cycle time.

The results of the experimental studies show that PP optimization using the NSGA-II highlights its shorter and smoother qualities, with the optimal trajectory leading to reductions of approximately 30% in the path length and 7% in the machining cycle time.

It can be highlighted that the current algorithm's operator speed restricts its applicability to offline PP. So, in future research, exploring innovative methods for improving the execution time in the NSGA-II can be considered. Additionally, the study does not consider the robot's kinematics. The literature review provided in this study reveals the importance of PP techniques in CNC machining processes, and provides a possible roadmap for future research. Moving forward, further improvements and efficiency gains can be attained using real-time PP problems and adaptive control systems.

Author Contributions: Conceptualization, G.D. and R.A.V.; methodology, G.D.; software, G.D.; validation, G.D. and R.A.V.; formal analysis, G.D.; investigation, G.D.; resources, G.D.; data curation, G.D.; writing—original draft preparation, G.D. and R.A.V.; writing—review and editing, G.D. and R.A.V.; visualization, G.D.; supervision, G.D. and R.A.V.; project administration, G.D. and R.A.V. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are provided within the article.

Conflicts of Interest: Author Gamze Demir was employed by the company R&D Department. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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