

Research and Development

The discussion on economic growth suggests that positive long-run growth implies that knowledge (information) always grows. However, education and R&D (research and development) are the only reliable sources of constant growth of knowledge. Motivated by this observation, this note focuses on the value of R&D according to three different perspectives: the society, a monopolist firm, and competitive firms.

The Economy

Let us analyze a simple economy. In this economy, consider a market where the demand function is linear:

$$Q = a - bP.$$

Q is the demand by the consumers when the price of the good is P . The given constants a and b are positive and arbitrary.

The marginal cost of production is c . Assume that marginal cost is initially high denoted by \bar{c} . But R&D reduces the marginal cost of production to \underline{c} by technological progress. Of course we assume $\bar{c} > \underline{c}$. This is called “process innovation”. So we do not analyze “product innovation” in this note.

Remark: The reduction of marginal cost from \bar{c} to \underline{c} is interpreted as technological progress or innovation.

Now let us see how much value is attributed to the decrease in marginal cost from \bar{c} to \underline{c} according to three different perspectives: (i) Monopoly, (ii) Competitive firms, (iii) Society.

The Monopoly

Of course, the values attributed to technological progress by the society and firms are different. To see how much value firms attribute to technological progress, first suppose there is a single firm in the market. So we consider the case of monopoly now. The competitive case will be analyzed later on.

Remark: Never memorize the mathematical expressions below. My experience clearly shows that students who memorize these expressions always fail in their exams. Try to understand the economic mechanism.

The profit of the monopolist is

$$\pi = (P - c)Q$$

But this is equal to

$$\pi = (P - c)(a - bP)$$

since the demand is

$$Q = a - bP.$$

In order to maximize π , the firm should solve

$$\frac{d\pi}{dP} = a - 2bP + bc = 0$$

whose solution in price P is

$$P^M(c) = \frac{a + bc}{2b}.$$

This is the optimal pricing rule for a monopoly. It tells us the profit maximizing price when the marginal cost of production is c . Now let us put this optimal price into the profit π to see

$$\pi = (P^M(c) - c)(a - bP^M(c)) = \left(\frac{a - bc}{2b}\right)\left(\frac{a - bc}{2}\right) = \frac{(a - bc)^2}{4b}.$$

This is how much the monopolist earns when the marginal cost is c . This means we can calculate the increase in profit due to innovation. The monopolist earns

$$\pi = \frac{(a - b\bar{c})^2}{4b}$$

when the marginal cost is high and

$$\pi = \frac{(a - b\underline{c})^2}{4b}$$

when the marginal cost is low after innovation. Therefore, the increase in profit from a reduction in marginal cost (i.e. technological progress) is

$$\frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{4b}.$$

Therefore, this increase in profit is also the value attributed to the reduction in marginal cost by the monopoly. So let us write

$$V^M = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{4b}$$

which is the value of technological progress from the perspective of a monopolist.

Competitive Firm

Now let us analyze how competitive firms value technological progress. Due to competitiveness, price is equal to marginal cost and profit is zero before innovation. But if a single firm finds a way to lower its marginal cost then it can charge a lower price. In this case, no other firm can compete with this firm making the low-cost high-technology firm a monopoly. Now we shall see how much extra profits a competitive firm earns from being a monopolist by technological progress.

Assume the marginal cost is \bar{c} for all firms which are perfectly competitive. So there is no innovation yet. In this case

$$P = \bar{c} \text{ and } \pi = 0.$$

That is to say, the price is equal to marginal cost and profits of all firms are zero. Why? When the price of a firm is higher than \bar{c} , its customers would be stolen by another firm offering a slightly lower price. When a firm charges a price lower than \bar{c} , it makes losses. So, either case, $P = \bar{c}$. But this also ensures zero profit since the price of each unit of good is equal to the cost of production: there is no extra money left to firms.

Now suppose a firm improves the technology and reduces its own marginal cost to \underline{c} . The marginal cost of all other firms is still \bar{c} . Therefore, other firms cannot offer prices lower than \bar{c} . In this case the competitive firm with the superior technology chooses its price as

$$P^C = \min\{\bar{c}, P^M(\underline{c})\}.$$

Recall that $P^M(\underline{c})$ is the monopoly price after innovation and \bar{c} is the lowest price that low technology firms can offer. Now let us see why the competitive firm with a superior technology would choose the minimum of $P^M(\underline{c})$ and \bar{c} .

First of all, of course, the high-technology firm would like to set its price to the monopoly price

$$P^M(\underline{c}) = \frac{a + b\underline{c}}{2b}$$

that we discussed earlier. But this is possible only when other firms cannot compete with this monopoly price. The mathematical condition for monopoly price to be sufficiently competitive for other firms is

$$P^M(\underline{c}) = \frac{a + b\underline{c}}{2b} \leq \bar{c}.$$

This inequality says other firms would find with the monopoly price too competitive since their marginal cost is very high. In this case, the firm with a superior technology choose its price as

$$P^C = P^M(\underline{c}).$$

This case is called the “drastic change” because there is a sudden fall in prices due to innovation. But what if

$$P^M(\underline{c}) = \frac{a + b\underline{c}}{2b} > \bar{c}$$

is true? When this inequality holds, the monopoly price exceeds the marginal cost of other firms. So the low-cost high-tech firm would lose its market to competitive firms if it chooses $P^M(\underline{c})$. So it must offer a lower price than $P^M(\underline{c})$. This reasoning shows that the price of the monopolist decrease until $P^C = \bar{c}$. This is called “non-drastic case” because there is no change in market prices despite technological progress. The monopolist still captures all the market. The optimal price choice of the competitive firm with a superior technology in both cases can be summarized as

$$P^C = \min\{\bar{c}, P^M(\underline{c})\}.$$

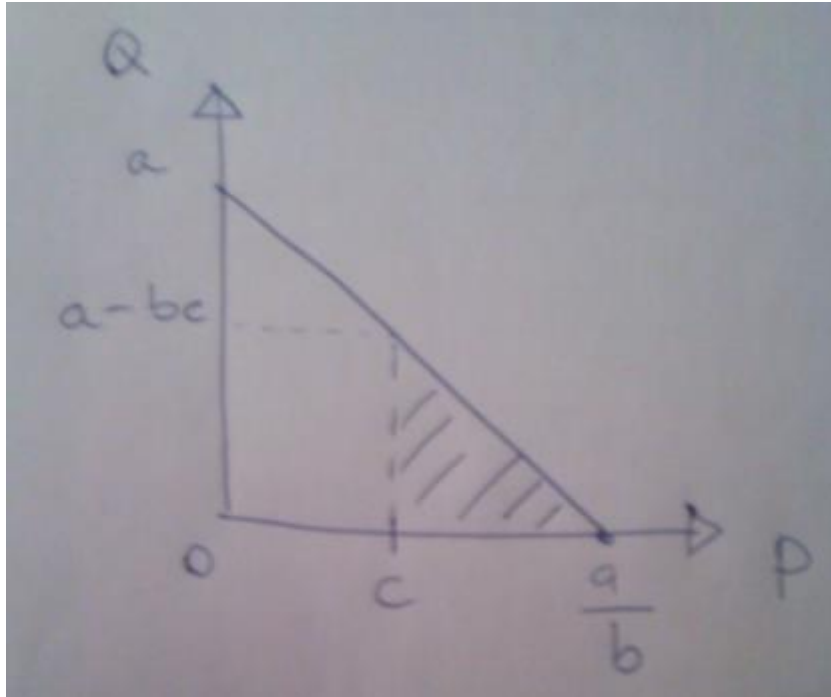
The value attributed to the reduction in marginal cost by the competitive firm is

$$V^C = (P^C - \underline{c})(a - bP^C).$$

Society

Assume that the society only cares about the consumer surplus. Therefore, the value of technological progress is the increase in consumer surplus due to lower costs of production.

Let us see how the consumer surplus should be computed when the marginal cost of production is c . The downward sloped line in the below figure is the demand function $Q = a - bP$ and the shaded area is the consumer surplus.



The shaded area is

$$S(c) = \frac{(a - bc)^2}{2b}$$

due to the Pythagoras Theorem (Pisagor Teoremi).

Now we can compute the consumer surplus when the marginal cost is \bar{c} and \underline{c} . The difference between these two surpluses gives us the value of technological change for the society. In particular,

$$S(\underline{c}) - S(\bar{c}) = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b}$$

is the increase in consumer surplus due to technological progress. Hence, the value attributed to technological progress by the society is

$$V^S = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b}$$

since this is how much technological progress increases consumer surplus.

Comparison

Now we can compare the value attributed to technological progress according to the society, monopoly, and competitive firms. More specifically we shall see that

$$V^S > V^C > V^M > 0.$$

Remark: This means the highest value to technological progress is given by the society and the lowest value is given by the monopoly. The competitive value of technological progress is always in between.

Let us see some numerical examples of this ordering.

Example: Suppose that initially, the parameters of the economy are

$$a = 100, b = 1, \bar{c} = 60.$$

Now assume that technological progress reduces the marginal cost down to

$$\underline{c} = 10.$$

Let us see how much the society values this reduction in marginal cost:

$$V^S = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b} = \frac{90^2 - 40^2}{2} = 3250.$$

The value of this technological progress for a monopolist is

$$V^M = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{4b} = 1625.$$

The value of the technological progress for a competitive firm depends on whether the price change will be drastic or not. Since

$$\frac{a + b\underline{c}}{2b} - \bar{c} = 55 - 60 = -5 < 0$$

the price change is drastic.

Remark: This means other firms with high marginal cost cannot compete with the monopoly price with low marginal cost.

So the value of technological progress for a competitive firm is

$$V^C = \frac{(a - b\underline{c})^2}{4b} = \frac{8100}{4} = 2025.$$

Conclude that

$$V^S > V^C > V^M.$$

Example: Suppose that initially, we have

$$a = 10, b = 1, \bar{c} = 4$$

but technological progress lowers the marginal cost down to

$$\underline{c} = 2.$$

Let us see how much the society values this reduction in marginal cost:

$$V^S = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b} = \frac{8^2 - 6^2}{2} = 14.$$

The value of this technological progress for a monopolist is

$$V^M = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{4b} = 7.$$

The value of technological progress for a competitive firm depends on whether the price change will be drastic or not. Since

$$\frac{a + b\underline{c}}{2b} - \bar{c} = 6 - 4 = 2 > 0$$

the price change is non-drastring. Hence the value of technological progress for a competitive firm is

$$V^C = (\bar{c} - \underline{c})(a - b\bar{c}) = 2 \times 6 = 12.$$

Again, conclude that

$$V^S > V^C > V^M > 0.$$

Now we can return to the general case. In these examples we have seen that

$$V^S > V^C > V^M > 0$$

Indeed, this is not special to our examples and this is the case in general.

Proposition: $V^S > V^C > V^M > 0$ is always true.

Proof: See the appendix.

The proof is appendix since the proof is very long, complex, and out of our scope. But the proof of the following claims is simpler.

Proposition: $V^S = 2V^M$ is always true.

Remark: This means the value of technological progress for the society is two times higher than the value for a monopoly.

Proof: Recall that

$$V^S = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b}$$

and

$$V^M = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{4b}$$

which means $V^S = 2V^M$.

Exercises*

1) Suppose that

$$Q = 10 - P.$$

Q is the demand by the consumers when the price of the good is P . Assume that the economy is perfectly competitive.

a) If the marginal cost is $\bar{c} = 4$ what is the equilibrium price? What is the profit of each firm in equilibrium?

b) Assume a single firm reduces its marginal cost down to $\underline{c} = 2$. Then the equilibrium price changes. Is the price change drastic or non-drastic?

- c) What is the value of this technological progress for the competitive firm?
- d) What is the value of this technological progress for the society?
- d) Interpret the difference in values attributed to technological progress.
- 2) Let the demand function be $Q = 1 - P$ where Q is the demand by the consumers when the price of the good is P . Assume that there is a single firm - a monopoly - producing output. The marginal cost is $\bar{c} = 1/2$.
- a) What is the equilibrium price charged by the monopolist? What is the equilibrium quantity?
- b) What is the maximum amount of money that the monopolist would pay to reduce the marginal cost down to zero?
- c) What is the maximum amount of money that the society would pay to universities so that universities can find a way to reduce the marginal cost down to zero?
- d) Interpret the difference in values attributed to technological progress.

Appendix

In this appendix, we shall prove that $V^S > V^C > V^M > 0$ is always true. The proof consists of two steps.

Step 1: $V^S > V^C$.

Proof: First remember that the social value of technological progress is

$$V^S = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b}.$$

As for V^C , there are two cases to consider: (i) drastic change, and (ii) non-drastic change. Let us start with the first case.

Drastic change: In this case

$$P^M(\underline{c}) = \frac{a + b\underline{c}}{2b} < \bar{c}$$

and

$$V^c = \frac{(a - b\underline{c})^2}{4b}$$

As a consequence,

$$V^s - V^c = \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b} - \frac{(a - b\underline{c})^2}{4b}$$

Simplifying this difference gives

$$V^s - V^c = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{2b}.$$

Now let us use

$$\frac{a + b\underline{c}}{2b} < \bar{c}$$

to see that

$$\begin{aligned} V^s - V^c &= \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\bar{c})^2}{2b} > \frac{(a - b\underline{c})^2}{4b} - \frac{\left(a - b \frac{a + b\underline{c}}{2b}\right)^2}{2b} \\ &= \frac{(a - b\underline{c})^2}{4b} - \frac{\left(\frac{a - b\underline{c}}{2}\right)^2}{2b} = 0 \end{aligned}$$

proving the claim for the drastic change.

Non-drastic change: In this case

$$P^M(\underline{c}) = \frac{a + b\underline{c}}{2b} \geq \bar{c}$$

and

$$V^c = (\bar{c} - \underline{c})(a - b\bar{c}).$$

As a consequence,

$$\begin{aligned}
V^S - V^C &= \frac{(a - b\underline{c})^2}{2b} - \frac{(a - b\bar{c})^2}{2b} - (\bar{c} - \underline{c})(a - b\bar{c}) \\
&= \frac{(\bar{c} - \underline{c})(2a - b\underline{c} - b\bar{c})}{2} - (\bar{c} - \underline{c})(a - b\bar{c}) \\
&= (\bar{c} - \underline{c}) \left(\frac{(2a - b\underline{c} - b\bar{c})}{2} - (a - b\bar{c}) \right) \\
&= (\bar{c} - \underline{c}) \left(\frac{(2a - b\underline{c} - b\bar{c})}{2} - \frac{2(a - b\bar{c})}{2} \right) = (\bar{c} - \underline{c}) \left(\frac{b\bar{c} - b\underline{c}}{2} \right) > 0.
\end{aligned}$$

Deduce that $V^S - V^C > 0$ whether the price change is drastic or not.

Step 2: $V^C > V^M > 0$.

Proof: Since we are dealing with V^C , the drastic and non-drastic cases should be analyzed separately. In case of drastic change

$$V^C - V^M = \frac{(a - b\underline{c})^2}{4b} - \frac{(a - b\underline{c})^2}{4b} + \frac{(a - b\bar{c})^2}{4b} = \frac{(a - b\bar{c})^2}{4b} > 0$$

If, however, the change is non-drastic then

$$\begin{aligned}
V^C - V^M &= (\bar{c} - \underline{c})(a - b\bar{c}) - \frac{(a - b\underline{c})^2}{4b} + \frac{(a - b\bar{c})^2}{4b} \\
&= \frac{4b(\bar{c} - \underline{c})(a - b\bar{c})}{4b} - \frac{(a - b\underline{c})^2}{4b} + \frac{(a - b\bar{c})^2}{4b} \\
&= \frac{4b(\bar{c} - \underline{c})(a - b\bar{c}) - b(\bar{c} - \underline{c})(2a - b\bar{c} - b\underline{c})}{4b} = \\
&= b(\bar{c} - \underline{c}) \frac{4(a - b\bar{c}) - (2a - b\bar{c} - b\underline{c})}{4b} = \frac{2a - 3b\bar{c} + b\underline{c}}{4b} \geq \frac{a - b\bar{c}}{4b} > 0
\end{aligned}$$

since the non-drastic case means

$$\frac{a + b\underline{c}}{2b} \geq \bar{c}.$$