ISSN 1311-8080 Volume 29 No. 2 2006

International Journal of Pure and Applied Mathematics

Editor-in-Chief
Drumi Bainov

Managing Editor
Svetoslav Nenov

Editorial Board

J. Abadie (France)
H. Adeli (USA)
I. Argyros (USA)
O. Axelsson (The Netherlands)
D. Bainov (Bulgaria)
A. Baker (USA)
M. Balas (USA)
V. Barbu (Romania)
P. Bates (USA)
G. Beer (USA)
J. Bona (USA)
G. Buttazzo (Italy)
F. Calogero (Italy)
J. Cash (UK)
C. Castaing (France)
E. Cheney (USA)
J. Conway (USA)
G. Cooperman (USA)
G. Da Prato (Italy)
C. Dafermos (USA)
F. Deutsch (USA)
E. DiBenedetto (USA)
A. Dishliev (Bulgaria)
I. Duff (UK)
C. Elliott (UK)
D. Evans (UK)
R. Ewing (USA)
R. Finn (USA)
T. Fokas (UK)
H. Frankowska (France)
R. Freund (USA)
W. Gautschi (USA)
B. Gavish (USA)
C. Gear (USA)
I. Gladwell (USA)
G. Glimm (USA)
R. Graham (USA)
B. Gramsch (Germany)
P. Gresho (USA)
C. Groetsch (USA)
C. Hess (France)
M. Iri (Japan)
M. Ito (Japan)
J. Jeffers (UK)
H. Kawarada (Japan)
N. Kenmochi (Japan)
D. Kolev (Bulgaria)
D. Larson (USA)
G. Leitmann (USA)
T.-T. Li (P.R. China)
P.-L. Lions (France)
M. Luskin (USA)
V. Milman (Israel)
C. Mitchell (UK)
B. Mordukhovich (USA)
K. Morgan (UK)
D. Motreanu (France)
Z. Nashed (USA)
S. Nenov (Bulgaria)
H. Niederreiter (Singapore)
G. Nürnberger (Germany)
J. Osborn (USA)
M. Ôtani (Japan)
D. Pallaschke (Germany)
C. Pao (USA)
N. Pavel (USA)
G. Pinder (USA)
S. Reich (Israel)
M. Renardy (USA)
L. Rowen (Israel)
P. Sabatier (France)
M. Schechter (USA)
R. Showalter (USA)
C.-W. Shu (USA)
M. Shub (USA)
P. Souganidis (USA)
H.M. Srivastava (Canada)
S. Suzuki (Japan)
E. Tadmor (USA)
A. Tani (Japan)
R. Thomas (USA)
K. Tamma (USA)
R. Varga (USA)
W. Vasconcelos (USA)
A. Vatsala (USA)
N. Velkov (Bulgaria)
R. Verma (USA)
J. Vines (France)
T. Ward (USA)
G. Watson (Scotland)
R. Wets (USA)
O. Widlund (USA)
M. Yasugi (Japan)
L. Zadeh (USA)
J. Zou (Hong Kong)

Academic Publications

ISSN 1311-8080 IJPAM, Volume 29 No. 2 2006

Contents

R. Safari
A NEW PION-NUCLEUS OPTICAL POTENTIAL VALID FOR Δ-RESONANCE REGION 141

Z.-F. Zhang, L. Jun, C.-C. Zhao, Z.-G. Ren, S. Bao
ON THE ADJACENT VERTEX-DISTINGUISHING EDGE CHROMATIC NUMBER OF $W_m \vee W_n$ 153

J.-W. Li, Z.-F. Zhang, Z.-W. Wang, B. Wei, L. Yan
ON THE ADJACENT VERTEX-DISTINGUISHING EQUITABLE-TOTAL CHROMATIC NUMBER OF $P_m \vee F_n$ 159

W.-J. Wang, Z.-W. Wang, Z.-F. Zhang
ON THE VERTEX DISTINGUISHING EQUITABLE EDGE CHROMATIC NUMBERS OF SOME JOIN GRAPHS 171

S. Friedland, E. Krop
EXACT CONDITIONS FOR COUNTABLE INCLUSION-EXCLUSION IDENTITY AND EXTENSIONS 177

E. Ballico
DEGREE 0 LINE BUNDLES ON A SMOOTH CURVE X SUPPORTED BY FEW POINTS OF X 183

C.-S. Lin
ON SEMIINNER PRODUCTS AND SEMINORMS IN LINEAR SPACES 187

E. Ballico
NON-LOCALLY FREE STABLE COHERENT SYSTEMS ON INTEGRAL PROJECTIVE CURVES: AN ASYMPTOTIC EXISTENCE THEOREM 201

A.L. Frenkel, D. Halpern
STRONGLY NONLINEAR NATURE OF INTERFACIAL-SURFACTANT INSTABILITY OF COUETTE FLOW 205

Q. Guo, J.-G. Liu
OPTIMIZATION MODEL OF THE HIGHWAY TOLL PLAZA TRAFFIC CAPACITY 225

M.M. Chawla
A LINEARLY IMPLICIT P-STABILIZED ONE-STEP SIMPSON-TYPE METHOD FOR SECOND ORDER INITIAL VALUE PROBLEMS 233

X.-H. Lu, J.-J. Tian, X.-S. Liu, Z.-F. Zhang
A NOTE ON THE TOTAL COLORING OF GRAPHS $S_m \vee P_n \vee P'_n$ 243

İ. Zorlutuna, Y. Küçük
WEAKLY SEMI-θ-CONTINUOUS FUNCTIONS 249

E. Grycko
PREDICTING DEBYE TEMPERATURE FROM THE BOILING POINT AND THE CRYSTALLOGRAPHIC LATTICE CONSTANT 259

C. Guler, M. Bayram, N. Guzel
ON THE SOLUTION OF AN INVERSE SCATTERING PROBLEM ON THE SEMI-AXIS WITH A SELF ADJOINT POTENTIAL MATRIX 269

International Journal of Pure and Applied Mathematics
Volume 29 No. 2 2006, 269-280

ON THE SOLUTION OF AN INVERSE SCATTERING PROBLEM ON THE SEMI-AXIS WITH A SELF ADJOINT POTENTIAL MATRIX

Coskun Guler[1], Mustafa Bayram[2 §], Nuran Guzel[3]

[1]Department of Mathematical Engineering
Chemical and Metallurgy Faculty
Yıldız Technical University
Davutpaşa, Istanbul, 34210, TURKEY
e-mail: cguler@yildiz.edu.tr

[2,3]Department of Mathematics
Faculty of Sciences and Arts
Yıldız Technical University
Davutpaşa, Istanbul, 34210, TURKEY
e-Mail: msbayram@yildiz.edu.tr

Abstract: In this study, we have constructed an inverse scattering problem on the semi-axis with a self adjoint potential matrix. We first tried to find potential function $C(x)$. For this, integral representation of the given system has been written, and then some relationships established between $C(x)$ and kernel function of the integral representation, from the aim of the relationships, we obtained Volterra integral equation. This Volterra equation has been solved by analytically and numerically, after that results are compared.

AMS Subject Classification: 26A33
Key Words: inverse scattering problem, Volterra integral equation, Adomian method, potential matrix, Dirac equation system

1. Introduction

In this paper, solution of the inverse scattering problem on the semi-axis for

Received: May 31, 2006
© 2006 Academic Publications
§Correspondence author

differential equation systems which including

$$\begin{aligned} i\frac{du_1(x,\lambda)}{dx} + c(x)u_2(x,\lambda) &= \lambda u_1(x,\lambda), \\ -i\frac{du_2(x,\lambda)}{dx} + c(x)u_1(x,\lambda) &= \lambda u_2(x,\lambda), \quad i=\sqrt{-1}, \end{aligned} \tag{1}$$

are investigated by using Adomian method. Boundary conditions for the problem (1) are

$$u_1(0,\lambda) = u_2(0,\lambda), \tag{2}$$

$$|c_{k,j}(x)| \le ce^{-\varepsilon x}, \quad k,j=1,2, \quad 0 \le x < \infty. \tag{3}$$

where $\varepsilon > 0$ and c are constants, $c_{k,j}(x)$ are complex valued functions.

$$C(x) = \begin{pmatrix} 0 & c(x) \\ c(x) & 0 \end{pmatrix}$$

is a symmetric matrix. Determining $c(x)$ coefficient with respect to $S(\lambda)$ scattering function which is defined on the semi-axis for the solution of the equation system (1) which provides conditions (2)-(3) to be unique is called inverse scattering problem.

Inverse scattering problems have been studied by many people, Zakharov and Shabat applied "inverse scattering" method to the Schrödinger equation [11] at the form

$$iq_t = q_{xx} + \chi q|q|^2, \quad \chi > 0.$$

Dirac equation system

$$\mathbf{T}\frac{dy(x,\lambda)}{dx} + \Omega(x)y(x,\lambda) = \lambda y(x,\lambda),$$
$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; y(x,\lambda) = \begin{pmatrix} y_1(x,\lambda) \\ y_2(x,\lambda) \end{pmatrix}, \; \Omega(x) = \begin{pmatrix} P(x) & Q(x) \\ Q(x) & -P(x) \end{pmatrix}$$

is treated as auxiliary linear equation.

Inverse scattering problems for Dirac system and first degree ordinary differential equation system studied by M.G. Gasymov, B.M. Levitan [4], L.P. Nizhnik, at al [9], A.B. Shabat [10], D.J. Kaup [8], R. Beals, R.R. Coifman [2], P.J. Coudrey [3], N.Sh. Iskenderov [7], C. Guler [5], [6]. If we use transformation

$$y_1(x,\lambda) = \frac{1}{2}(u_1(x,\lambda) + u_2(x,\lambda)), \quad y_2(x,\lambda) = \frac{1}{2i}(u_2(x,\lambda) - u_1(x,\lambda)),$$

in the system (1), it turns into Dirac equation system

$$\mathbf{T}\frac{dy(x,\lambda)}{dx} + \Omega(x)y(x,\lambda) = \lambda y(x,\lambda)$$

that mass of system is $m = 0$ which $\mathbf{T}$ is

$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ y(x,\lambda) = \begin{pmatrix} y_1(x,\lambda) \\ y_2(x,\lambda) \end{pmatrix},\ \Omega(x) = \begin{pmatrix} c(x) & 0 \\ 0 & -c(x) \end{pmatrix}.$$

2. An Inverse Scattering Problem and Solution Method

Integral representation $u(x,\lambda)$ for the system (1) is the restricted solution of system (1) when λ is a real number. Then,

$$\begin{aligned} u_1(x,\lambda) &= Ae^{-i\lambda x} + o(1), \\ u_2(x,\lambda) &= Be^{i\lambda x} + o(1),\ x \to +\infty, \end{aligned} \tag{4}$$

where A and B are constants.

System (1) is equivalent to the following integral equation system,

$$\begin{aligned} u_1(x,\lambda) &= Ae^{-i\lambda x} + \frac{1}{i}\int_x^\infty e^{-i\lambda(x-t)}c(t)u_2(t,\lambda)dt, \\ u_2(x,\lambda) &= Be^{i\lambda x} - \frac{1}{i}\int_x^\infty e^{i\lambda(x-t)}c(t)u_1(t,\lambda)dt. \end{aligned} \tag{5}$$

If we substitute (4) into the (1) and after applying the successive approximation method,we obtained solution of the system (1) as

$$\begin{aligned} u_1(x,\lambda) &= Ae^{-i\lambda x} + A\int_x^\infty H_{11}(x,s)e^{-i\lambda s}ds + B\int_x^\infty H_{12}(x,s)e^{i\lambda s}ds, \\ u_2(x,\lambda) &= Be^{i\lambda x} + A\int_x^\infty H_{21}(x,s)e^{-i\lambda s}ds + B\int_x^\infty H_{22}(x,s)e^{i\lambda s}ds. \end{aligned} \tag{6}$$

An expression (6) is known as integral representation of system (1). There is the following relationship between function$H_{ij}(x,s)$ and potential matrix $\mathbf{C}(x)$ in the integral representation.

$$2i\begin{bmatrix} 0 & H_{12}(x,x) \\ -H_{21}(x,x) & .0 \end{bmatrix} = \begin{bmatrix} 0 & c(x) \\ c(x) & 0 \end{bmatrix}, \tag{7}$$

where $|H_{ij}(x,s)| \le ce^{-\varepsilon\frac{x+s}{2}}$, $0 \le x \le s$, $i,j = 1,2$ and c is constant.

Proof of Expression (7). The following equation is obtained by substituting $u_1(x,\lambda)$ that is in the integral equation (5) into the integral representation (6) and doing several calculations

$$A\int_x^\infty [H_{11}(x,s) - \frac{1}{i}\int_x^\infty H_{21}(t,s-x-t)c(t)dt]e^{-i\lambda s}ds$$

$$+B\int_x^\infty [H_{12}(x,s) - \frac{1}{2i}c\left(\frac{x+s}{2}\right) - \frac{1}{i}\int_x^{\frac{x+s}{2}} H_{22}(t,s+x-t)c(t)dt]e^{i\lambda s}ds = 0\,,$$

where, $e^{-i\lambda s} \neq 0$, $e^{-i\lambda s} \neq 0$ and, A and B are not equal to zero for every time. Therefore,

$$H_{11}(x,s) - \frac{1}{i}\int_x^\infty H_{21}(t,s-x+t)c(t)dt = 0\,,$$

$$H_{12}(x,s) - \frac{1}{2i}c\left(\frac{x+s}{2}\right) - \frac{1}{i}\int_x^{\frac{x+s}{2}} H_{22}(t,s+x-t)c(t)dt = 0\,.$$

For $s = x$ we obtain

$$H_{12}(x,x) = \frac{1}{2i}c(x)\,,$$

and the equality

$$H_{21}(x,x) = -\frac{1}{2i}c(x)\,.$$

is obtained by doing necessary calculations after substituting $u_2(x,\lambda)$ which is at the integral equation (5) into the integral representation (6). Thus, correctness of the (7) is shown. □

2.1. $S(\lambda)$ Scattering Function

Let us treat condition (2) in the integral representation (6)

$$A\left(1 + \int_0^\infty [H_{11}(0,t) - H_{21}(0,t)]e^{-i\lambda t}dt\right)$$

$$= B\left(1+\int_0^\infty [H_{22}(0,t)-H_{12}(0,t)]e^{i\lambda t}dt\right),$$

where

$$S(\lambda)=\frac{1+\int_0^\infty [H_{11}(0,t)-H_{21}(0,t)]e^{-i\lambda t}dt}{1+\int_0^\infty [H_{22}(0,t)-H_{12}(0,t)]e^{i\lambda t}dt}. \tag{8}$$

Function $S(\lambda)$ is called scattering function that provides $B = S(\lambda)A$. Function $S(\lambda)$ is a Meromorf function for $-\varepsilon_0 \leq \mathrm{Im}\lambda \leq \varepsilon_0$. The pole points of the function $S(\lambda)$ and its inverse are not real. Function $S(\lambda)$, nulls of the numerator of the function for $\lambda_j, j = 1,...,\gamma$, and nulls of the denominator of the function for $\lambda_k,\ k = 1,...,\alpha$ are called scattering data for self-adjoint potential matrix.

Inverse scattering problem is finding $C(x)$ coefficient potential function of the system (1) with respect to given scattering data.

The following integral equation system is obtained by the aim of the integral representation (6) and scattering data

$$H_{11}(x,x+\xi)+\int_0^\infty H_{12}(x,x+t)f(-2x-\xi-t)dt=0, \tag{9}$$

$$g(2x+\xi)H_{12}(x,x+\xi)+\int_0^\infty H_{11}(x,x+t)g(2x+\xi+t)dt=0. \tag{10}$$

Integral equation system (9)-(10) is known as Gelfand-Levitan-Machenko equation system, where $x \geq 0,\ \xi > 0$ and

$$f(t)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}[S(\lambda)-1]e^{-i\lambda t}d\lambda, \tag{11}$$

$$g(t)=\frac{1}{2\pi}\int_{-\infty-i\eta}^{\infty-i\eta}[S^{-1}(\lambda)-1]e^{-i\lambda t}d\lambda, \tag{12}$$

functions $f(-x)$ and $g(x)$ are restricted functions providing following conditions,

$$|f(-x)| \leq De^{-\frac{\xi}{4}x},\ |g(x)| \leq De^{-\frac{\xi}{4}x},\ D \text{ is constant.}$$

Functions $f(-x)$ and $g(x)$ are calculated by solving integrals (11)-(12). Integral equation system (9)-(10) is obtained by the aim of these functions. The solution of the system is exist and unique. Functions $H_{11}(x, x+\xi)$ and $H_{12}(x, x+\xi)$ are found from the solution of system (9)-(10). The $C(x)$ coefficients are found by using expression (7). We have already called scattering data to the given $S(\lambda)$ scattering function and this functions nulls and pole points. To solve (9)-(10) Gelfand-Levitan-Machenko integral equation system could not be easy and could not be possible analytically by the aim of the scattering data. Because of the above reasons we need to use numerical method to solve the problem. If we use transformation $u = x+\xi$, $s = x+t$ in the (9)-(10), we have

$$H_{11}(x,u) + \int_x^\infty H_{12}(x,s) f(-u-s) ds = 0\,, \tag{13}$$

$$g(x+u) H_{12}(x,u) + \int_x^\infty H_{11}(x,s) g(u+s) ds = 0\,. \tag{14}$$

As can be seen, equations (13)-(14) are linear Volterra Integral equation systems.

We have obtained series solution of the (13)-(14) Volterra Integral equations using Adomian method [1]. Therefore a problem treated as an example about the theme and solved analytically first, then solved by using Adomian method, after that results are compared.

Now letus explain solving linear Volterra integral equation system by using Adomian method shortly

$$F(t) = G(t) + \int_0^t K(t,s)\, F(s) ds\,, \tag{15}$$

where

$$F(t) = (f_1(t), ..., f_n(t))^T\,,$$
$$G(t) = (g_1(t), ..., g_n(t))^T\,,$$
$$K(t,s) = [K_{ij}(t,s)] \quad \text{for } i = 1, ..., n;\ j = 1, ..., n\,.$$

Solution of (15) is

$$f_i(t) = g_i(t) + \int_0^t \sum_{j=1}^n K_{ij}(t,s)\, f_j(s) ds\,, \tag{16}$$

$$f_{i\lambda}(t) = \sum_{m=0}^{\infty} \lambda^{(m)} f_{im}(t)\,, \tag{17}$$

$$\sum_{m=0}^{\infty} \lambda^{(m)} f_{im}(t) = g_i(t) + \int_0^t \sum_{j=1}^{n} K_{ij}(t,s)\,(\sum_{m=0}^{\infty} \lambda^{(m)} f_{im}(s))ds\,. \tag{18}$$

$$f_{i0}(t) = g_i(t),$$

$$f_{i,m+1}(t) = \int_0^t \sum_{j=1}^{n} K_{ij}(t,s)\, f_{jm}(s)ds, \quad i = 1,...n;\ m = 0,1,2...\,, \tag{19}$$

$$f_i(t) \cong \sum_{k=0}^{N} f_{ik}(t), \qquad i = 1,...,n\,, \tag{20}$$

$$f_{i,m+1}(t) = g_i(t) + \int_0^t \sum_{j=1}^{n} K_{ij}(t,s)\, f_{jm}(s)ds,$$

$$i = 1,...n,\ \ m = 0,1,2....\tag{21}$$

Thus, solution of the given linear Volterra integral equation system (15) is obtained as (21).

3. An Example

Let us solve the inverse scattering problem which is defined by

$$S(\lambda) = \frac{\lambda + i}{\lambda - i}, \quad -\varepsilon_0 < \operatorname{Im}\lambda < \varepsilon_0\,,$$

system (1) and conditions (2)-(3). We are going to solve the problem first analytically, and then using Adomian method.

3.1. Analytical Solution

By using equation (11) and (12),

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda+i}{\lambda-i} - 1\right]e^{-i\lambda t}d\lambda\,, \quad g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda-i}{\lambda+i} - 1\right]e^{-i\lambda t}d\lambda$$

are obtained. Since

$$\int_{-\infty}^{\infty} f(\lambda)d\lambda = 2\pi i \sum \mathrm{Res}(f(t)),$$

we have

$$\mathrm{Res}\,(f(t), i) = 2ie^{t}, \quad f(t) = -2e^{t}.$$

Thus, function $g(t)$ becomes

$$\int_{-\infty}^{\infty} g(\lambda)d\lambda = 2\pi i \sum \mathrm{Res}\,(g(t))\,,$$

$$\mathrm{Res}\,(g(t), -i) = -2ie^{-t}, \quad g(t) = 2e^{-t}.$$

Therefore,

$$f(-x) = -2e^{-x}, \quad g(x) = 2e^{-x}.$$

If we substitute functions $f(-x)$ and $g(x)$ into the (13)-(14) Volterra integral equation system, we treat

$$\begin{aligned} &H_{11}(x,u) + \int_{x}^{\infty} H_{12}(x,s)\left[-2e^{-(u+s)}\right]ds = 0\,, \\ &-2e^{-(x+u)} + H_{12}(x,u) + \int_{x}^{\infty} H_{11}(x,s)\left[2e^{-(u+s)}\right]ds = 0\,. \end{aligned} \tag{22}$$

Then, we obtain

$$\begin{aligned} H_{11}(x,u) &= 2e^{-u}\int_{x}^{\infty} H_{12}(x,s)e^{-s}ds\,, \\ H_{12}(x,u) &= -2e^{-(x+u)} - 2e^{-u}\int_{x}^{\infty} H_{11}(x,s)e^{-s}ds\,, \end{aligned} \tag{23}$$

and if

$$A_1 = \int_{x}^{\infty} H_{12}(x,s)e^{-s}ds\,, \quad A_2 = \int_{x}^{\infty} H_{11}(x,s)e^{-s}ds,$$

then

$$H_{11}(x,u) = 2e^{-u}A_1, \quad H_{12}(x,u) = -2e^{-(x+u)} - 2e^{-u}A_2\,. \tag{24}$$

If we substitute (24) into (23), we find

$$A_1 = \frac{-e^{-3x}}{1+e^{-4x}}, \quad A_2 = \frac{-e^{-5x}}{1+e^{-4x}}.$$

We substitute A_1 and A_2 into (24), and obtain:

$$H_{11}(x,x+\xi) = \frac{-2e^{-(4x+\xi)}}{1+e^{-4x}}, \quad H_{11}(x,u) = \frac{-2e^{-u}e^{-3x}}{1+e^{-4x}},$$

$$H_{12}(x,x+\xi) = \frac{-2e^{-(2x+\xi)}}{1+e^{-4x}}, \quad H_{12}(x,u) = \frac{-2e^{-u}e^{-x}}{1+e^{-4x}},$$

where $x+\xi = u$. For $\xi \to 0$, we obtain

$$H_{11}(x,x) = \frac{-2e^{-4x}}{1+e^{-4x}}, \quad H_{12}(x,x) = \frac{-2e^{-2x}}{1+e^{-4x}}\,. \tag{25}$$

Thus, we obtain the analytical solution of Volterra integral equation system (22). As can be seen expression (6), when $c(x) = 2iH_{12}(x,x)$, $c(x)$ potential function of equation system (1) is obtained as

$$c(x) = \frac{-4ie^{-2x}}{1+e^{-4x}}, \quad x \geq 0\,. \tag{26}$$

3.2. Numerical Solution

Let us solve linear Volterra integral equation system (22) by using Adomian Decomposition Method. We first modify the equation (22) as

$$H_{11}(x,u) = \int_x^\infty 2e^{-(u+s)}H_{12}(x,s)ds, \tag{27}$$

$$H_{12}(x,u) = -2e^{-(x+u)} - \int_x^\infty 2e^{-(u+s)}H_{11}(x,s)ds\,. \tag{28}$$

Then we use Adomian method which is given as follows,

$$H_{110}(x,u) = 0, \quad H_{120}(x,u) = -2e^{-(x+u)},$$

$$H_{11,m+1}(x,u) = \int_x^\infty 2e^{-(u+s)} H_{12m}(x,s)ds,$$
$$H_{12,m+1}(x,u) = -\int_x^\infty 2e^{-(u+s)} H_{11m}(x,s)ds, \quad m = 0,1,2\ldots.$$

After first iteration, we have

$$H_{11}^1(x,u) = H_{110}(x,u) + H_{111}(x,u) = -2e^{-u}e^{-3x},$$
$$H_{12}^1(x,u) = H_{120}(x,u) + H_{121}(x,u) = -2e^{-u}e^{-x}.$$

After second iteration,

$$H_{11}^2(x,u) = H_{110}(x,u) + H_{111}(x,u) + H_{112}(x,u) = -2e^{-u}e^{-3x},$$
$$H_{12}^2(x,u) = H_{120}(x,u) + H_{121}(x,u) + H_{122}(x,u) = -2e^{-u}e^{-x} + 2e^{-u}e^{-5x}.$$

Therefore, after fifth iteration, we have

$$H_{11}^5(x,u) = -2e^{-u}e^{-3x} + 2e^{-u}e^{-7x} - 2e^{-u}e^{-11x},$$
$$H_{12}^5(x,u) = -2e^{-u}e^{-x} + 2e^{-u}e^{-5x} - 2e^{-u}e^{-9x}.$$

Then, we obtained numerical solution of the Volterra integral equation system (22) as

$$H_{11}(x,u) = -2e^{-u}e^{-3x}(1 - e^{-4x} + e^{-8x} - \ldots),$$
$$H_{12}(x,u) = -2e^{-u}e^{-x}(1 - e^{-4x} + e^{-8x} - \ldots).$$

If we substitute $H_{12}(x,u)$ into the equation (7), then we obtain

$$c(x) = -4ie^{-2x}(1 - e^{-4x} + e^{-8x} - \cdots),$$

where we replace u by x. We have illustrated analytical and numerical solution of the given problem in Table 1.

4. Conclusion

The fundamental goal of this study has been to construct an approximation to a numerical solution of the inverse scattering problem. We introduced a numerical method (see Table 1) for numerical presentation of solution of the problem under consideration. The numerical values in Table 1 coincide with the exact solution.

x_i	Exact value of $c(x)$	Numerical value $c(x)$	Absolute error
0.0	-2.00000000i	-4.00000000i	2.00000000i
0.1	-1.96065599i	-1.98472763i	0.02407164i
0.2	-1.85001490i	-1.85029376i	0.00027886i
0.3	-1.687101371	-1.68710449i	0.00000312i
0.4	-1.49539983i	-1.49539987i	0.00000004i
0.5	-1.29610854i	-1.29610854i	0.00000000
0.6	-1.10457230i	-1.10457230i	0.00000000
0.7	-0.92984398i	-0.92984398i	0.00000000
0.8	-0.77595637i	-0.77595637i	0.00000000
0.9	-0.64360973i	-0.64360973i	0.00000000
1.0	-0.53160445i	-0.53160445i	0.00000000

Table 1: Comparison of analytical and numerical value of $c(x)$

References

[1] E. Babolian, J. Biazar, A.R. Vahidi, On the decomposition method for system of linear equations and systems of linear Volterra integral equations, *App. Math. And Comp.*, **147** (2004), 19-27.

[2] R. Beels, R.R. Coifman, Scattering and inverse Scattering for first order systems, *Communications a pure and Applied Mathematics*, **XXXVII** (1984), 39-90.

[3] P.J. Coudrey, The inverse spectral problem for a system of three coupled first order equations, *Proceeding of the Royal Iris Academy*, **83A** (1983), 23-31.

[4] M.B. Gasymov, B.M. Levitan, *Dokl. Akad. Nauk SSSR, Soviet Math. Dokl.*, **167** (1966), 495.

[5] C. Guler, An inverse scattering problem for ordinary differential equations system with four equations on the semi-axis, *Journal of Yildiz Technical University* (1998), 54-64.

[6] C. Guler, The inverse scattering problem for a system of n differential equations on half-axis, *Far East Journal of Applied Mathematics*; Pushpa Publishing House, Indiana, **10**, No. 1 (2003), 55-64.

[7] N.Sh. Iskenderov, Direct and inverse scattering problem for a system of three hyperbolic equations of the first order on semi-axis with scattering waves, *Preprint/AN Ukr. SSR, Institute of Mathematics*, Kiev (1985), 85-87.

[8] D.J. Kaup, The three-wave interaction-Ajondispersive, *Studies in Applied Mathematics*, **55** (1976), 9-44.

[9] L.P. Nizhnik , F.L. Vu , An inverse scattering problem on the semi-axis with a nonselfadjoint potential matrix, *Ukr. Math. Zh.* (1974), 469-486.

[10] A.B. Shabat, An inverse scattering problem, *Diff. Uravv.*, **15** (1978), 1824-1834.

[11] V.E. Zakharov, A.B. Shabat, *Zh. Exp. Teo. Fiz.* **61** (1971), 118, Translation: *Sov. Phys.-JETP*, **34** (1972), 62.