Dynamical (harmonic) interface stress field in the half-plane covered by the prestretched layer under a strip load

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The manuscript was received on 30 December 2003 and was accepted after revision for publication on 5 October 2004.

DOI: 10.1243/030932405X12867

Abstract: Within the framework of the piecewise homogeneous body model, by employing the three-dimensional linearized theory of elastic waves in initially stressed bodies the dynamical problem of the stress distribution in a half-plane covered with a prestretched layer is investigated. It is assumed that the free face plane of the covered layer is subjected to a uniformly distributed harmonic load acting on a strip extending to infinity in the x_3 direction, which is perpendicular to the x_1–x_2 plane and is of width $2a$ in the x_1 direction. The plane-strain state in the x_1–x_2 plane is analysed.

The corresponding boundary-value problems are investigated by employing the exponential Fourier integral transformation. The numerical results regarding the interface normal stress distribution are presented. The influences of the problem parameters and pre-stretching of the covered layer on this distribution are analysed. Practical engineering application fields of the results are suggested.

Keywords: prestretching, layer, half-plane, dynamical stresses, harmonic loading

1 INTRODUCTION

Dynamical problems of an elastic body with initial stresses have a wide range of applications in almost all areas of natural sciences and engineering such as the mechanics of composites, seismology, and biomechanics. Intensive investigations of these problems started approximately in the second half of the twentieth century and, from that time to the present, numerous theoretical and experimental investigations have been made in this field. Except for references [**1**] to [**3**] and some others, almost all these investigations refer to the influence of initial stresses on the speed and on the dispersion of various types of wave. A systematic consideration and analyses of the above-mentioned results which were obtained before 1986 has been published in the monographs in references [**4**] and [**5**]. A review of the later research has been given in references [**6**] to [**8**] from which it follows that a considerable part of the obtained results refer to layered composite materials with initial stresses. However, investigations regarding the study of the influence of the initial stresses on the dynamical stress state in a layered medium also have great significance in the theoretical and application sense. Note that the related static problems were considered in references [**9**] and [**10**] and in the papers listed therein.

In references [**1**] and [**2**], some attempts were made in this field and Lamb's problem for the case where a harmonic linear load acts perpendicular to the free surface was considered. In references [**1**] and [**2**] it was assumed that the initial stress state is determined in the framework of the linear theory of elasticity and some concrete investigations were made for the cases where the material of the stratified layer is more rigid than the material of the half-plane. In the present paper the investigations reported in references [**1**] and [**2**] are developed for the case where a harmonic normal load acts on a strip of finite width of the prestretched covered layer. Moreover, in the present

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paper, not only the case where the modulus of elasticity of the layer is greater than that for the half-plane but also the case where the modulus of elasticity of the half-plane material is greater than that of the stratified layer are considered. Note that the latter case is more complicated than the former case because, in the latter case, it is suitable for generalized Rayleigh wave propagation in a covered half-plane [**6**, **7**, **11**]. In connection with this, many difficulties arise while obtaining the numerical results. Moreover, it should be noted that the related dynamic problem for a half-plane was reported in references [**12**] to [**15**], for example. In these papers, investigations were made using the linear theory of elasticity and major attention was focused on the displacement distribution analyses in the half-plane (ground). However, in the present paper the study is made within the framework of the piecewise homogeneous body model using the three-dimensional linearized theory of elastic waves in initially stressed (or prestressed) bodies (TLTEWISB). The concrete numerical results regarding the stresses acting on the interface plane are presented. The influence of the problem parameters and the magnitude of the prestretching of the covered layer on these stresses are analysed.

The results of the present investigation can be applied in the following areas of modern engineering:

(a) the dynamics of a vehicle–track system the basic material of which is covered with the other material;
(b) the propagation of vibration through covered ground caused by the foundation of a vibrating machine;
(c) the response of buildings to the vibration of covered ground caused by an earthquake, etc.

Some aspects of similar applications have been given and explained in references [**16**] to [**18**].

In recent years a new scientific area called acousto-elasticity has been introduced in the theory of waves. It addresses the propagation of elastic waves and vibrations in initially stressed bodies and includes practical issues such as determination of the physico-mechanical properties and non-destructive analysis of the residual stress–strain state in structural members of engineering design. A review of the relevant investigations has been given in reference [**19**]. Consequently, the results of the present investigation can be used as theoretical results for the study of non-destructive residual stress analysis in a covered layer of structural members.

It should be noted that, for application of the results of the present investigations, certain conditions must be satisfied. These conditions will be explained in the next section of the paper. Throughout the investigations, repeated indices are summed over their ranges.

2 FORMULATION OF THE PROBLEM

Consider a half-plane covered by a prestretched layer (Fig. 1). The Lagrangian coordinates $ox_1x_2x_3$, which in the natural state coincide with the Cartesian coordinates, are associated with the covered layer. Note that the covered layer and half-plane occupy the regions $\{-\infty \leqslant x_1 \leqslant \infty, -h \leqslant x_2 \leqslant 0, -\infty \leqslant x_3 \leqslant \infty\}$ and $\{-\infty \leqslant x_1 \leqslant +\infty, -\infty \leqslant x_2 \leqslant -h, -\infty \leqslant x_3 \leqslant \infty\}$ respectively. Assume that the free surface (plane) of the layer is subjected to a uniformly distributed harmonic load acting on a strip extending to infinity in the x_3 direction, which is perpendicular to the x_1–x_2 plane and is of width $2a$ in the x_1 direction.

Below the following notation is used: the values related to the layer and half-plane are denoted by superscripts (1) and (2) respectively; the values related to the initial stresses are denoted by superscripts (m),0 where $m = 1, 2$.

Consider the case where the initial stresses in the layer and half-plane are determined as

$$\sigma_{11}^{(m),0} = \text{constant}_m \neq 0, \qquad m = 1, 2$$
$$\sigma_{ij}^{(m),0} = 0 \qquad \text{for } i = j \neq 1 \tag{1}$$

The linear elastic material of the layer and the half-plane are assumed to be both homogeneous and isotropic and $\sigma_{11}^{(m),0}$ is the unique non-zero component of the initial stress tensor of both the layer and the half-plane; therefore, $\sigma_{11}^{(m),0}$ is denoted in short form as $\sigma_0^{(m)}$. Following from the above, it can be assumed that the plane deformation state in the ox_1x_2 plane prevails. According to references [**4**] and [**5**], for the considered case the TLTEWISB equations are

$$\frac{\partial \sigma_{ij}^{(m)}}{\partial x_j} + \sigma_0^{(m)} \frac{\partial^2 u_i^{(m)}}{\partial x_1^2} = \rho_0^{(m)} \frac{\partial^2 u_i^{(m)}}{\partial t^2}$$
$$i, j = 1, 2, \qquad m = 1, 2 \tag{2}$$

In equation (2) the density of the mth material in the natural state is denoted by $\rho_0^{(m)}$. The other notation in equation (2) is conventional.

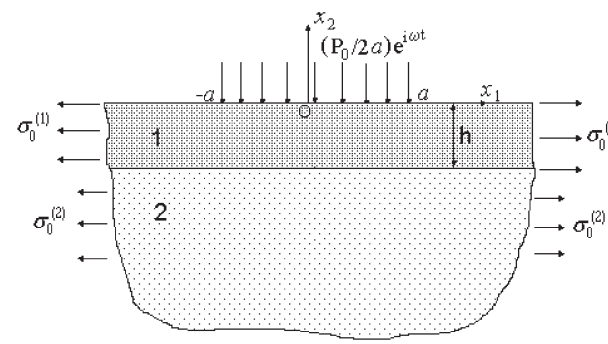


Fig. 1 The geometry of the structure of the half-plane covered by the layer

For an isotropic compressible material the mechanical relations can be written

$$\sigma_{ij}^{(\underline{m})} = \lambda^{(\underline{m})}\theta^{(\underline{m})}\delta_{ij} + 2\mu^{(\underline{m})}\varepsilon_{ij}^{(\underline{m})}$$
$$\theta^{(\underline{m})} = \varepsilon_{11}^{(\underline{m})} + \varepsilon_{22}^{(\underline{m})} \tag{3}$$

where $\lambda^{(m)}$ and $\mu^{(m)}$ are the Lamé constants and

$$\varepsilon_{ij}^{(m)} = \frac{1}{2}\left(\frac{\partial u_i^{(m)}}{\partial x_j} + \frac{\partial u_j^{(m)}}{\partial x_i}\right) \tag{4}$$

It is assumed that the complete contact conditions

$$u_i^{(1)}|_{x_2=-h} = u_i^{(2)}|_{x_2=-h}$$
$$\sigma_{i2}^{(1)}|_{x_2=-h} = \sigma_{i2}^{(2)}|_{x_2=-h}, \qquad i = 1, 2 \tag{5}$$

between the layer and the half-plane are satisfied. In the free face plane of the covered layer the conditions

$$\sigma_{12}^{(1)}|_{x_2=0} = 0$$
$$\sigma_{22}^{(1)}|_{x_2=0} = \begin{cases} -\dfrac{p_0}{2a}\delta(x_1)\,\mathrm{e}^{\mathrm{i}\omega t} & \text{for } |x_1| \leqslant a \\ 0 & \text{for } |x_1| > a \end{cases} \tag{6}$$

hold. In addition to the above-written conditions, it is also assumed that, as $x_2 \to -\infty$, there is no reflection, which means that all waves travel in the negative x_2 direction. In other words, it is assumed that

$$|u_i^{(2)}|, |\sigma_{ij}^{(2)}| < M = \text{constant, as } x_2 \to -\infty \tag{7}$$

With the above equation the formulation of the problem is completed.

Note that within the framework of the following restrictions the above formulation can be considered as mathematical modelling of the forced vibration of the structural members by the external forces given in equation (6):

1. The plane-strain state conditions must be satisfied.
2. The size of the region on which the external force acts (i.e. a) in equation (6) and the thickness of the covered layer (i.e. h) must be significantly less than the minimal characteristic size (denoted by d) of the structural member. In other words, the conditions $a \ll d$ and $h \ll d$ must be satisfied.
3. The values of the external force p_0 in equation (6) must be significantly less than the values of the residual (initial) stresses $\sigma_0^{(m)}$, i.e. $p_0 \ll \sigma_0^{(m)}$.

The case where $\sigma_0^{(m)} \equiv 0$ in the above mathematical formulation transforms to the corresponding case in the linear theory of elasticity.

3 METHOD OF SOLUTION

Substituting equations (3) and (4) in equation (2) the linearized equations of motion can be derived in terms of displacement perturbations according to

$$(\lambda^{(\underline{m})} + 2\mu^{(\underline{m})} + \sigma_0^{(\underline{m})})\frac{\partial^2 u_1^{(\underline{m})}}{\partial x_1^2} + \mu^{(\underline{m})}\frac{\partial^2 u_1^{(\underline{m})}}{\partial x_2^2} + (\lambda^{(\underline{m})} + \mu^{(\underline{m})})\frac{\partial^2 u_2^{(\underline{m})}}{\partial x_1\,\partial x_2} = \rho_0^{(\underline{m})}\frac{\partial^2 u_1^{(\underline{m})}}{\partial t^2}$$
$$(\lambda^{(\underline{m})} + \mu^{(\underline{m})})\frac{\partial^2 u_1^{(\underline{m})}}{\partial x_1\,\partial x_2} + (\mu^{(\underline{m})} + \sigma_0^{(\underline{m})})\frac{\partial^2 u_2^{(\underline{m})}}{\partial x_1^2} + (\lambda^{(\underline{m})} + 2\mu^{(\underline{m})})\frac{\partial^2 u_2^{(\underline{m})}}{\partial x_2^2} = \rho_0^{(\underline{m})}\frac{\partial^2 u_2^{(\underline{m})}}{\partial t^2} \tag{8}$$

From references [**1**] and [**2**], the displacement can be preserved in terms of potentials $\varphi^{(m)}$ and $\psi^{(m)}$ by

$$u_1^{(m)} = \frac{\partial\varphi^{(m)}}{\partial x_1} + \frac{\partial\psi^{(m)}}{\partial x_2}, \qquad u_2^{(m)} = \frac{\partial\varphi^{(m)}}{\partial x_2} - \frac{\partial\psi^{(m)}}{\partial x_1} \tag{9}$$

By direct verification it is proved that the potentials must satisfy the equations

$$\nabla^2\varphi^{(\underline{m})} + \frac{\sigma_0^{(\underline{m})}}{\lambda^{(\underline{m})} + 2\mu^{(\underline{m})}}\frac{\partial^2\varphi^{(\underline{m})}}{\partial x_1^2} = \frac{1}{(c_1^{(\underline{m})})^2}\frac{\partial^2\varphi^{(\underline{m})}}{\partial t^2}$$
$$\nabla^2\psi^{(\underline{m})} + \frac{\sigma_0^{(\underline{m})}}{\mu^{(\underline{m})}}\frac{\partial^2\psi^{(\underline{m})}}{\partial x_1^2} = \frac{1}{(c_2^{(\underline{m})})^2}\frac{\partial^2\psi^{(\underline{m})}}{\partial t^2} \tag{10}$$

where

$$c_1^{(\underline{m})} = \sqrt{\frac{\lambda^{(\underline{m})} + 2\mu^{(\underline{m})}}{\rho_0^{(\underline{m})}}}, \qquad c_2^{(\underline{m})} = \sqrt{\frac{\mu^{(\underline{m})}}{\rho_0^{(\underline{m})}}} \tag{11}$$

are the speeds of dilatation and distortion waves respectively. It follows from equation (10) that, under the condition $\sigma_0^{(m)} = 0$, equations (10) coincide with the corresponding equations derived in the classical linear theory of elastodynamics [**11**].

Now the solution to equations (10) is considered. Because the strip load is harmonic in time, only the stationary case will be considered; all the dependent variables become harmonic and can be represented as

$$\{u_n^{(\underline{m})}, \sigma_{nj}^{(\underline{m})}, \varepsilon_{nj}^{(\underline{m})}\}(x_1, x_2, t) = \{\bar{u}_n^{(\underline{m})}, \bar{\sigma}_{nj}^{(\underline{m})}, \bar{\varepsilon}_{nj}^{(\underline{m})}\}(x_1, x_2)\,\mathrm{e}^{\mathrm{i}\omega t} \tag{12}$$

where an overbar denotes the amplitude of the relevant quantity. Below this overbar will be omitted.

If equation (11) is employed in equations (2) to (10), by replacing $\partial^2 u_i^{(m)}/\partial t^2$ by $-\omega^2 u_i^{(m)}$ the same equations and conditions for the amplitude of the sought quantities are obtained. Consequently, introducing the dimensionless coordinates $x_i \to x_i/h$, the parameter $a \to a/h$, and the dimensionless frequency

$$\Omega = \frac{\omega h}{c_2^{(1)}} \tag{13}$$

the equations for the potentials $\varphi^{(m)}$ and $\psi^{(m)}$ can be obtained as

$$\nabla^2\varphi^{(\underline{m})} + \frac{\sigma_0^{(\underline{m})}}{\lambda^{(\underline{m})}+2\mu^{(\underline{m})}}\frac{\partial^2\varphi^{(\underline{m})}}{\partial x_1^2} + \frac{(c_2^{(1)})^2}{(c_1^{(\underline{m})})^2}\Omega^2\varphi^{(\underline{m})} = 0$$

$$\nabla^2\psi^{(\underline{m})} + \frac{\sigma_0^{(\underline{m})}}{\mu^{(\underline{m})}}\frac{\partial^2\psi^{(\underline{m})}}{\partial x_1^2} + \frac{(c_2^{(1)})^2}{(c_2^{(\underline{m})})^2}\Omega^2\psi^{(\underline{m})} = 0 \tag{14}$$

For the solution to equations (14) the exponential Fourier transformation

$$f_{\mathrm{F}}(sx_2) = \int_{-\infty}^{+\infty} f(x_1,x_2)\,\mathrm{e}^{\mathrm{i}sx_1}\,\mathrm{d}x_1 \tag{15}$$

is employed. After applying equation (15) to equations (14) the functions $\varphi_{\mathrm{F}}^{(m)}$ and $\psi_{\mathrm{F}}^{(m)}$ are determined as

$$\begin{aligned}
\bar{\varphi}_{\mathrm{F}}^{(1)} &= A_1^{(1)}(s)\,\mathrm{e}^{\gamma_1^{(1)}x_2} + A_2^{(1)}(s)\,\mathrm{e}^{-\gamma_1^{(1)}x_2}\\
\bar{\varphi}_{\mathrm{F}}^{(2)} &= A_1^{(2)}(s)\,\mathrm{e}^{\gamma_1^{(2)}x_2}\\
\bar{\psi}_{\mathrm{F}}^{(1)} &= B_1^{(1)}(s)\,\mathrm{e}^{\gamma_2^{(1)}x_2} + B_2^{(1)}(s)\,\mathrm{e}^{-\gamma_2^{(1)}x_2}\\
\bar{\psi}_{\mathrm{F}}^{(2)} &= B_1^{(2)}(s)\,\mathrm{e}^{\gamma_2^{(2)}x_2}
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
(\gamma_1^{(\underline{m})})^2 &= s^2\left(1+\frac{\sigma_0^{(\underline{m})}}{\lambda^{(\underline{m})}+2\mu^{(\underline{m})}}\right) - \frac{(c_2^{(1)})^2}{(c_1^{(\underline{m})})^2}\Omega^2\\
(\gamma_2^{(\underline{m})})^2 &= s^2\left(1+\frac{\sigma_0^{(\underline{m})}}{\mu^{(\underline{m})}}\right) - \frac{(c_2^{(1)})^2}{(c_2^{(\underline{m})})^2}\Omega^2
\end{aligned} \tag{17}$$

The unknowns $A_1^{(1)}, A_2^{(1)}, A_1^{(2)}, B_1^{(1)}, B_2^{(1)}$, and $B_1^{(2)}$ which enter equations (16) are determined from the contact conditions (5) and boundary conditions (6). By the usual mathematical procedure, expressions are also determined for $u_{n\mathrm{F}}^{(m)}$, $\varepsilon_{nj\mathrm{F}}^{(m)}$, and $\sigma_{nj\mathrm{F}}^{(m)}$ $(n,j,m=1,2)$ from which the original of the sought values is represented as

$$\begin{aligned}
&\{u_{\underline{n}}^{(\underline{m})}, \sigma_{\underline{nj}}^{(\underline{m})}, \varepsilon_{\underline{nj}}^{(\underline{m})}\}(x_1,x_2)\\
&\quad = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\{u_{\underline{n\mathrm{F}}}^{(\underline{m})}, \sigma_{\underline{nj\mathrm{F}}}^{(\underline{m})}, \varepsilon_{\underline{nj\mathrm{F}}}^{(\underline{m})}\}(sx_2)\,\mathrm{e}^{-\mathrm{i}sx_1}\,\mathrm{d}s
\end{aligned} \tag{18}$$

The expressions for $u_{n\mathrm{F}}^{(m))}$, $\varepsilon_{nj\mathrm{F}}^{(m)}$, and $\sigma_{nj\mathrm{F}}^{(m)}$ are cumbersome; therefore they are not given here. To determine the stresses acting on the interface the integrals

$$\sigma_{n2}^{(m)}(x_1,-1) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\sigma_{n2\mathrm{F}}^{(m)}(-s)\,\mathrm{e}^{-\mathrm{i}sx_1}\,\mathrm{d}s \tag{19}$$

are calculated numerically.

Using the problem symmetry the integrals are represented as

$$\begin{aligned}
\sigma_{22}^{(m)}(x_1,-1) &= \frac{1}{\pi}\int_0^{+\infty}\sigma_{22\mathrm{F}}^{(m)}(-s)\cos(sx_1)\,\mathrm{d}s\\
\sigma_{12}^{(m)}(x_1,-1) &= \frac{-\mathrm{i}}{\pi}\int_0^{+\infty}\sigma_{12\mathrm{F}}^{(m)}(-s)\sin(sx_1)\,\mathrm{d}s
\end{aligned} \tag{20}$$

Note that the expression for $\sigma_{12\mathrm{F}}^{(m)}$ has the form $\mathrm{i}|\sigma_{12\mathrm{F}}^{(m)}|$; therefore the values of the second integral in equation (20) are also real.

Now some remarks are made on the calculation of the improved integrals (20). Under the calculation procedure these improved integrals are replaced by the corresponding definite integrals; i.e. the relation $\int_0^{\infty}(.)\,\mathrm{d}s = \int_0^{s_*}(.)\,\mathrm{d}s$ is used. The values of s_* are determined from the convergence criterion of the improved integrals.

Consider the determination of the singular points of the integrated expressions in equations (20). As the unknowns in equation (16) can be expressed as

$$\begin{aligned}
&(A_1^{(1)}(s); A_2^{(1)}(s); \ldots; B_2^{(2)}(s))\\
&\quad = \frac{1}{\det\|\alpha_{ij}(s)\|}(\det\|\beta_{ij}^{A_1^{(1)}(s)}\|; \det\|\beta_{ij}^{A_2^{(1)}(s)}\|; \ldots;\\
&\qquad\qquad \det\|\beta_{ij}^{B_2^{(2)}(s)}\|)
\end{aligned} \tag{21}$$

these singular points coincide with the roots of the equations

$$\det\|\alpha_{ij}(s)\| = 0, \qquad i,j = 1,2,3,4,5,6 \tag{22}$$

with respect to s, where α_{ij} are the coefficients of the unknowns in the algebraic equation system obtained from contact (5) and boundary (6) conditions for those. Note that the expressions for

$$\|\beta_{ij}^{A_1^{(1)}(s)}\|, \|\beta_{ij}^{A_2^{(1)}(s)}\|, \ldots, \|\beta_{ij}^{B_2^{(2)}(s)}\|$$

are obtained from $\|\alpha_{ij}(s)\|$ by replacing the corresponding column of $\|\alpha_{ij}(s)\|$ with the right-hand side of the above-mentioned algebraic equation system.

Numerical analysis shows that the order of the roots of equation (22) with respect to s is one. Therefore, the order of all singular points of the integrated expressions in equations (20) is also one. Taking this situation into account for the solution to equation

(22) with respect to s, the well-known bisection method is employed.

Denote the roots of equation (22) by

$$s_1 < s_2 < \cdots < s_k < \cdots < s_N \tag{23}$$

The number N in equation (23) depends mainly on the values of the dimensionless frequency Ω, mechanical and geometrical parameters of the layer and half-plane.

Thus, after determination of the roots (23) the integration interval $[0, s_*]$ in equation (20) is separated into parts according to

$$\int_0^{s_*} (.)\,\mathrm{d}s = \int_0^{s_1-\varepsilon} (.)\,\mathrm{d}s + \int_{s_1+\varepsilon}^{s_2-\varepsilon} (.)\,\mathrm{d}s + \cdots + \int_{s_N+\varepsilon}^{s_*} (.)\,\mathrm{d}s \tag{24}$$

Consequently, calculation of the integrals (20) is made in Cauchy's principal value sense and, in equation (24), ε is a very small value which is numerically determined from the convergence requirement of the integrals (20). Further, each of the integration intervals $[s_k + \varepsilon, s_{k+1} - \varepsilon]$ is also divided into a certain number of shorter integration intervals and for each of these the Gauss integration algorithm is employed. All these procedures are carried out automatically on a personal computer using the programs written in MATLAB.

4 NUMERICAL RESULTS AND DISCUSSION

It is assumed that $\sigma_0^{(1)} > 0$, $\sigma_0^{(2)} = 0$, and $\nu^{(1)} = \nu^{(2)} = 0.25$ where $\nu^{(1)}$ and $(\nu^{(2)})$ are Poisson's ratios of the layer and half-plane materials respectively. Further, it is assumed that $\rho_0^{(1)} = \rho_0^{(2)}$, and the parameter $\eta_1 = \sigma_0^{(1)}/\mu^{(1)}$ is introduced. Moreover, the notation $\sigma_{22}^{(1)}|_{x_2=-1} = \sigma_{22}^{(2)}|_{x_2=-1} = \sigma_{22}$ is also introduced. Note that, in the present paper, only the distribution of the normal stress σ_{22} acting on the interface will be considered and through the parameter η_1 the influence of the pre-stretching of the covered layer on this distribution will be investigated. Consider the cases $E^{(1)}/E^{(2)} = 1$, $E^{(1)}/E^{(2)} > 1$, and $E^{(1)}/E^{(2)} < 1$ separately, where $E^{(1)}$ and $E^{(2)}$ are Young's moduli of elasticity for the layer and half-plane materials respectively.

4.1 The case where $E^{(1)}/E^{(2)} = 1$

As noted above, it is assumed that in the half-plane there is no initial stretching. This case corresponds to the problem considered in reference [**12**]. However, in reference [**12**], the Lamé constants of the half-plane were assumed to be complex and through the imaginary parts of these constants the attenuation property of the ground was taken into account. Furthermore, in reference [**12**], only the displacement distribution was considered. Therefore, below no comparison of the present results with those of reference [**12**] will be made.

Figures 2(a), (b), and (c) show the graphs of the dependences between σ_{22}/P_0 and x_1/h for various Ω for $a/h = 0.05$, $a/h = 1.0$, and $a/h = 3.0$ respectively.

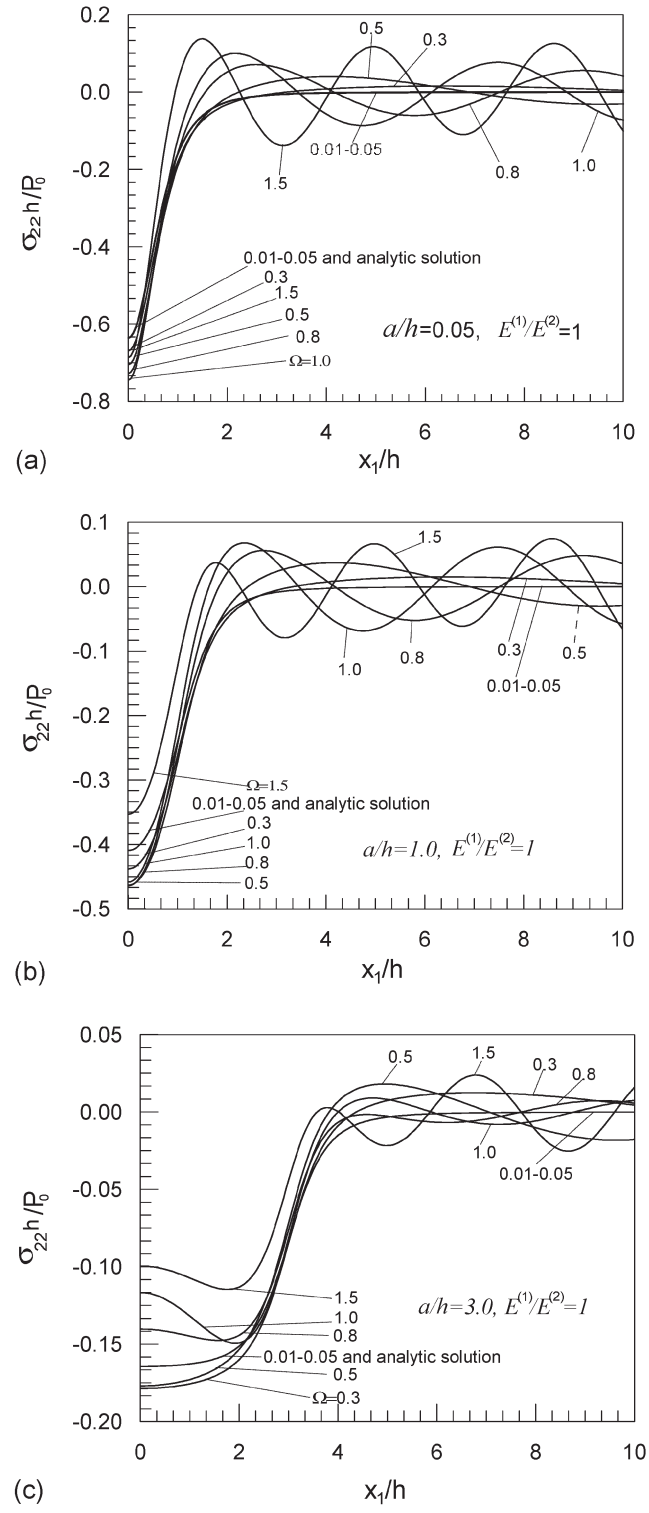


Fig. 2 The distribution of σ_{22}/P_0 with respect to x_1/h in $x_2/h = 1$ for the half-plane: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$

In these figures the graphs of the same dependences for the corresponding static problem are also given. It is known that the latter problem has an analytical solution given, for example, in reference [**20**] and elsewhere. According to the well-known mechanical consideration, with $\Omega \to 0$ the values of σ_{22} must approach the corresponding values obtained in the static loading. This situation is also observed in Fig. 2 and the graphs constructed for the static problem coincide with those constructed for $\Omega \in [0.01, 0.05]$ with very high accuracy.

This statement warrants the validity of the algorithms and programs used in the present investigations. Also, the graphs given in Fig. 2 indicate that the values of σ_{22} at the point $x_1 = 0$ depend nonmonotonically on Ω and the vibrational character of the distribution of σ_{22} with respect to x_1/h becomes clearer with increasing Ω. The concentration of the stress σ_{22} increases with increasing a.

The distribution of the stress σ_{22} in the interphase plane will be considered later because the adhesion strength of the covered layer–half-space system depends mainly on the values and distributions of this stress.

4.2 The case where $E^{(1)}/E^{(2)} > 1$

The numerical results regarding this case show that the values of σ_{22} decrease with increasing $E^{(1)}/E^{(2)}$. Therefore, here only the cases for which $E^{(1)}/E^{(2)} = 5$ and 10 respectively are considered. Firstly, it is assumed that $\eta^{(1)} = 0$ and the distribution of the stress σ_{22} is investigated with respect to x_1/h. The graphs of this distribution are given in Figs 3 and 4 for $E^{(1)}/E^{(2)} = 5$ and 10 respectively. Figures 3(a) and 4(a), Figs 3(b) and 4(b), and Figs 3(c) and 4(c) correspond to the values of $a/h = 0.05$, 1.0, and 3.0 respectively. It follows from these graphs that for the considered change in the range of Ω the absolute values of σ_{22} at $x_1/h = 0$ decrease with increasing Ω. However, the vibrational character of the distribution of σ_{22}/P_0 with respect to x_1/h becomes clearer with increasing Ω. Moreover, the attenuation rate of σ_{22} with x_1/h decreases with increasing Ω. The increase in the a/h values causes a decrease in the concentration of the stress σ_{22}.

The influence of the initial stretching of the covered layer on the values of the stresses σ_{22} is considered. For this purpose the notation

$$\psi_{22} = \frac{(\sigma_{22}|_{\eta_1=0} - \sigma_{22}|_{\eta_1>0}) \times 10^3}{P_0} \tag{25}$$

is introduced. The graphs of the dependences between ψ_{22} and x_1/h are given in Figs 5(a), (b), and (c) for $\Omega = 1.0$, $E^{(1)}/E^{(2)} = 5$ with various η_1 and with $a/h = 0.05$, 1.0, and 3.0 respectively. It follows from these graphs that there exist some interface points at which the initial stretching of the covered layer does not influence the values of σ_{22}. However, in the interface areas between these points, as a

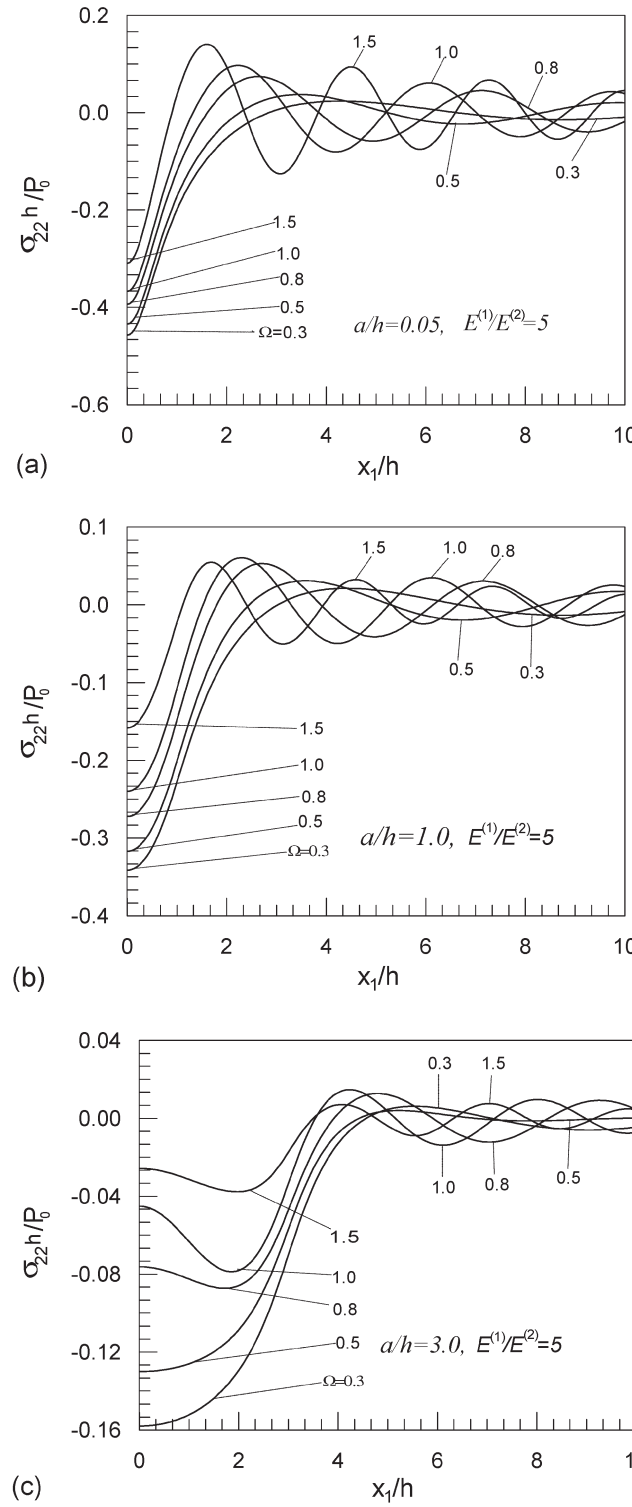
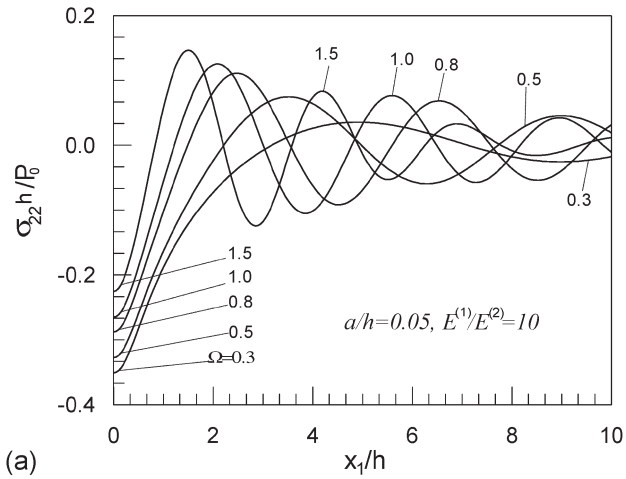
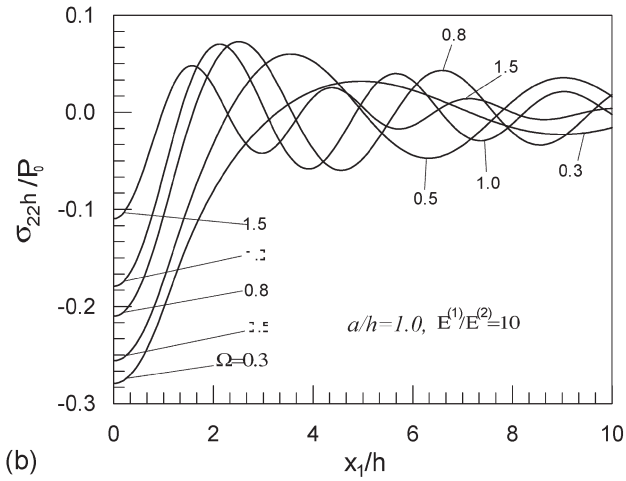


Fig. 3 The distribution of σ_{22}/P_0 with respect to x_1/h in the interface plane for various Ω in the case where $E^{(1)}/E^{(2)} = 5$ and $\eta_1 = 0.000$: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$





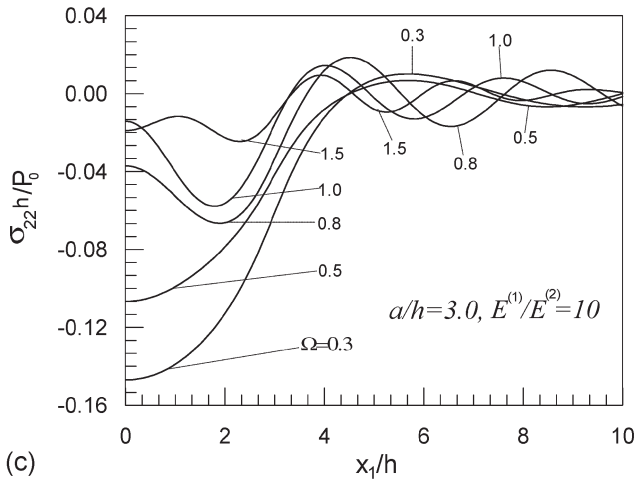
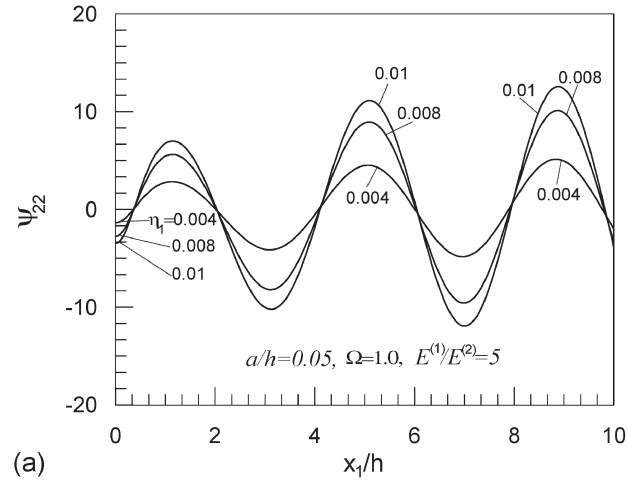
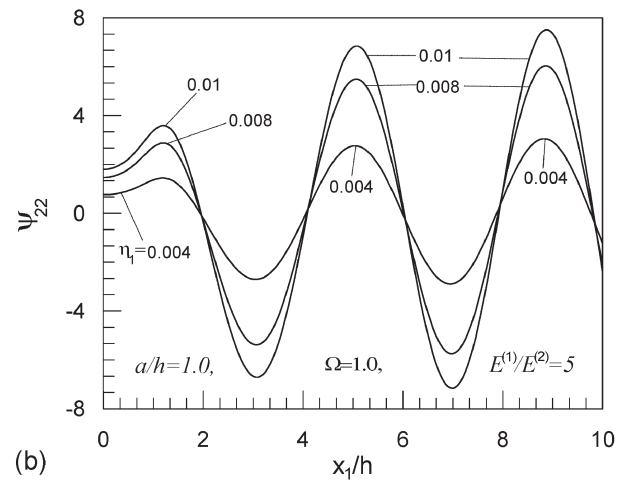


Fig. 4 The distribution of σ_{22}/P_0 with respect to x_1/h in the interface plane for various Ω in the case where $E^{(1)}/E^{(2)} = 10$ and $\eta_1 = 0.000$: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$





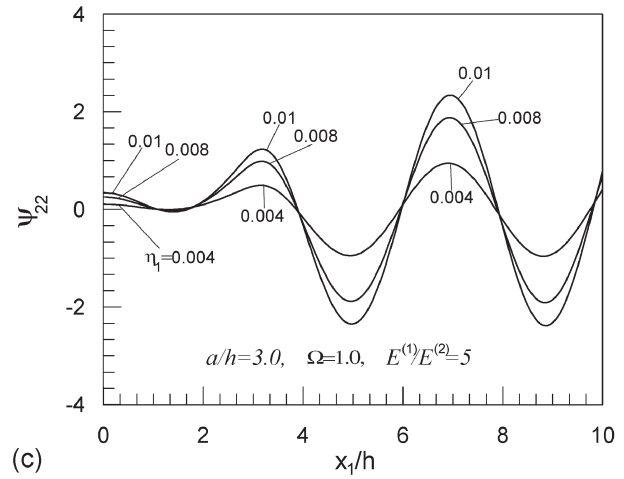


Fig. 5 The distribution ψ_{22} [equation (25)] with respect to x_1/h in the interface plane for various η_1 in the case where $E^{(1)}/E^{(2)} = 5$ and $\Omega = 1.0$: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$

result of the initial stretching, the values of σ_{22} decrease and increase alternatively. The influence of η_1 on the values of σ_{22} increases monotonically with increasing η_1.

Figure 6 shows the graphs of the dependences between ψ_{22} and x_1/h for various Ω with $a/h = 1$, $E^{(1)}/E^{(2)} = 5$ and $\eta_1 = 0.01$. According to these graphs, it can be concluded that the frequency of

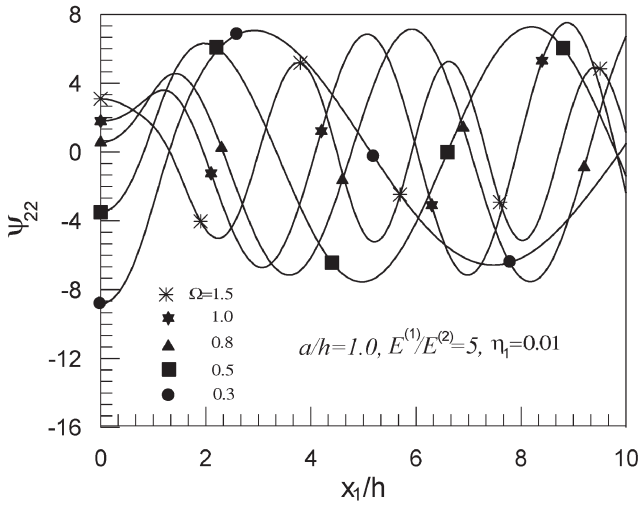


Fig. 6 The distribution ψ_{22} [equation (25)] with respect to x_1/h in the interface plane for various Ω in the case where $E^{(1)}/E^{(2)} = 5$, $a/h = 1.0$, and $\eta_1 = 0.01$

the 'vibrational' character of this distribution increases monotonically with increasing Ω. Here the results regarding the influence of η_1 on the values of σ_{22} are examined only in the case where $E^{(1)}/E^{(2)} = 5$. Similar results are also obtained for $E^{(1)}/E^{(2)} = 10$ and other values of $E^{(1)}/E^{(2)} (> 1)$.

Note that with decreasing a/h the results obtained in the present section approach the corresponding results obtained in reference [**1**].

4.3 The case where $E^{(1)}/E^{(2)} < 1$

As in the previous cases, first the numerical results obtained for $\eta_1 = 0$ are considered. These results indicate that the considered stress distributions approach each other asymptotically with decreasing $E^{(1)}/E^{(2)}$ and it follows from the numerical results, which are not given here, that the corresponding graphs constructed for the cases $E^{(1)}/E^{(2)} \leqslant 0.05$ coincide. Consequently, for the case where $E^{(1)}/E^{(2)} = 0.05$ the half-space can be taken as the absolute rigid basis on which the layer lies. Taking into account that this last case is very interesting in the practical sense, further only the numerical results obtained for $E^{(1)}/E^{(2)} = 0.05$ will be considered.

Thus the graphs given in Figs 7(a), (b), and (c) show the dependences between σ_{22}/P_0 and x_1/h for various Ω and for $a/h = 0.05$, 1.0, and 3.0 respectively. It follows from these results that the absolute maximum value of the considered stress increases monotonically with increasing Ω. However, these values of stress σ_{22} decrease with increasing a/h. These results also indicate that the vibrational character of the considered distribution which has been observed in the cases where $E^{(1)}/E^{(2)} \geqslant 1$ disappears for $E^{(1)}/E^{(2)} = 0.05$.

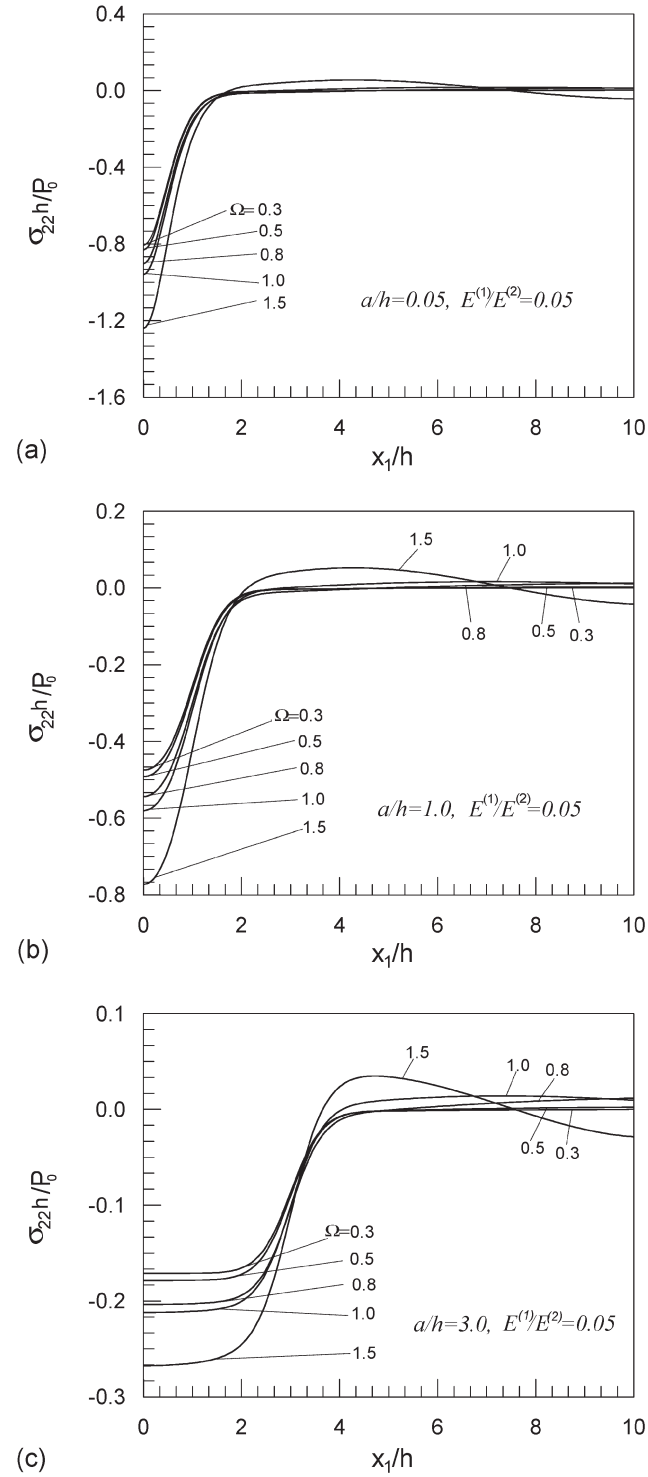


Fig. 7 The distribution of σ_{22}/P_0 with respect to x_1/h in the interface plane for various Ω in the case where $E^{(1)}/E^{(2)} = 0.05$ and $\eta_1 = 0.000$: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$

Now the influence of the prestretching of the layer on the considered stress distribution is considered. In Figs 8(a), (b), and (c) the graphs of the dependences between ψ_{22} and x_1/h are given for various η_1 and for $a/h = 0.05$, 1.0, and 3.0 respectively in the case

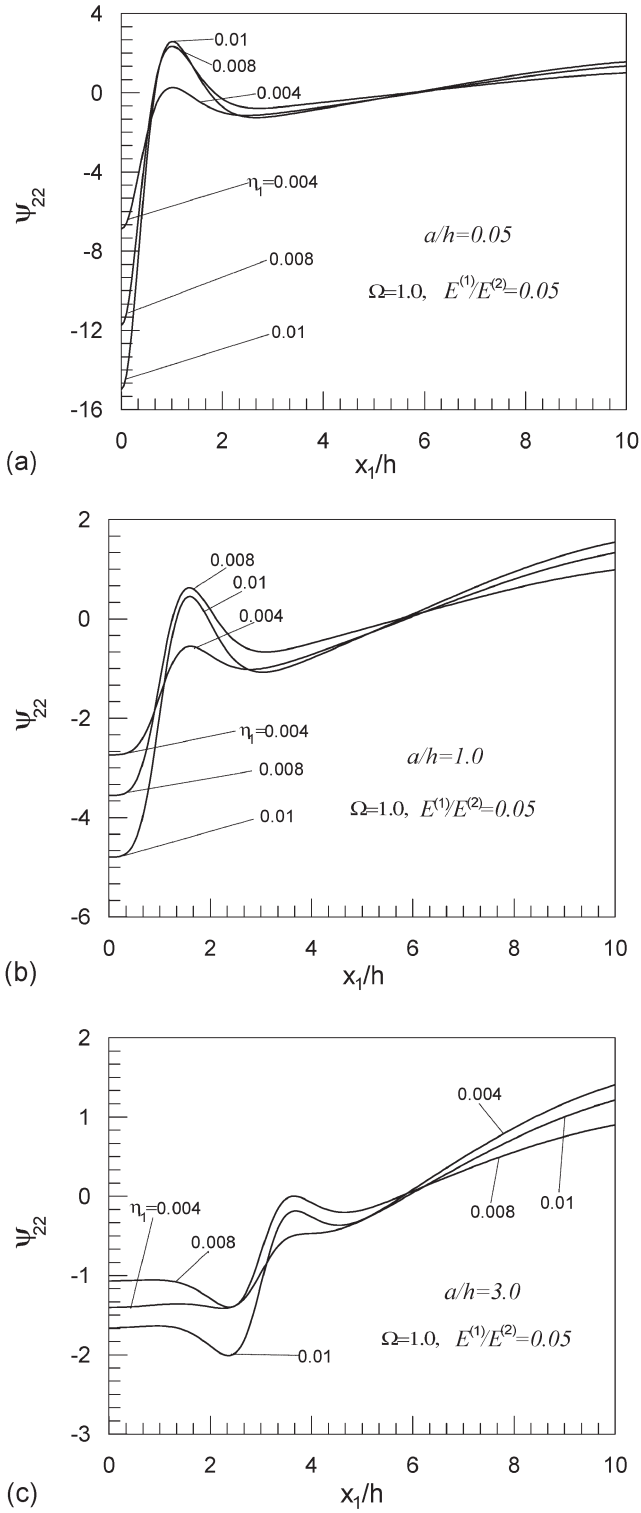


Fig. 8 The distribution ψ_{22} [equation (25)] with respect to x_1/h in the interface plane for various η_1 in the case where $E^{(1)}/E^{(2)} = 0.05$ and $\Omega = 1.0$: (a) for $a/h = 0.05$; (b) for $a/h = 1.0$; (c) for $a/h = 3.0$

where $\Omega = 1.0$. These graphs show that, as a result of the initial stretching of the layer, the values of σ_{22} decrease around the point $x_1/h = 0$. The influence of this prestretching on the stress h has been

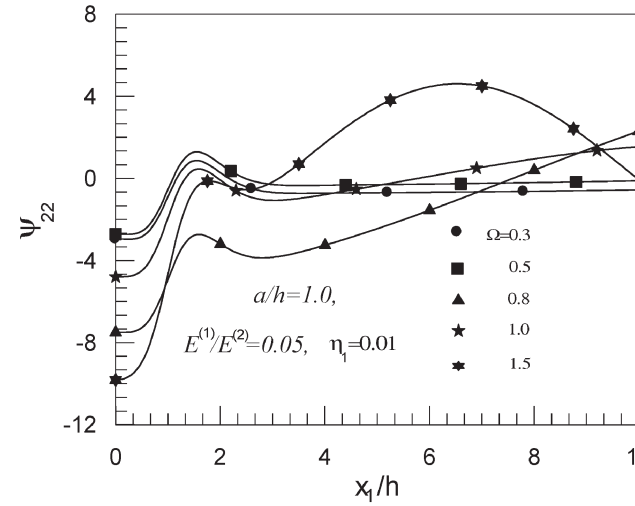


Fig. 9 The distribution ψ_{22} [equation (25)] with respect to x_1/h in the interface plane for various Ω in the case where $E^{(1)}/E^{(2)} = 0.05$, $a/h = 1.0$, and $\eta_1 = 0.01$

observed in the distribution that increases with increasing η_1. The vibrational character of the distribution of ψ_{22} which has been observed in the cases where $E^{(1)}/E^{(2)} > 1$ disappears in the present case. Furthermore, the considered influence becomes more complicated as the values a/h increase.

Figure 9 shows the graphs of the dependences between ψ_{22} and x_1/h for various Ω with $a/h = 1$ and $\eta_1 = 0.01$. It follows from these graphs that the dependence between ψ_{22} and Ω has a non-monotonic character.

5 CONCLUSIONS

In the present paper the dynamical problem of the stress distribution in a half-plane covered with a pre-stretched layer has been investigated within the framework of the piecewise homogeneous body model by employing the TLTEWISB. The formulation of the problem and all solution procedures have been made for the case where the half-space and covered layer have homogeneous uniaxial initial stresses.

It is assumed that the free face plane of the covered layer is subjected to a uniformly distributed harmonic load acting on a strip extending to infinity in the x_3 direction, which is perpendicular to the x_1–x_2 plane and is of width $2a$ in the x_1 direction. The plane-strain state in the x_1–x_2 plane is analysed. Moreover, it is assumed that, between the layer and the half-plane, complete contact conditions are satisfied. The materials of the layer and the half-plane are taken to be homogeneous, isotropic, and linearly elastic. The corresponding boundary-value problems are investigated by employing the exponential Fourier integral transformation. The algorithm for obtaining

concrete numerical results has been developed. These results have been obtained for the case where the half-plane does not have any initial stresses but the covered layer is pre-stretched initially. The influences of the quantities of this prestretching on the distribution of the normal stress acting on the interface plane are analysed.

Analyses are made separately for the cases where $E^{(1)}/E^{(2)} = 1$, $E^{(1)}/E^{(2)} > 1$, and $E^{(1)}/E^{(2)} < 1$, where $E^{(1)}$ and $E^{(2)}$ are the moduli of elasticity of the layer and half-plane materials respectively. In each case, the influence of the dimensionless velocity $\Omega = \omega h/c_2^{(1)}$ (where ω is the frequency of the external force, h is the thickness of the layer, and $c_2^{(1)}$ is the distortion wave speed for the layer material) on the distribution of the above-mentioned normal stress is investigated first. After this examination, the influence of the pre-stretching of the layer on this distribution is analysed. The initial stretching is estimated through the parameter $\eta_1 = \sigma_0^{(1)}/\mu^{(1)}$, where $\sigma_0^{(1)}$ is the initial normal stress acting in the x_1 direction in the layer and $\mu^{(1)}$ is the shear modulus of the layer material.

According to the numerical results obtained, in particular, the following conclusions can be drawn:

1. In the cases where $E^{(1)}/E^{(2)} > 1$ for the considered range of change in Ω (i.e. for $0 \leqslant \Omega \leqslant 1.5$) in the vicinity of the strip loading the absolute values of interface normal stress decrease with increasing Ω as well as with increasing a/h.
2. The values of the interface normal stress decrease with increasing $E^{(1)}/E^{(2)}$.
3. In cases where $E^{(1)}/E^{(2)} > 1$ the distribution of the considered stress with respect to x_1/h has a vibrational character and this character becomes stronger with increasing Ω.
4. In cases where $E^{(1)}/E^{(2)} < 1$ the absolute maximum of the considered stress increases and the corresponding distribution graphs approach each other asymptotically with decreasing $E^{(1)}/E^{(2)}$. The corresponding graphs obtained for $E^{(1)}/E^{(2)} \leqslant 0.05$ coincide.
5. In cases where $E^{(1)}/E^{(2)} \ll 1$ the vibrational character of the considered stress distribution disappears. The results obtained for the case where $E^{(1)}/E^{(2)} = 0.05$ can be taken as the results regarding the case where the considered layer lies on an absolutely rigid basis.
6. In the case where $E^{(1)}/E^{(2)} = 0.05$ the absolute maximum value of the stress increases (decreases) monotonically with increasing Ω and increasing a/h.
7. In all the considered cases the influence of the pre-stretching of the layer on the considered stress distribution increases monotonically with increasing η_1.
8. In cases where $E^{(1)}/E^{(2)} > 1$ the influence of the pre-stretching of the layer on the considered stress distribution has a vibrational character with respect to x_1/h and this character becomes stronger with increasing Ω.
9. In cases where $E^{(1)}/E^{(2)} < 1$ the above-mentioned vibrational character of the influence of the pre-stretching of the layer on the stress distribution disappears.

The results obtained in the case where $\eta_1 \equiv 0$ correspond to those attained within the framework of the classical linear theory of elasticity. Because, in this case, equations (2) of the linearized theory of elasticity coincide with the corresponding equations of the *classical* linear theory of elasticity, consequently the numerical results presented in the paper are also significant from this viewpoint.

Moreover, the presented results have the following application for mathematical modelling of problems of ground vibration in engineering. To investigate the motion transmitted to the ground through the foundation of a vibrating machine the ground is modelled by a system composed of a mass, spring, and dashpot [**16**, **21**, **22**]. Consequently, the above numerical results can be taken as standard for the corresponding results obtained within the framework of the above-mentioned approximate modelling of the ground (half-space).

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APPENDIX

Notation

$2a$	width of the area on which the external load acts
$c_2^{(1)}$	speed of the distortion waves in the layer material
$E^{(1)}$	Young's modulus of the layer material
$E^{(2)}$	Young's modulus of the half-plane material
h	thickness of the covered layer
$P_0/2a$	magnitude of the external load acting per unit length in the x_1 direction
$u_i^{(1)}$	components of the displacement vector of the layer
$u_i^{(2)}$	components of the displacement vector of the half-plane
δ_i^j	Kronecker symbols $= \delta_{ij}$
$\varepsilon_{ij}^{(1)}$	components of the strain tensor of the layer
$\varepsilon_{ij}^{(2)}$	components of the strain tensor of the half-plane
η_1	parameter characterizing the magnitude of the initial stretching of the layer $= \sigma_0^{(1)}/\mu^{(1)}$
$\lambda^{(1)}, \mu^{(1)}$	Lamé constants of the layer material
$\lambda^{(2)}, \mu^{(2)}$	Lamé constants of the half-plane material
$\sigma_{ij}^{(1)}$	components of the stress tensor of the layer
$\sigma_{ij}^{(2)}$	components of the stress tensor of the half-plane
$\sigma_0^{(1)}$	initial normal stress acting in the layer
σ_{22}	interface normal stress
ω	frequency of the external loading
Ω	dimensionless frequency of the external loading $= \omega h/c_2^{(1)}$