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Weak Solution of High Order of Accuracy Difference Scheme for the Nonlinear System of sine-Gordon Equations

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Abstract

The Sine-Gordon equation is a well-known nonlinear hyperbolic partial differential equation that arises in various physical applications, including nonlinear optics, field theory, and crystal dislocation dynamics. In this study an unconditionally stable fourth-order of accuracy difference scheme that corresponds to the nonlinear system of sine-Gordon equations within the framework of Sobolev spaces is considered. Our primary objective is to investigate the existence and uniqueness of the weak solution by employing variational analysis techniques. We establish the necessary mathematical foundation and derive the corresponding a priori estimates to ensure the well-posedness of the proposed scheme. The results contribute to the theoretical development of high-accuracy numerical methods for solving nonlinear wave equations and provide valuable insights into the stability and convergence properties of difference schemes in variational settings.

Keywords: sine-Gordon equation, partial differential equations, Sobolev space, weak solution.

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