Neutrosophic 2-absorbing primary ideals on lattices

SALIH YAZAR*, *Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, 34220, Istanbul, Turkiye.*

SERKAN ONAR, *Department of Mathematical Engineering, Yildiz Technical University, Davutpaşa-Istanbul, Turkiye.*

E. MEHMET ÖZKAN, *Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, 34220, Istanbul, Turkiye.*

KOSTAQ HILA, *Department of Mathematics & Computer Science,University of Gjirokastra, Gjirokastra 6001, Albania.*

Abstract

Recent advances in neutrosophic lattice theory have highlighted its potential to model uncertainty, indeterminacy, and inconsistency more effectively than classical and intuitionistic fuzzy approaches. Building upon foundational studies that introduced neutrosophic lattices and their extensions through soft structures, concept lattices, triangular norms, and lattice-valued neutrosophic sets, this study contributes to the growing body of research on neutrosophic ideals. We introduce and investigate several key notions in the context of lattices, including neutrosophic prime ideals, prime neutrosophic ideals, neutrosophic 2-absorbing ideals, 2-absorbing neutrosophic ideals, neutrosophic primary ideals, and primary neutrosophic ideals. A special focus is placed on the characterization of neutrosophic 2-absorbing primary ideals, accompanied by an analysis of the transitions among these concepts and their interrelationships with traditional lattice ideals. Furthermore, these structures are examined within lattices formed by Cartesian products, offering new perspectives on their behavior in complex settings.

Keywords: lattice; neutrosophic prime ideal; neutrosophic primary ideal; neutrosophic 2-absorbing ideal

1 Introduction

Fuzzy sets, introduced by Zadeh in 1965, extend the classical notion of set membership by allowing each element to belong to a set with a degree of membership in the interval [0, 1] [24]. This approach represents a significant advancement in modeling uncertainty. Later, in 1986, Atanassov introduced intuitionistic fuzzy sets, which added dimension by associating each element with a degree of membership (truth), non-membership (falsity), and hesitation [2]. Intuitionistic fuzzy sets can be viewed as a proper generalization of fuzzy sets.

*E-mail: slhyzr101@gmail.com

Vol. 33, No. 6, © The Author(s) 2025. Published by Oxford University Press. All rights reserved.
For permissions, please e-mail: journals.permission@oup.com.
https://doi.org/10.1093/jigpal/jzaf088

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

In the literature concerning fuzzy ideals in lattice structures, various generalizations have been introduced. The paper titled "Generalizations of Prime Fuzzy Ideals of a Lattice" [19] extends the classical notion of prime ideals and characterizes fuzzy prime ideals through their α-level sets, thereby enriching the structural understanding of fuzzy ideals and paving the way for more flexible ideal types. Building upon these developments, neutrosophic sets, first proposed by Smarandache [18], have emerged as a powerful tool by allowing degrees of truth, indeterminacy, and falsity to be defined independently within [0, 1], thus exceeding the constraints of classical and intuitionistic fuzzy approaches. Among their various formulations, single-valued neutrosophic sets [5] stand out by combining theoretical generality with computational feasibility, making them well-suited for extending ideal-related notions in lattice structures. Within this neutrosophic context, 2-absorbing ideal-type structures can be redefined and explored to achieve greater theoretical depth and practical expressiveness.

The study of neutrosophic lattices has witnessed significant growth in recent years. The foundational contributions of Kandasamy and Smarandache [10, 11] introduced neutrosophic lattices and pseudo lattice graphs, laying the groundwork for subsequent theoretical and applied studies. Building upon this foundation, Uluçay *et al.* [12] investigated neutrosophic soft lattices, while Singh [14–17] made extensive contributions through three-way fuzzy and bipolar neutrosophic concept lattices, interval-valued representations, and complex neutrosophic lattices, demonstrating their applicability in domains such as air quality analysis and multi-criteria decision-making. Complementary developments by Zhang *et al.* [26] on inclusion relations and Hu and Zhang [7] on neutrosophic triangular norms and residuated lattices have further expanded the structural framework.

Parallel to these theoretical advancements, significant attention has been given to neutrosophic ideals and filters. Borzooei and Jun [4], along with Jun and Xin [9], explored neutrosophic LI-ideals within lattice implication algebras, while Zedam, Milles, and Bennoui [25], as well as Bennoui *et al.* [3], examined multiple types of neutrosophic ideals and filters on lattices. James and Mathew [8] enriched this direction through lattice-valued neutrosophic sets, and Topal and Smarandache [21] introduced a novel neutrosophic approach to adjectival phrases from a lattice-theoretic perspective.

On the application side, Yang and Qin [23] proposed neutrosophic three-way concept lattices for conflict analysis, while Vimala *et al.* [22] applied lattice-ordered neutrosophic soft sets to aid large-scale companies in selecting the best NGOs for fund distribution during the COVID-19 pandemic. Most recently, Agarwal *et al.* [1] extended the scope by studying ideals and filters on neutrosophic topologies generated by neutrosophic relations, underscoring the versatility of neutrosophic frameworks in both theoretical and practical settings.

Taken together, these studies illustrate the dynamic evolution of neutrosophic lattice theory. From foundational structures and conceptual modeling to the characterization of ideals and real-world applications, neutrosophic lattices have developed into a robust interdisciplinary field. Motivated by these advancements, the present work extends the study of lattice ideals by introducing and characterizing 2-absorbing ideal-type structures within the framework of single-valued neutrosophic sets, thereby bridging theoretical generalizations with practical modeling capacity in complex uncertain environments.

The rest of this paper is organized as follows. Section 2 presents the necessary preliminaries and definitions. In Section 3, we introduce the concepts of neutrosophic prime ideal and prime neutrosophic ideal, and present their fundamental properties. Section 4 is devoted to the introduction of neutrosophic 2-absorbing ideals, and their relationship with neutrosophic prime ideals is examined. In Section 5, we define 2-absorbing neutrosophic ideals and explore their connection with prime neutrosophic ideals. In Section 6, we introduce the definition of radical for neutrosophic, followed by the concepts of the neutrosophic primary ideal and the neutrosophic 2-absorbing primary ideal.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Then, we discuss their relationship with the neutrosophic prime ideals and neutrosophic 2-absorbing ideals. In Section 7, we introduce the concepts of primary neutrosophic ideal and 2-absorbing primary neutrosophic ideal. Then, we discuss their relationship with the prime neutrosophic ideals and 2-absorbing neutrosophic ideals. In Section 8, we investigate how these concepts extend to lattices formed by the Cartesian product of individual lattices and examine whether their structural properties are preserved. Finally, Section 9 provides a general evaluation of the findings and outlines potential directions for future research.

2 Preliminaries

2.1 Ideals on lattices

In this section, we will first provide information about our working area, which is lattices, and then about the concept of ideals and their types on lattices. Let us start with our working area. First, we take a partially ordered set. Then, if the supremum and infimum of any two elements selected from this partially ordered set are defined, we call this set a lattice. Let us express this definition as follows:

DEFINITION 2.1
[6]. A partially ordered set $(\Pi; \leq)$ is called a lattice if for every $x, y \in \Pi$, $inf\{x, y\}$, and $sup\{x, y\}$ are defined.

We will revise the definition as in [6], as follows:

DEFINITION 2.2
[6] $(\Pi, \wedge, \vee)$ is a called lattice if for $\forall x, y \in \Pi$, $x \wedge y \in \Pi$, $x \vee y \in \Pi$ where $x \wedge y = inf\{x, y\}$, $x \vee y = sup\{x, y\}$. Also, $\wedge$ denotes the meet operation, and $\vee$ denotes the join operation.

In the context of our research, the lattice Π that we will consider in our investigation will have zero as its smallest element and one as its largest. So, $0, 1 \in (\Pi, \wedge, \vee)$.

EXAMPLE 2.3
[6] Let $\Pi = \{0, \alpha, \beta, 1\}$ and let α and β be elements that cannot be comparable (if $\alpha \leq \beta$ or $\beta \leq \alpha$, then α and β can be comparable; otherwise, they cannot be comparable). If so,

$$\leq = \{(0,0), (0,\alpha), (0,\beta), (0,1), (\alpha,\alpha), (\alpha,1), (\beta,\beta), (\beta,1), (1,1)\}$$

we can write ((x, y) means $x \leq y$). According to this relation, we can draw our diagram as in Fig. 1.

Throughout this work, unless otherwise stated, Π is a lattice. Now, let's define an ideal of Π.

DEFINITION 2.4
[20] A nonempty subset Λ of a lattice Π is called an ideal, if for $x, y \in \Pi$, the following conditions hold. (i) If $x, y \in \Lambda$, then $x \vee y \in \Lambda$, and (ii) if $x \leq y$ and $y \in \Lambda$, then $x \in \Lambda$. When $x \wedge y \in \Lambda$ implying either $x \in \Lambda$ or $y \in \Lambda$, then a proper ideal Λ (i.e. $\Lambda \neq \Pi$) is named as a prime ideal Π.

In our examples in this study, we will use principal ideals. Roughly speaking, we can say the following: we select a point on the lattice diagram and take all the points connected downward from

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

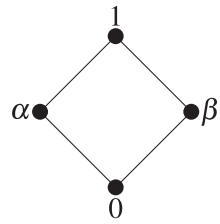


Figure 1. A representative from the family of 4-element lattices.

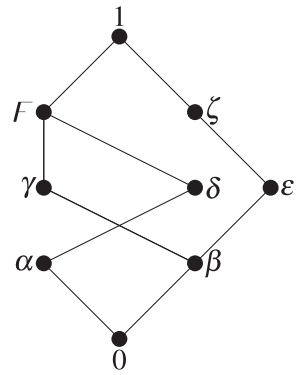


Figure 2. A representative from the family of 9-element lattices.

that point. We call this set of points a principal ideal. We will symbolically denote it as $(\alpha] = \{x | x \leq \alpha\}$, as shown in [6].

EXAMPLE 2.5
Given in Fig. 2 is a lattice Π, and $(\Gamma] = \{0, \alpha, \beta, \gamma, \delta, \Gamma\}$, $(\epsilon] = \{0, \beta, \epsilon\}$ are ideals on Π.

DEFINITION 2.6
[20] Let Π be a lattice. An ideal Λ of Π is called a 2-absorbing ideal, if for $x, y, z \in \Pi$, $x \wedge y \wedge z \in \Lambda$ implies that either $x \wedge y \in \Lambda$ or $y \wedge z \in \Lambda$ or $z \wedge x \in \Lambda$.

DEFINITION 2.7
[20] Let Π be a lattice. An ideal Λ of Π is called a primary ideal if, for $x, y \in \Pi$, $x \wedge y \in \Lambda$ implies that either $x \in \Lambda$ or $y \in \sqrt{\Lambda}$, where $\sqrt{\Lambda}$ denotes the radical of Λ (i.e. the intersection of all prime ideals containing Λ). If there does not exist a prime ideal containing an ideal Λ in a lattice Π, then we define $\sqrt{\Lambda} = \Pi$.

2.2 Single valued neutrosophic set

DEFINITION 2.8
[5] Let X be a nonempty set. A single valued neutrosophic set *SVNS* A of X is an object of the following form $A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$, where the functions $T_A : X \to [0, 1]$,

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$I_A : X \to [0,1]$, and $F_A : X \to [0,1]$ define the truth membership function, the indeterminacy membership function, and the falsity membership function of the element $x \in X$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The family of all single-valued neutrosophic sets on X is denoted by $SVNS(X)$.

DEFINITION 2.9
[5] If $A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ and $B = \{< x, T_B(x), I_B(x), F_B(x) > | x \in X\}$ are any two *SVNS* of X,then

(1) $A \subseteq B \iff T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x), \forall x \in X$;
(2) $A = B \iff T_A(x) = T_B(x), I_A(x) = I_B(x)$ and $F_A(x) = F_B(x), \forall x \in X$.
(3) $A^{\complement} = \{< x, F_A(x), 1 - I_N(x), T_A(x) > | x \in X\}$;
(4) $A \cup B = \{< x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x) > | x \in X\}$,where
$T_{A\cup B}(x) = max\{T_A(x), T_B(x)\}, I_{A\cup B}(x) = max\{I_A(x), I_B(x)\}$ and $F_{A\cup B}(x) = min\{F_A(x), F_B(x)\}$;
(5) $A \cap B = \{< x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x) > | x \in X\}$,where
$T_{A\cap B}(x) = min\{T_A(x), T_B(x)\}, I_{A\cap B}(x) = min\{I_A(x), I_B(x)\}$ and $F_{A\cap B}(x) = max\{F_A(x), F_B(x)\}$.

DEFINITION 2.10
[3] *SVNS* A of X is called a single valued neutrosophic ideal (*SVNSI*) of Π if the following conditions are satisfied for all $x, y \in X$:

$$(i) T_A(x \vee y) \geq min\{T_A(x), T_A(y)\}; (ii) T_A(x \wedge y) \geq max\{T_A(x), T_A(y)\};$$
$$(iii) I_A(x \vee y) \geq min\{I_A(x), I_A(y)\}; (iv) I_A(x \wedge y) \geq max\{I_A(x), I_A(y)\};$$
$$(v) F_A(x \vee y) \leq max\{F_A(x), F_A(y)\}; (vi) F_A(x \wedge y) \leq min\{F_A(x) F_A(y)\}.$$

DEFINITION 2.11
[5] Let X be a set, $A \in SVNS(X)$ and let $\alpha \in [0,1]$. Then the set $A_\alpha = \{x | T_A(x) \geq \alpha, I_A(x) \geq \alpha,$ and $F_A(x) \leq \alpha, x \in \Pi\}$ is called a α-the level subset of A.

3 Neutrosophic prime ideals and prime neutrosophic ideals of lattices

In this section, we will introduce the concepts of neutrosophic prime ideal (*NPI*) and prime neutrosophic ideal (*PNI*). Let's start with the first definition, *NPI*.

DEFINITION 3.1
[3] A proper *SVNI* $N =< T_N, I_N, F_N >$ of a lattice Π is called a *NPI*, if for all $x, y \in \Pi$, $T_N(x \wedge y) \leq T_N(x) \vee T_N(y)$, $I_N(x \wedge y) \leq I_N(x) \vee I_N(y)$ and $F_N(x \wedge y) \geq F_N(x) \wedge F_N(y)$.

DEFINITION 3.2
[13] Let Π be a nonempty fixed set. Let Λ be a nonempty subset of Π. The neutrosophic characteristic function of the set Λ, denoted by $\chi_\Lambda(\alpha) = (\chi_{T_\Lambda}(\alpha), \chi_{I_\Lambda}(\alpha), \chi_{F_\Lambda}(\alpha))$, is defined as

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

follows:

$$\chi_{\Lambda}(\alpha) = \begin{cases} (1,1,0), & \text{if } \alpha \in \Lambda \\ (0,0,1), & \text{if } \alpha \notin \Lambda \end{cases}$$

Here, χ_{T_Λ}, $\chi_{I\Lambda}$, and $\chi_{F\Lambda}$ represent the truth-membership, indeterminacy-membership, and falsity-membership functions of the neutrosophic characteristic function, respectively.

THEOREM 3.3
Λ is a prime ideal of Π if and only if $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a *NPI* of Π.

PROOF. ($\Rightarrow$): Let χ_Λ be a *SVNI* of Π. Assume that Λ is a prime ideal of Π. Let $x, y \in \Pi$.
(i) If $x, y \in \Lambda$, then $x \wedge y \in \Lambda$ and we have

$$\chi_{T_\Lambda}(x \wedge y) = 1 \leq 1 = 1 \vee 1 = \chi_{T_\Lambda}(x) \vee \chi_{T_\Lambda}(y),$$
$$\chi_{I_\Lambda}(x \wedge y) = 1 \leq 1 = 1 \vee 1 = \chi_{I_\Lambda}(x) \vee \chi_{I_\Lambda}(y), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) = 0 \geq 0 = 0 \wedge 0 = \chi_{F_\Lambda}(x) \wedge \chi_{F_\Lambda}(y).$$

(ii) if $x, y \notin \Lambda$, then as Λ is a prime ideal, $x \wedge y \notin \Lambda$ and we have

$$\chi_{T_\Lambda}(x \wedge y) = 0 \leq 0 = 0 \vee 0 = \chi_{T_\Lambda}(x) \vee \chi_{T_\Lambda}(y),$$
$$\chi_{I_\Lambda}(x \wedge y) = 0 \leq 0 = 0 \vee 0 = \chi_{I_\Lambda}(x) \vee \chi_{I_\Lambda}(y), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) = 1 \geq 1 = 1 \wedge 1 = \chi_{F_\Lambda}(x) \wedge \chi_{F_\Lambda}(y).$$

(iii) If $x \in \Lambda, y \notin \Lambda$. Then $x \wedge y \in \Lambda$. We have

$$\chi_{T_\Lambda}(x \wedge y) = 1 \leq 1 = 1 \vee 0 = \chi_{T_\Lambda}(x) \vee \chi_{T_\Lambda}(y),$$
$$\chi_{I_\Lambda}(x \wedge y) = 1 \leq 1 = 1 \vee 0 = \chi_{I_\Lambda}(x) \vee \chi_{I_\Lambda}(y), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) = 0 \geq 0 = 0 \wedge 1 = \chi_{F_\Lambda}(x) \wedge \chi_{F_\Lambda}(y).$$

Hence χ_Λ is a *NPI* of Π. ($\Leftarrow$): Assume that χ_Λ is a *NPI* of Π. Let $x \wedge y \in \Lambda$. Then

$$\chi_{T_\Lambda}(x \wedge y) = 1 \leq \chi_{T_\Lambda}(x) \vee \chi_{T_\Lambda}(y) \text{ and}$$
$$\chi_{I_\Lambda}(x \wedge y) = 1 \leq \chi_{I_\Lambda}(x) \vee \chi_{I_\Lambda}(y) \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) = 0 \geq \chi_{F_\Lambda}(x) \wedge \chi_{F_\Lambda}(y).$$

So either $x \in \Lambda$ or $y \in \Lambda$. In this way, Λ must be a prime ideal of Π. □

DEFINITION 3.4
A *SVNI* $N =< T_N, I_N, F_N >$ of Π is called a *PNI* of Π if for any two *SVNI* $M =< T_M, I_M, F_M >$ and $K =< T_K, I_K, F_K >$ of a lattice Π if $M \cap K \subseteq N$ imply that either $M \subseteq N$ or $K \subseteq N$.

THEOREM 3.5
Λ is a prime ideal of Π if and only if χ_Λ is a *PNI* of Π.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

PROOF. ($\Rightarrow$): Assume that Λ is a prime ideal of Π. Let $M =< T_M, I_M, F_M >$ and $K =< T_K, I_K, F_K >$ be *SVNI* of Π. Assume that $M \cap K \subseteq \chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$. If $M \nsubseteq \chi_\Lambda, K \nsubseteq \chi_\Lambda$, then there exist $x, y \in \Pi$ such that

$$\chi_{T_\Lambda}(x) < T_M(x), \chi_{I_\Lambda}(y) < I_M(x), \chi_{F_\Lambda}(x) > F_M(x) \text{ and}$$
$$\chi_{T_\Lambda}(y) < T_K(y), \chi_{I_\Lambda}(y) < I_K(y), \chi_{F_\Lambda}(y) > F_K(y).$$

By definition of χ_Λ, we conclude that $x, y \notin \Lambda$. For, if say $x \in \Lambda$, then $\chi_{T_\Lambda}(x) = 1$, $\chi_{I_\Lambda}(x) = 1$ and $\chi_{F_\Lambda}(x) = 0$ lead to $1 < T_M(x)$, $1 < I_M(x)$ and $0 > F_M(x)$, which is not possible. Since Λ is a prime ideal of Π, we get $x \wedge y \notin \Lambda$. Hence $\chi_{T_\Lambda}(x \wedge y) = 0$. Since M, K are *SVNI*s of Π, we have

$$T_M(x) \leq T_M(x \wedge y), I_M(x) \leq I_M(x \wedge y), F_M(x) \geq F_M(x \wedge y) \text{ and}$$
$$T_K(y) \leq T_K(x \wedge y), I_K(y) \leq I_K(x \wedge y), F_K(y) \geq F_K(x \wedge y).$$

As the image of any element under an *SVNS* is a non-negative number, from the above, we get

$$\begin{aligned}
\chi_{T_\Lambda}(x \wedge y) &= 0 \\
&\leq \chi_{T_\Lambda}(x) \wedge \chi_{T_\Lambda}(y) \\
&< T_M(x) \wedge T_K(y) \\
&\leq T_M(x \wedge y) \wedge T_K(x \wedge y) \\
&= (T_M \cap T_K)(x \wedge y) \leq \chi_{T_\Lambda}(x \wedge y) \\
&= 0.
\end{aligned}$$

$$\begin{aligned}
\chi_{I_\Lambda}(x \wedge y) &= 0 \\
&\leq \chi_{I_\Lambda}(x) \wedge \chi_{I_\Lambda}(y) \\
&< I_M(x) \wedge I_K(y) \\
&\leq I_M(x \wedge y) \wedge I_K(x \wedge y) \\
&= (I_M \cap I_K)(x \wedge y) \leq \chi_{I_\Lambda}(x \wedge y) \\
&= 0.
\end{aligned}$$

$$\begin{aligned}
\chi_{F_\Lambda}(x \wedge y) &= 1 \\
&\geq \chi_{F_\Lambda}(x) \vee \chi_{F_\Lambda}(y) \\
&> F_M(x) \vee F_K(y) \\
&\geq F_M(x \wedge y) \vee F_K(x \wedge y) \\
&= (F_M \cup F_K)(x \wedge y) \geq \chi_{F_\Lambda}(x \wedge y) \\
&= 1.
\end{aligned}$$

Thus, we get $0 < 0$, which is not possible. Hence either $M \subseteq \chi_\Lambda$ or $K \subseteq \chi_\Lambda$. ($\Leftarrow$): Assume that A is a *PNI* of Π. Suppose that for some $x, y \in \Pi, x \wedge y \in \Lambda$ but $x, y \notin \Lambda$. Define *SVNI*s M and K of

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Π as follows.

$$M(\alpha) = \begin{cases} (1,1,0), & \alpha \in (x] \\ (0,0,1), & otherwise \end{cases}$$

$$K(\alpha) = \begin{cases} (1,1,0), & \alpha \in (y] \\ (0,0,1), & otherwise \end{cases}$$

Then $M \cap K \subseteq \chi_\Lambda$ but neither $M \subseteq \chi_\Lambda$ nor $K \subseteq \chi_\Lambda$, a contradiction. Hence Λ is a prime ideal of Π. □

EXAMPLE 3.6
Let's examine the lattice Π illustrated in Fig. 1. In this lattice, the ideal Λ taken $\Lambda = (0]$ is not a prime ideal of Π, because $\alpha \wedge \beta = 0 \in \Lambda$ but $\alpha \notin \Lambda$, $\beta \notin \Lambda$. Therefore $\chi_{T_\Lambda}(\alpha \wedge \beta) = 1$ and $\chi_{T_\Lambda}(\alpha) = \chi_{T_\Lambda}(\beta) = 0$, $\chi_{I_\Lambda}(\alpha \wedge \beta) = 1$ and $\chi_{I_\Lambda}(\alpha) = \chi_{I_\Lambda}(\beta) = 0$, $\chi_{F_\Lambda}(\alpha \wedge \beta) = 0$ and $\chi_{F_\Lambda}(\alpha) = \chi_{F_\Lambda}(\beta) = 1$. Hence $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is not a *NPI* of Π.

EXAMPLE 3.7
Let's examine the lattice Π illustrated in Fig. 1. Define *SVNI*s $M =< T_M, I_M, F_M >$ and $K =< T_K, I_K, F_K >$ of a lattice Π as follows. $M = \{(0;1,1,0), (\alpha;1/2,1/2,1/2), (\beta;0,0,1), (1;0,0,1)\}$ $K = \{(0;1,1,0), (\alpha;0,0,1), (\beta;1/3,1/5,2/3), (1;0,0,1)\}$ Then when $\Lambda = (0]$, $M \cap K \subseteq \chi_\Lambda$, but $M \nsubseteq \chi_\Lambda$ and $K \nsubseteq \chi_\Lambda$. Thus χ_Λ is not a *PNI* of Π. If $\Lambda = (\alpha]$ or $\Lambda = (\beta]$, χ_Λ is a *PNI*.

4 Neutrosophic 2-absorbing ideals

In this section, we will introduce the concept of neutrosophic 2-absorbing ideal (*N*2*AI*) of a lattice. Then, we will discuss its relationship with the *NPI* concept introduced in the previous section.

DEFINITION 4.1
A proper *SVNI* $N =< T_N, I_N, F_N >$ of a lattice Π is called an *N*2*AI* of Π, if for all $x, y, z \in \Pi$,

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x) \text{ and}$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x) \text{ and}$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x).$$

LEMMA 4.2
Λ is a 2-absorbing ideal of Π if and only if $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a *N*2*AI* of Π.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

PROOF. ($\Rightarrow$): Assume that Λ is a 2-absorbing ideal of Π. Let $x,y,z \in \Pi$. If $x \wedge y \wedge z \in \Lambda$, then as Λ is 2-absorbing, either $x \wedge y \in \Lambda$ or $y \wedge z \in \Lambda$ or $z \wedge x \in \Lambda$. Thus in this case,

$$\chi_{T_\Lambda}(x \wedge y \wedge z) \leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{T_\Lambda}(y \wedge z) \vee \chi_{T_\Lambda}(z \wedge x),$$
$$\chi_{I_\Lambda}(x \wedge y \wedge z) \leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{I_\Lambda}(y \wedge z) \vee \chi_{I_\Lambda}(z \wedge x), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y \wedge z) \geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{F_\Lambda}(y \wedge z) \wedge \chi_{F_\Lambda}(z \wedge x).$$

If $x \wedge y \wedge z \notin \Lambda$, then clearly, $x \wedge y \notin \Lambda$, $y \wedge z \notin \Lambda$, and $z \wedge x \notin \Lambda$. Thus in this case,

$$\chi_{T_\Lambda}(x \wedge y \wedge z) \leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{T_\Lambda}(y \wedge z) \vee \chi_{T_\Lambda}(z \wedge x),$$
$$\chi_{I_\Lambda}(x \wedge y \wedge z) \leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{I_\Lambda}(y \wedge z) \vee \chi_{I_\Lambda}(z \wedge x), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y \wedge z) \geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{F_\Lambda}(y \wedge z) \wedge \chi_{F_\Lambda}(z \wedge x).$$

Hence χ_Λ is a $N2AI$ of Π. ($\Leftarrow$): Assume that χ_Λ is an $N2AI$ of Π. Let $x,y,z \in \Pi$ be such that $x \wedge y \wedge z \in \Lambda$, but $x \wedge y \notin \Lambda$, $y \wedge z \notin \Lambda$ and $z \wedge x \notin \Lambda$. This implies that $\chi_{T_\Lambda}(x \wedge y \wedge z) = 1$ and $\chi_{T_\Lambda}(x \wedge y) = \chi_{T_\Lambda}(y \wedge z) = \chi_{T_\Lambda}(z \wedge x) = 0$. Then,

$$\chi_{T_\Lambda}(x \wedge y \wedge z) \not\leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{T_\Lambda}(y \wedge z) \vee \chi_{T_\Lambda}(z \wedge x),$$
$$\chi_{I_\Lambda}(x \wedge y \wedge z) \not\leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{I_\Lambda}(y \wedge z) \vee \chi_{I_\Lambda}(z \wedge x), \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y \wedge z) \not\geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{F_\Lambda}(y \wedge z) \wedge \chi_{F_\Lambda}(z \wedge x)$$

a contradiction, as χ_Λ is a $N2AI$. □

DEFINITION 4.3
Let X be a set, $A \in SVNS(X)$ and let $\alpha, \beta, \gamma \in [0,1]$. Then the set $A_{(\alpha,\beta,\gamma)} = \{x | T_A(x) \geq \alpha, I_A(x) \geq \beta, \text{ and } F_A(x) \leq \gamma, x \in \Pi\}$ is called a (α,β,γ)-the level subset of A.

LEMMA 4.4
A $SVNI$ $N =< T_N, I_N, F_N >$ of Π is a $N2AI$ if and only if for each $\alpha, \beta, \gamma \in [0,1]$, the level set $A_{(\alpha,\beta,\gamma)}$ is a 2-absorbing ideal of Π.

PROOF. ($\Rightarrow$): Let $N =< T_N, I_N, F_N >$ be a $N2AI$ of Π. Let $\alpha, \beta, \gamma \in [0,1]$. Let $x,y,z \in \Pi$ be such that $x \wedge y \wedge z \in A_{(\alpha,\beta,\gamma)}$. Then $\alpha \leq T_N(x \wedge y \wedge z)$, $\beta \leq I_N(x \wedge y \wedge z)$, and $\gamma \geq F_N(x \wedge y \wedge z)$. Since N is a $N2AI$,

$$\alpha \leq T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x) \text{ and}$$
$$\beta \leq I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x) \text{ and}$$
$$\gamma \geq F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x),$$

then either

$$\alpha \leq T_N(x \wedge y) \text{ or } \alpha \leq T_N(y \wedge z) \text{ or } \alpha \leq T_N(z \wedge x) \text{ and}$$
$$\beta \leq I_N(x \wedge y) \text{ or } \beta \leq I_N(y \wedge z) \text{ or } \beta \leq I_N(z \wedge x) \text{ and}$$
$$\gamma \geq F_N(x \wedge y) \text{ or } \gamma \geq F_N(y \wedge z) \text{ or } \gamma \geq F_N(z \wedge x).$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Thus either $x \wedge y$ or $y \wedge z$ or $z \wedge x \in A_{(\alpha,\beta,\gamma)}$. So, $A_{(\alpha,\beta,\gamma)}$ is a 2-absorbing ideal of Π. ($\Leftarrow$): $A_{(\alpha,\beta,\gamma)}$ be a 2-absorbing ideal of Π,for each $\alpha, \beta, \gamma \in [0,1]$. Let $x, y, z \in \Pi$ and $\alpha = T_N(x \wedge y \wedge z)$, $\beta = I_N(x \wedge y \wedge z)$, $\gamma = F_N(x \wedge y \wedge z)$. Then $x \wedge y \wedge z \in A_{(\alpha,\beta,\gamma)}$. Since $A_{(\alpha,\beta,\gamma)}$ is a 2-absorbing ideal of Π, either $x \wedge y$ or $y \wedge z$ or $z \wedge x \in A_{(\alpha,\beta,\gamma)}$. This implies that

$$\alpha \leq T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_A(z \wedge x) \text{ and}$$
$$\beta \leq I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x) \text{ and}$$
$$\gamma \geq F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x).$$

Thus N is a $N2AI$ of Π. □

LEMMA 4.5
Let $N =< T_N, I_N, F_N >$ be a *NPI* of Π. Then N is a $N2AI$ of Π.

PROOF. Let N be a *NPI* of Π. Then for all $x, y \in \Pi$, $T_N(x \wedge y) \leq T_N(x) \vee T_N(y)$, $I_N(x \wedge y) \leq I_N(x) \vee I_N(y)$ and $F_N(x \wedge y) \geq F_N(x) \wedge F_N(y)$. Hence for all $x, y, z \in \Pi$, we have

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(z)$$
$$T_N(x \wedge y \wedge z) \leq T_N(y \wedge z) \vee T_N(x)$$
$$T_N(x \wedge y \wedge z) \leq T_N(z \wedge x) \vee T_N(y) \text{ and}$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(z)$$
$$I_N(x \wedge y \wedge z) \leq I_N(y \wedge z) \vee I_N(x)$$
$$I_N(x \wedge y \wedge z) \leq I_N(z \wedge x) \vee I_N(y) \text{ and}$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(z)$$
$$F_N(x \wedge y \wedge z) \geq F_N(y \wedge z) \wedge F_N(x)$$
$$F_N(x \wedge y \wedge z) \geq F_N(z \wedge x) \wedge F_N(y).$$

Hence

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(z) \vee T_N(y \wedge z) \vee T_N(x) \vee T_N(z \wedge x) \vee T_N(y),$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(z) \vee I_N(y \wedge z) \vee I_N(x) \vee I_N(z \wedge x) \vee I_N(y),$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(z) \wedge F_N(y \wedge z) \wedge F_N(x) \wedge F_N(z \wedge x) \wedge F_N(y).$$

By the definition of a *NPI*, it follows that for any $x, y \in \Pi$, $T(x) \leq T(x \wedge y)$, $I(x) \leq I(x \wedge y)$, and $F(x) \geq F(x \wedge y)$. Hence, we can write

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x),$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x),$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x).$$

Thus $N =< T_N, I_N, F_N >$ is a $N2AI$ of Π. □

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

EXAMPLE 4.6
Let's examine the lattice Π illustrated in Fig. 1. Let $N =< T_N, I_N, F_N >$ be the *SVNS* defined by $T_N(0) = 1$, $I_N(0) = 5/6$, $F_N(0) = 0$, $T_N(\alpha) = 0$, $I_N(\alpha) = 3/5$, $F_N(\alpha) = 1$, $T_N(\beta) = 1/2$, $I_N(\beta) = 1/4$, $F_N(\beta) = 1/2$, $T_N(1) = 0$, $I_N(1) = 1/2$, $F_N(1) = 1$. Then N is a $N2AI$ of Π. However, N is not a *NPI* as

$$0 = F_N(0) = F_N(\alpha \wedge \beta) \not\geq 1 \wedge 1/2 = F_N(\alpha) \wedge F_N(\beta).$$

LEMMA 4.7
The intersection of any two distinct *NPI* of Π is an $N2AI$ of Π.

PROOF. Let $N =< T_N, I_N, F_N >$, $M =< T_M, I_M, F_M >$, be two distinct *NPI*s of Π. We know that $N \cap M =< T_{N\cap M}, I_{N\cap M}, F_{N\cap M} >$ where $T_{N\cap M} = T_N \wedge T_M$, $I_{N\cap M} = I_N \wedge I_M$, and $F_{N\cap M} = F_N \vee F_M$. So, for any $\alpha \in \Pi$,

$$T_{A\cap B}(\alpha) = T_A(\alpha) \wedge T_B(\alpha), I_{A\cap B}(\alpha) = I_A(\alpha) \wedge I_B(\alpha) \text{ and } F_{A\cap B}(\alpha) = F_A(\alpha) \vee F_B(\alpha).$$

Let $x, y, z \in \Pi$. We have

$$\begin{aligned} T_{N\cap M}(x \wedge y \wedge z) &= T_N(x \wedge y \wedge z) \wedge T_M(x \wedge y \wedge z) \text{ and} \\ I_{N\cap M}(x \wedge y \wedge z) &= I_N(x \wedge y \wedge z) \wedge I_M(x \wedge y \wedge z) \text{ and} \\ F_{N\cap M}(x \wedge y \wedge z) &= F_N(x \wedge y \wedge z) \vee F_M(x \wedge y \wedge z). \end{aligned}$$

Since every *NPI* is $N2AI$, we get

$$\begin{aligned} T_{N\cap M}(x \wedge y \wedge z) &\leq [T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x)] \\ &\quad \wedge [T_M(x \wedge y) \vee T_M(y \wedge z) \vee T_M(z \wedge x)] \text{ and} \\ I_{N\cap M}(x \wedge y \wedge z) &\leq [I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x)] \\ &\quad \wedge [I_M(x \wedge y) \vee I_M(y \wedge z) \vee I_M(z \wedge x)] \text{ and} \\ F_{N\cap M}(x \wedge y \wedge z) &\geq [F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x)] \\ &\quad \vee [F_M(x \wedge y) \wedge F_M(y \wedge z) \wedge F_M(z \wedge x)]. \end{aligned}$$

Since N and M are *NPI*, we can write

$$\begin{aligned} T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x) &\leq T_N(x) \vee T_N(y) \vee T_N(z) \\ T_M(x \wedge y) \vee T_M(y \wedge z) \vee T_M(z \wedge x) &\leq T_M(x) \vee T_M(y) \vee T_M(z) \\ I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x) &\leq I_N(x) \vee I_N(y) \vee I_N(z) \\ I_M(x \wedge y) \vee I_M(y \wedge z) \vee I_M(z \wedge x) &\leq I_M(x) \vee I_M(y) \vee I_M(z) \\ F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x) &\geq F_N(x) \wedge F_N(y) \wedge F_N(z) \\ F_M(x \wedge y) \wedge F_M(y \wedge z) \wedge F_M(z \wedge x) &\geq F_M(x) \wedge F_M(y) \wedge F_M(z). \end{aligned}$$

Hence, we can write

$$\begin{aligned}
T_{N\cap M}(x\wedge y\wedge z) &\leq [T_N(x)\vee T_N(y)\vee T_N(z)]\wedge[T_M(x)\vee T_M(y)\vee T_M(z)]\\
&= [T_N(x)\wedge T_M(x)]\vee[T_N(x)\wedge T_M(y)]\vee[T_N(x)\wedge T_M(z)]\\
&\vee [T_N(y)\wedge T_M(x)]\vee[T_N(y)\wedge T_M(y)]\vee[T_N(y)\wedge T_M(z)]\\
&\vee [T_N(z)\wedge T_M(x)]\vee[T_N(z)\wedge T_M(y)]\vee[T_N(z)\wedge T_M(z)] \text{ and}\\
I_{N\cap M}(x\wedge y\wedge z) &\leq [I_N(x)\vee I_N(y)\vee I_N(z)]\wedge[I_M(x)\vee I_M(y)\vee I_M(z)]\\
&= [I_N(x)\wedge I_M(x)]\vee[I_N(x)\wedge I_M(y)]\vee[I_N(x)\wedge I_M(z)]\\
&\vee [I_N(y)\wedge I_M(x)]\vee[I_N(y)\wedge I_M(y)]\vee[I_N(y)\wedge I_M(z)]\\
&\vee [I_N(z)\wedge I_M(x)]\vee[I_N(z)\wedge I_M(y)]\vee[I_N(z)\wedge I_M(z)] \text{ and}\\
F_{N\cap M}(x\wedge y\wedge z) &\geq [F_N(x)\wedge F_N(y)\wedge F_N(z)]\vee[F_M(x)\wedge F_M(y)\wedge F_M(z)]\\
&= [F_N(x)\vee F_M(x)]\wedge[F_N(x)\vee F_M(y)]\wedge[F_N(x)\vee F_M(z)]\\
&\wedge [F_N(y)\vee F_M(x)]\wedge[F_N(y)\vee F_M(y)]\wedge[F_N(y)\vee F_M(z)]\\
&\wedge [F_N(z)\vee F_M(x)]\wedge[F_N(z)\vee F_M(y)]\wedge[F_N(z)\vee F_M(z)].
\end{aligned}$$

For any *SVNI K*, we have $T_K(\alpha)\leq T_K(\alpha\wedge\beta)$, $I_K(\alpha)\leq I_K(\alpha\wedge\beta)$, and $F_K(\alpha)\geq F_K(\alpha\wedge\beta)$, for all $\alpha,\beta\in\Pi$. This implies

$$\begin{aligned}
T_N(x)\wedge T_M(y)&\leq T_N(x\wedge y)\wedge T_M(x\wedge y)=T_{N\cap M}(x\wedge y) \text{ and}\\
I_N(x)\wedge I_M(y)&\leq I_N(x\wedge y)\wedge I_M(x\wedge y)=I_{N\cap M}(x\wedge y) \text{ and}\\
F_N(x)\vee F_M(y)&\geq F_N(x\wedge y)\vee F_M(x\wedge y)=F_{N\cap M}(x\wedge y).
\end{aligned}$$

So, we can write

$$\begin{aligned}
T_{N\cap M}(x\wedge y\wedge z) &\leq T_{N\cap M}(x)\vee T_{N\cap M}(x\wedge y)\vee T_{N\cap M}(x\wedge z)\\
&\vee T_{N\cap M}(y\wedge x)\vee T_{N\cap M}(y)\vee T_{N\cap M}(y\wedge z)\\
&\vee T_{N\cap M}(z\wedge x)\vee T_{N\cap M}(z\wedge y)\vee T_{N\cap M}(z) \text{ and}\\
I_{N\cap M}(x\wedge y\wedge z) &\leq I_{N\cap M}(x)\vee I_{N\cap M}(x\wedge y)\vee I_{N\cap M}(x\wedge z)\\
&\vee I_{N\cap M}(y\wedge x)\vee I_{N\cap M}(y)\vee I_{N\cap M}(y\wedge z)\\
&\vee I_{N\cap M}(z\wedge x)\vee I_{N\cap M}(z\wedge y)\vee I_{N\cap M}(z) \text{ and}\\
F_{N\cap M}(x\wedge y\wedge z) &\geq F_{N\cap M}(x)\wedge F_{N\cap M}(x\wedge y)\wedge F_{N\cap M}(x\wedge z)\\
&\wedge F_{N\cap M}(y\wedge x)\wedge F_{N\cap M}(y)\wedge F_{N\cap M}(y\wedge z)\\
&\wedge F_{N\cap M}(z\wedge x)\wedge F_{N\cap M}(z\wedge y)\wedge F_{N\cap M}(z).
\end{aligned}$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

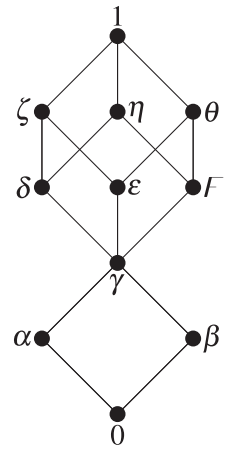


Figure 3. A representative from the family of 11-element lattices.

Since $N \cap M$ is a *SVNI*, for all $x, y \in \Pi$, we have $T_{N\cap M}(x) \leq T_{N\cap M}(x \wedge y)$, $I_{N\cap M}(x) \leq I_{N\cap M}(x \wedge y)$ and $F_{N\cap M}(x) \geq F_{N\cap M}(x \wedge y)$. So we can write

$$T_{N\cap M}(x \wedge y \wedge z) \leq T_{N\cap M}(x \wedge y) \vee T_{N\cap M}(y \wedge z) \vee T_{N\cap M}(z \wedge x) \text{ and}$$

$$I_{N\cap M}(x \wedge y \wedge z) \leq I_{N\cap M}(x \wedge y) \vee I_{N\cap M}(y \wedge z) \vee I_{N\cap M}(z \wedge x) \text{ and}$$

$$F_{N\cap M}(x \wedge y \wedge z) \geq F_{N\cap M}(x \wedge y) \wedge F_{N\cap M}(y \wedge z) \wedge F_{N\cap M}(z \wedge x).$$

Thus $N \cap M$ is a $N2AI$ of Π. □

EXAMPLE 4.8
Let's examine the lattice Π illustrated in Fig. 3.

Define $N =< T_N, I_N, F_N >$ and $M =< T_M, I_M, F_M >$ as follows.

We note that N and M are $N2AI$s of Π. But $N \cap M$ is not a $N2AI$ of Π. Because;

$$N \cap M = \{(0; 1, 1, 0), (\alpha; 1/3, 1/4, 2/3), (\beta; 1/3, 1/3, 2/3), (\gamma; 1/3, 1/2, 2/3), (\delta; 0, 0, 1),$$
$$(\epsilon; 0, 0, 1), (\Gamma; 0, 0, 1), (\zeta; 0, 0, 1), (\eta; 0, 0, 1), (\theta; 0, 0, 1), (1, 0, 0, 1)\}$$

we can write

$$T_{N\cap M}(\zeta \wedge \eta \wedge \theta) \nleq T_{N\cap M}(\zeta \wedge \eta) \vee T_{N\cap M}(\eta \wedge \theta) \vee T_{N\cap M}(\zeta \wedge \theta) \text{ and}$$

$$I_{N\cap M}(\zeta \wedge \eta \wedge \theta) \nleq I_{N\cap M}(\zeta \wedge \eta) \vee I_{N\cap M}(\eta \wedge \theta) \vee I_{N\cap M}(\zeta \wedge \theta) \text{ and}$$

$$F_{N\cap M}(\zeta \wedge \eta \wedge \theta) \ngeq F_{N\cap M}(\zeta \wedge \eta) \wedge F_{N\cap M}(\eta \wedge \theta) \wedge F_{N\cap M}(\zeta \wedge \theta).$$

5 2-absorbing neutrosophic ideals

In this section, we will introduce the concept of a 2-absorbing neutrosophic ideal (2*ANI*). Then, we will discuss its relationship with the *PNI* concept introduced in the previous section.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$A(0) = (1, 1, 0)$	$B(0) = (1, 1, 0)$
$A(\alpha) = (1/2, 1/4, 1/2)$	$B(\alpha) = (1/3, 1/2, 2/3)$
$A(\beta) = (2/3, 2/5, 1/3)$	$B(\beta) = (1/3, 1/3, 2/3)$
$A(\gamma) = (1/2, 1/2, 1/2)$	$B(\gamma) = (1/3, 2/3, 2/3)$
$A(\delta) = (1/2, 4/9, 1/2)$	$B(\delta) = (0, 0, 1)$
$A(\epsilon) = (0, 0, 1)$	$B(\epsilon) = (1/3, 2/3, 2/3)$
$A(F) = (0, 0, 1)$	$B(F) = (0, 0, 1)$
$A(\zeta) = (0, 0, 1)$	$B(\zeta) = (0, 0, 1)$
$A(\eta) = (0, 0, 1)$	$B(\eta) = (0, 0, 1)$
$A(\theta) = (0, 0, 1)$	$B(\theta) = (0, 0, 1)$
$A(1) = (0, 0, 1)$	$B(1) = (0, 0, 1)$

DEFINITION 5.1
A proper *SVNI* $N =< T_N, I_N, F_N >$ of Π is called a (2*ANI*) of Π if whenever $M \cap K \cap L \subseteq N$ for $M, K, L \in SVNS(\Pi)$, then either $M \cap K \subseteq N$ or $K \cap L \subseteq N$ or $L \cap M \subseteq N$.

EXAMPLE 5.2
Let's consider the *SVNI*s of the lattice Π depicted in Fig. 1. $N = \{(0; 7/8, 7/8, 1/8), (\alpha; 1/3, 1/3, 2/3), (\beta; 3/4, 3/4, 1/4), (1; 1/3, 1/3, 2/3)\}$ $M = \{(0; 1, 1, 0), (\alpha; 1/4, 1/4, 3/4), (\beta; 4/5, 4/5, 1/5), (1; 1/4, 1/4, 3/4)\}$ $K = \{(0; 1, 1, 0), (\alpha; 3/4, 3/4, 1/4), (\beta; 2/3, 2/3, 1/3), (1; 2/3, 2/3, 1/3)\}$ $L = \{(0; 4/5, 4/5, 1/5), (\alpha; 3/4, 3/4, 1/4), (\beta; 4/5, 4/5, 1/5), (1; 3/4, 3/4, 1/4)\}$ We note that N is an $N2AI$ but N is not 2*ANI*. Because $M \cap K \nsubseteq N$, $K \cap L \nsubseteq N$, and $L \cap K \nsubseteq N$.

LEMMA 5.3
Let Λ be an ideal of Π. If $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a 2*ANI* of Π, then Λ is a 2-absorbing ideal of Π.

PROOF. Assume that $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a 2*ANI* of Π. Let $\alpha \wedge \beta \wedge \gamma \in \Lambda$ for some $\alpha, \beta, \gamma \in \Pi$. Assume that $\alpha \wedge \beta \notin \Lambda$, $\beta \wedge \gamma \notin \Lambda$, and $\gamma \wedge \alpha \notin \Lambda$. Then clearly, $\alpha, \beta, \gamma \in \Pi$. Define *SVNI*s

$$N(x) = \begin{cases} (1, 1, 0), & x \in (\alpha] \\ (0, 0, 1), & otherwise \end{cases}$$

$$M(x) = \begin{cases} (1, 1, 0), & x \in (\beta] \\ (0, 0, 1), & otherwise \end{cases}$$

$$K(x) = \begin{cases} (1, 1, 0), & x \in (\gamma] \\ (0, 0, 1), & otherwise \end{cases}$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$$(N \cap M \cap K)(x) = \begin{cases} (1,1,0), & x \in (\alpha \wedge \beta \wedge \gamma] \\ (0,0,1), & otherwise \end{cases}$$

Then $N \cap M \cap K \subseteq \chi_\Lambda$ but $N \cap M \nsubseteq \chi_\Lambda$, $M \cap K \nsubseteq \chi_\Lambda$, and $K \cap N \nsubseteq \chi_\Lambda$. This contradicts the assumption that χ_Λ is a $2ANI$ of Π. □

LEMMA 5.4
Every prime *SVNI* of a lattice Π is a $2ANI$ of Π.

PROOF. Let $N =< T_N, I_N, F_N >$ be a *PNI* of Π. Suppose that $M, K, L \in SVNS(\Pi)$ and $M \cap K \cap L \subseteq N$. As N is a *PNI* of Π we have either (1) $M \cap K \subseteq N$ or $L \subseteq N$, or (2) $M \cap L \subseteq N$ or $K \subseteq N$, or (3)$K \cap L \subseteq N$ or $M \subseteq N$. Without loss of generality, suppose that $M \cap K \subseteq N$ or $L \subseteq N$. If $M \cap K \subseteq N$, then the proof is obvious, and if $L \subseteq N$, then $M \cap L \subseteq N$ and $K \cap L \subseteq N$. Thus N is a $2ANI$ of a lattice Π. □

LEMMA 5.5
The intersection of two *PNI*s of Π is a $2ANI$ of Π.

PROOF. Let N and M be two distinct *PNI*s of Π. Assume that K, L, H are *SVNI*s of Π such that $K \cap L \cap H \subseteq N \cap M$ but $K \cap L \nsubseteq N \cap M$, $K \cap H \nsubseteq N \cap M$, and $L \cap H \nsubseteq N \cap M$. $K \cap L \cap H \subseteq N$ and $K \cap L \cap H \subseteq M$. Since N and M are *PNI*s, we get; (i) $K \cap L \subseteq N$ or $H \subseteq N$ and (ii) $K \cap L \subseteq M$ or $H \subseteq M$. We have the following cases: **Case(1)**: If $K \cap L \subseteq N$ and $K \cap L \subseteq M$, then $K \cap L \subseteq N \cap M$, a contradiction. **Case(2)**: If $H \subseteq N$ and $H \subseteq M$, then we get $K \cap H \subseteq N \cap M$, a contradiction. **Case(3)**: Let $K \cap L \subseteq N$ and $H \subseteq M$. As N is a *PIFI*, we get either $K \subseteq N$ or $L \subseteq N$.Hence either $K \cap H \subseteq N \cap M$ or $L \cap H \subseteq N \cap M$, a contradiction in either case. **Case(4)**: Let $H \subseteq N$ and $K \cap L \subseteq M$. As M is a *PIFI*, we get either $K \subseteq M$ or $L \subseteq M$. Hence either $K \cap H \subseteq N \cap M$ or $L \cap H \subseteq N \cap M$, a contradiction in either case. Hence, at least one of $K \cap L$ or $L \cap H$ or $H \cap K$ must be a subset of $N \cap M$. Therefore $N \cap M$ is *PNI*. □

6 Neutrosophic primary ideals

In this section, we will introduce the definition of radical for neutrosophic, followed by the concepts of neutrosophic primary ideal (*NPRI*) and neutrosophic 2-absorbing primary ideal ($N2APRI$). Then, we will discuss their relationship with the *NPI* and $N2AI$ concepts introduced in the previous section.

DEFINITION 6.1
Let $N =< T_N, I_N, F_N >$ be a *SVNI* of a lattice Π. We define the neutrosophic prime (respectively, prime neutrosophic) radical of N as the intersection of all neutrosophic prime (respectively, prime neutrosophic) ideals containing N and we denote it by $\sqrt{N} =< \sqrt{T_N}, \sqrt{I_N}, \sqrt{F_N} >$. We note that for a *SVNI* N of L always $N \subseteq \sqrt{N}$. If $N \subseteq \sqrt{N}$, then $T_N \subseteq \sqrt{T_N}$, $I_N \subseteq \sqrt{I_N}$, and $F_N \supseteq \sqrt{F_A}$. Also, $\sqrt{T_N} = T_{\sqrt{N}}$, $\sqrt{I_N} = I_{\sqrt{N}}$, and $\sqrt{F_N} = F_{\sqrt{N}}$. Thus, we can write $\sqrt{N} =< T_{\sqrt{N}}, I_{\sqrt{N}}, F_{\sqrt{N}} >$.

DEFINITION 6.2
A proper *SVNI* $N =< T_N, I_N, F_N >$ of a lattice Π is called a *NPRI* of Π, if for $x, y \in \Pi$, $T_N(x \wedge y) \leq T_N(x) \vee \sqrt{T_N}(y)$, $I_N(x \wedge y) \leq I_N(x) \vee \sqrt{I_N}(y)$, and $F_N(x \wedge y) \geq F_N(x) \wedge \sqrt{F_N}(y)$.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

LEMMA 6.3
Λ is a primary ideal of Π if and only if $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a *NPRI* of Π.

PROOF. ($\Rightarrow$): Assume that Λ is a primary ideal of Π. Let $x, y \in \Pi$. (i) If $x \wedge y \in \Lambda$, then as Λ is a primary ideal of Π, either $x \in \Lambda$ or $y \in \sqrt{\Lambda}$. Hence,

$$\chi_{T_\Lambda}(x \wedge y) \leq \chi_{T_\Lambda}(x) \vee \sqrt{\chi_{T_\Lambda}}(y), \chi_{I_\Lambda}(x \wedge y) \leq \chi_{I_\Lambda}(x) \vee \sqrt{\chi_{I_\Lambda}}(y) \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) \geq \chi_{F_\Lambda}(x) \wedge \sqrt{\chi_{F_\Lambda}}(y).$$

(ii) If $x \wedge y \notin \Lambda$, then clearly $x \notin \Lambda$ and $y \notin \Lambda$. In this case also

$$\chi_{T_\Lambda}(x \wedge y) \leq \chi_{T_\Lambda}(x) \vee \sqrt{\chi_{T_\Lambda}}(y), \chi_{I_\Lambda}(x \wedge y) \leq \chi_{I_\Lambda}(x) \vee \sqrt{\chi_{I_\Lambda}}(y) \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) \geq \chi_{F_\Lambda}(x) \wedge \sqrt{\chi_{F_\Lambda}}(y).$$

Thus χ_Λ is a *NPRI* of Π. ($\Leftarrow$): Assume that χ_Λ is a *NPRI* of Π. Let $x \wedge y \in \Lambda$. Then

$$\chi_{T_\Lambda}(x \wedge y) \leq \chi_{T_\Lambda}(x) \vee \sqrt{\chi_{T_\Lambda}}(y), \chi_{I_\Lambda}(x \wedge y) \leq \chi_{I_\Lambda}(x) \vee \sqrt{\chi_{I_\Lambda}}(y) \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) \geq \chi_{F_\Lambda}(x) \wedge \sqrt{\chi_{F_\Lambda}}(y),$$

implies that either $\chi_{T_\Lambda}(x) = 1$ or $\sqrt{\chi_{T_\Lambda}}(y) = 1$, either $\chi_{I_\Lambda}(x) = 1$ or $\sqrt{\chi_{I_\Lambda}}(y) = 1$ and $\chi_{F_\Lambda}(x) = 0$ or $\sqrt{\chi_{F_\Lambda}}(y) = 0$. Thus either $x \in \Lambda$ or $y \in \sqrt{\Lambda}$. □

LEMMA 6.4
If $N =< T_N, I_N, F_N >$ is a *NPI* of Π, then $N =< T_N, I_N, F_N >$ is a *NPRI* of Π.

PROOF. Let $N =< T_N, I_N, F_N >$ be a *NPI* of Π. For all $x, y \in \Pi$,

$$T_N(x \wedge y) \leq T_N(x) \vee \sqrt{T_N}(y), I_N(x \wedge y) \leq I_N(x) \vee \sqrt{I_N}(y), \text{ and } F_N(x \wedge y) \geq F_N(x) \wedge \sqrt{F_N}(y).$$

Since $N \subseteq \sqrt{N}$, we get the result. □

EXAMPLE 6.5
Figure 4 illustrates the lattice. Let's examine the ideal $\Lambda = (\gamma]$. We observe that $\Phi = (F]$ represents the sole prime ideal of Π that includes Λ. Hence $\sqrt{\Lambda} = \Phi$. We can write $\sqrt{\chi_\Lambda} = \chi_{\sqrt{\Lambda}} = \chi_\Phi$, where $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$. Since Φ is a prime ideal, $\sqrt{\chi_\Lambda}$ is a *NPI* and so χ_Λ is a *NPRI*. But χ_Λ is not *NPI*. Because, since $\alpha, \beta \notin \Lambda$, $\chi_{T_\Lambda}(\alpha \wedge \beta) = 1 \not\leq 0 = 0 \vee 0 = \chi_{T_\Lambda}(\alpha) \vee \chi_{T_\Lambda}(\beta)$, $\chi_{I_\Lambda}(\alpha \wedge \beta) = 1 \not\leq 0 = 0 \vee 0 = \chi_{I_\Lambda}(\alpha) \vee \chi_{I_\Lambda}(\beta)$, and $\chi_{F_\Lambda}(\alpha \wedge \beta) = 0 \not\geq 1 = 1 \wedge 1 = \chi_{F_\Lambda}(\alpha) \wedge \chi_{F_\Lambda}(\beta)$.

THEOREM 6.6
Let N be an *SVNI* of Π.Then N is *NPRI* if and only if the level set $N_{(\alpha,\beta,\gamma)}$ is a primary ideal of Π.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

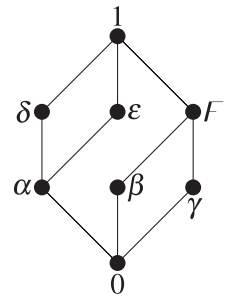


Figure 4. A representative from the family of 8-element lattices.

PROOF. ($\Rightarrow$): Assume that $N =< T_N, I_N, F_N >$ be a *SVNI* of Π. Let $x, y \in \Pi$ be such that $x \wedge y \in N_{(\alpha,\beta,\gamma)}$ and $x \notin N_{(\alpha,\beta,\gamma)}, y \notin \sqrt{N_{(\alpha,\beta,\gamma)}}$. Then we have

$$\alpha \leq T_N(x \wedge y), \alpha > T_N(x), \alpha > T_N(y)$$
$$\beta \leq I_N(x \wedge y), \beta > I_N(x), \beta > I_N(y)$$
$$\gamma \geq F_N(x \wedge y), \gamma < F_N(x), \gamma < F_N(y).$$

Since N is *NPRI*, we have

$$T_N(x \wedge y) \leq T_N(x) \vee \sqrt{T_N}(y) \text{ and}$$
$$I_N(x \wedge y) \leq I_N(x) \vee \sqrt{I_N}(y) \text{ and}$$
$$F_N(x \wedge y) \geq F_N(x) \wedge \sqrt{F_N}(y).$$

Thus, we get $\alpha < \alpha$, which is not possible. Hence $N_{(\alpha,\beta,\gamma)}$ is a primary ideal of Π. ($\Leftarrow$): Assume that $N_{(\alpha,\beta,\gamma)}$ is a primary ideal Π. Let $x, y \in \Pi$ be such that

$$T_N(x \wedge y) \not\leq T_N(x) \vee \sqrt{T_N}(y) \text{ and}$$
$$I_N(x \wedge y) \not\leq I_N(x) \vee \sqrt{I_N}(y) \text{ and}$$
$$F_N(x \wedge y) \not\geq F_N(x) \wedge \sqrt{F_N}(y).$$

Let $T_N(x \wedge y) = \alpha$, $I_N(x \wedge y) = \beta$, and $F_N(x \wedge y) = \gamma$. Then $\alpha > T_N(x)$, $\alpha > T_N(y)$, $\beta > I_N(x)$, $\beta > I_N(y)$, $\gamma < F_N(x)$, $\gamma < F_N(y)$. Since $N_{(\alpha,\beta,\gamma)}$ is a primary ideal, $x \wedge y \in N_{(\alpha,\beta,\gamma)}$ implies that either $x \in N_{(\alpha,\beta,\gamma)}$ or $y \in \sqrt{N_{(\alpha,\beta,\gamma)}}$, i.e. either $T_N(x) \geq \alpha$ or $\sqrt{T_N} \geq \alpha$, $I_N(x) \geq \beta$ or $\sqrt{I_N} \geq \beta$, and $F_N(x) \leq \gamma$ or $\sqrt{F_N} \leq \gamma$, a contradiction. □

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

DEFINITION 6.7
A proper *SVNI* $N =< T_N, I_N, F_N >$ of a lattice Π is called an *N2APRI* of Π, if for $x, y, z \in \Pi$,

$$T_N(x \wedge y \wedge z) \leq T_N(x) \vee \sqrt{T_N}(y) \vee \sqrt{T_N}(z) \text{ and}$$
$$I_N(x \wedge y \wedge z) \leq I_N(x) \vee \sqrt{I_N}(y) \vee \sqrt{I_N}(z) \text{ and}$$
$$F_N(x \wedge y \wedge z) \geq F_N(x) \wedge \sqrt{F_N}(y) \wedge \sqrt{F_N}(z).$$

LEMMA 6.8
A proper ideal Λ of Π is a 2-absorbing primary ideal, if and only if $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a *N2APRI* of Π.

PROOF. Suppose that Λ is a 2-absorbing primary ideal of Π. Let $x, y, z \in \Pi$. If $x \wedge y \wedge z \in \Lambda$, then $\chi_{T_\Lambda}(x \wedge y \wedge z) = 1$, $\chi_{I_\Lambda}(x \wedge y \wedge z) = 1$, and $\chi_{F_\Lambda}(x \wedge y \wedge z) = 0$. As Λ is a 2-absorbing primary, we have either

$$x \wedge y \in \Lambda \text{ or } y \wedge z \in \sqrt{\Lambda} \text{ or } z \wedge x \in \sqrt{\Lambda}.$$

Hence either

$$\chi_{T_\Lambda}(x \wedge y) = 1 \text{ or } \chi_{\sqrt{T_\Lambda}}(y \wedge z) = 1 \text{ or } \chi_{\sqrt{T_\Lambda}}(z \wedge x) = 1 \text{ and}$$
$$\chi_{I_\Lambda}(x \wedge y) = 1 \text{ or } \chi_{\sqrt{I_\Lambda}}(y \wedge z) = 1 \text{ or } \chi_{\sqrt{I_\Lambda}}(z \wedge x) = 1 \text{ and}$$
$$\chi_{F_\Lambda}(x \wedge y) = 0 \text{ or } \chi_{\sqrt{F_\Lambda}}(y \wedge z) = 0 \text{ or } \chi_{\sqrt{F_\Lambda}}(z \wedge x) = 0.$$

Thus

$$\chi_{T_\Lambda}(x \wedge y \wedge z) \leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{\sqrt{T_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{T_\Lambda}}(z \wedge x)$$
$$\chi_{I_\Lambda}(x \wedge y \wedge z) \leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{\sqrt{I_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{I_\Lambda}}(z \wedge x)$$
$$\chi_{F_\Lambda}(x \wedge y \wedge z) \geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{\sqrt{F_\Lambda}}(y \wedge z) \wedge \chi_{\sqrt{F_\Lambda}}(z \wedge x).$$

If $x \wedge y \wedge z \notin \Lambda$, then $\chi_{T_\Lambda}(x \wedge y \wedge z) = 0$, $\chi_{I_\Lambda}(x \wedge y \wedge z) = 0$, and $\chi_{F_\Lambda}(x \wedge y \wedge z) = 1$. It is clear that $x \wedge y \notin \Lambda$. Hence

$$\chi_{T_\Lambda}(x \wedge y \wedge z) \leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{\sqrt{T_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{T_\Lambda}}(z \wedge x)$$
$$\chi_{I_\Lambda}(x \wedge y \wedge z) \leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{\sqrt{I_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{I_\Lambda}}(z \wedge x)$$
$$\chi_{F_\Lambda}(x \wedge y \wedge z) \geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{\sqrt{F_\Lambda}}(y \wedge z) \wedge \chi_{\sqrt{F_\Lambda}}(z \wedge x).$$

Thus χ_Λ is a *N2APRI*. Conversely, suppose that χ_Λ is a *N2APRI*. Let $x \wedge y \wedge z \in \Lambda$. Then $\chi_{T_\Lambda}(x \wedge y \wedge z) = 1$, $\chi_{I_\Lambda}(x \wedge y \wedge z) = 1$, and $\chi_{F_\Lambda}(x \wedge y \wedge z) = 0$. Suppose that $x \wedge y \notin \Lambda$, $y \wedge z \notin \sqrt{\Lambda}$, and

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$z \wedge x \notin \sqrt{\Lambda}$. Since χ_λ is a $N2APRI$, we have

$$1 = \chi_{T_\Lambda}(x \wedge y \wedge z) \leq \chi_{T_\Lambda}(x \wedge y) \vee \chi_{\sqrt{T_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{T_\Lambda}}(z \wedge x)$$
$$1 = \chi_{I_\Lambda}(x \wedge y \wedge z) \leq \chi_{I_\Lambda}(x \wedge y) \vee \chi_{\sqrt{I_\Lambda}}(y \wedge z) \vee \chi_{\sqrt{I_\Lambda}}(z \wedge x)$$
$$0 = \chi_{F_\Lambda}(x \wedge y \wedge z) \geq \chi_{F_\Lambda}(x \wedge y) \wedge \chi_{\sqrt{F_\Lambda}}(y \wedge z) \wedge \chi_{\sqrt{F_\Lambda}}(z \wedge x).$$

Since each of $\chi_{T_\Lambda}(x \wedge y)$, $\chi_{\sqrt{T_\Lambda}}(y \wedge z)$, $\chi_{\sqrt{T_\Lambda}}(z \wedge x)$ belongs to $[0, 1]$, at least one of these numbers must be 1. This implies that either

$$x \wedge y \in \Lambda \text{ or } y \wedge z \in \sqrt{\Lambda} \text{ or } z \wedge x \in \sqrt{\Lambda}.$$

Thus Λ is a 2-absorbing primary ideal. □

LEMMA 6.9
If $N =< T_N, I_N, F_N >$ is a $NPRI$ of Π, then $N =< T_N, I_N, F_N >$ is a $N2APRI$ of Π.

PROOF. Let $N =< T_N, I_N, F_N >$ be a $NPRI$ of Π. Let $x, y, z \in \Pi$. As N is a $NPRI$, we have

$$\begin{aligned}
T_N(x \wedge y \wedge z) = T_N(x \wedge y \wedge y \wedge z) &\leq T_N(x \wedge y) \vee \sqrt{T_A}(y \wedge z) \\
&\leq T_N(x \wedge y) \vee \sqrt{T_N}(y \wedge z) \vee \sqrt{T_N}(z \wedge x) \\
I_N(x \wedge y \wedge z) = I_N(x \wedge y \wedge y \wedge z) &\leq I_N(x \wedge y) \vee \sqrt{I_N}(y \wedge z) \\
&\leq I_N(x \wedge y) \vee \sqrt{I_N}(y \wedge z) \vee \sqrt{I_N}(z \wedge x) \\
F_N(x \wedge y \wedge z) = F_N(x \wedge y \wedge y \wedge z) &\geq F_N(x \wedge y) \wedge \sqrt{F_N}(y \wedge z) \\
&\geq F_N(x \wedge y) \wedge \sqrt{F_N}(y \wedge z) \wedge \sqrt{F_N}(z \wedge x)
\end{aligned}$$

Thus N is a $N2APRI$. □

EXAMPLE 6.10
Figure 5 illustrates the lattice. Let's examine the ideal $\Lambda = (0]$. We note that the ideals $(\theta] = \{0, \beta, \gamma, \delta, \epsilon, \digamma, \zeta, \theta\}$ and $(\eta] = \{0, \alpha, \beta, \gamma, \epsilon, \eta\}$ are the only prime ideals of Π. Hence $\sqrt{\Lambda} = (\eta] \cap (\theta] = (\epsilon]$.As Λ is a 2-absorbing primary ideal and by Lemma 6.4, $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a $N2APRI$ of Π. However, χ_Λ is not $NPRI$. In view of the fact that $\chi_{T_\Lambda}(\alpha \wedge \delta) = 1 \nleq 0 \vee 0 = \chi_T(\alpha) \vee \chi_{\sqrt{T_\Lambda}}(\delta)$, $\chi_{I_\Lambda}(\alpha \wedge \delta) = 1 \nleq 0 \vee 0 = \chi_{I_\Lambda}(\alpha) \vee \chi_{\sqrt{I_\Lambda}}(\delta)$ and $\chi_{F_\Lambda}(\alpha \wedge \delta) = 0 \ngeq 1 \wedge 1 = \chi_{F_\Lambda}(\alpha) \vee \chi_{\sqrt{F_\Lambda}}(\delta)$.

LEMMA 6.11
If N is a $N2AI$ of Π, then N is a $N2APRI$ of Π.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

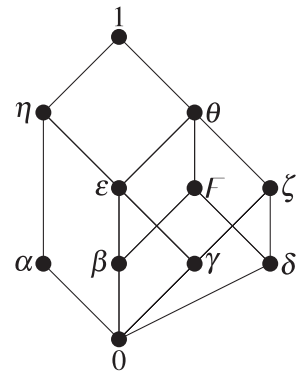


Figure 5. A representative from the family of 11-element lattices.

PROOF. Let $N =< T_N, I_N, F_N >$ be a *N2AI* of Π. Let $x, y, z \in \Pi$. Since $N =< T_N, I_N, F_N >$ is a *N2AI*, we get

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(z \wedge x)$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(z \wedge x)$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(z \wedge x)$$

Since $N \subseteq \sqrt{N}$, we get the result. □

EXAMPLE 6.12
Figure 6 illustrates the lattice. Let's examine the ideal $\Lambda = (0]$. The only prime ideals of Π are $(\iota], (\kappa], (\lambda]$. So, $\sqrt{\Lambda} = (\iota] \cap (\kappa] \cap (\lambda] = (\Gamma]$. Since Λ is a 2-absorbing primary ideal of Π and by Lemma 6.4, $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is a *N2APRI* of Π. We note that Λ is not a 2-absorbing ideal of Π, as $\delta \wedge \epsilon \wedge \Gamma = 0 \in \Lambda$, but $\delta \wedge \epsilon \notin \Lambda$, $\epsilon \wedge \Gamma \notin \Lambda$, and $\delta \wedge \Gamma \notin \Lambda$. We have

$$\chi_{T_\Lambda}(\delta \wedge \epsilon \wedge \Gamma) = 1 \nleq \chi_{T_\Lambda}(\delta \wedge \epsilon) \vee \chi_{T_\Lambda}(\epsilon \wedge \Gamma) \vee \chi_{T_\Lambda}(\Gamma \wedge \delta) = 0.$$
$$\chi_I(\delta \wedge \epsilon \wedge \Gamma) = 1 \nleq \chi_{I_\Lambda}(\delta \wedge \epsilon) \vee \chi_{I_\Lambda}(\epsilon \wedge \Gamma) \vee \chi_{I_\Lambda}(\Gamma \wedge \delta) = 0.$$
$$\chi_{F_\Lambda}(\delta \wedge \epsilon \wedge \Gamma) = 0 \ngeq \chi_{F_\Lambda}(\delta \wedge \epsilon) \wedge \chi_{F_\Lambda}(\epsilon \wedge \Gamma) \wedge \chi_{F_\Lambda}(\Gamma \wedge \delta) = 1.$$

This $\chi_\Lambda =< \chi_{T_\Lambda}, \chi_{I_\Lambda}, \chi_{F_\Lambda} >$ is not a *N2AI* of Π.

LEMMA 6.13
Let N be a *SVNI* of Π. If $\sqrt{N}$ is a *NPI*, then N is a *N2APRI*.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

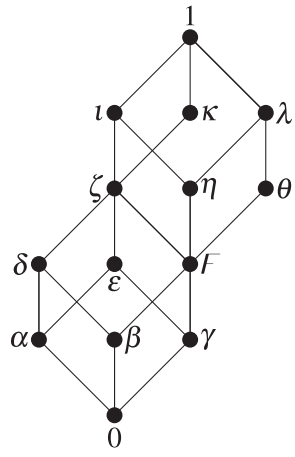


Figure 6. A representative from the family of 14-element lattices.

PROOF. Let N be a *SVNI* of Π. Suppose that $\sqrt{N}$ is a *NPRI*. If N is not a *N2APRI*, then there exist $x, y, z \in \Pi$ such that

$$T_N(x \wedge y \wedge z) \not\leq T_N(x \wedge y) \vee \sqrt{T_N}(y \wedge z) \vee \sqrt{T_N}(z \wedge x)$$
$$I_N(x \wedge y \wedge z) \not\leq I_N(x \wedge y) \vee \sqrt{I_N}(y \wedge z) \vee \sqrt{I_N}(z \wedge x)$$
$$F_N(x \wedge y \wedge z) \not\geq F_N(x \wedge y) \wedge \sqrt{F_N}(y \wedge z) \wedge \sqrt{F_N}(z \wedge x).$$

This implies that

$$T_N(x \wedge y) \vee \sqrt{T_N}(y \wedge z) \vee \sqrt{T_N}(z \wedge x) < T_N(x \wedge y \wedge z)$$
$$I_N(x \wedge y) \vee \sqrt{I_N}(y \wedge z) \vee \sqrt{I_N}(z \wedge x) < I_N(x \wedge y \wedge z)$$
$$F_N(x \wedge y) \wedge \sqrt{F_N}(y \wedge z) \wedge \sqrt{F_N}(z \wedge x) > F_N(x \wedge y \wedge z).$$

Since $\sqrt{N}$ is a *NPI*, we have

$$\sqrt{T_N}(x \wedge y \wedge z) = \sqrt{T_N}(y \wedge z) \vee \sqrt{T_N}(x) = \sqrt{T_N}(x \wedge z) \vee \sqrt{T_N}(y).$$
$$\sqrt{I_N}(x \wedge y \wedge z) = \sqrt{I_N}(y \wedge z) \vee \sqrt{I_N}(x) = \sqrt{I_N}(x \wedge z) \vee \sqrt{I_N}(y).$$
$$\sqrt{F_N}(x \wedge y \wedge z) = \sqrt{F_N}(y \wedge z) \wedge \sqrt{F_N}(x) = \sqrt{F_N}(x \wedge z) \wedge \sqrt{F_N}(y).$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Hence

$$\begin{aligned}\sqrt{T_N}(y\wedge z)\vee\sqrt{T_N}(x\wedge z)&=\sqrt{T_N}(y\wedge z)\vee\sqrt{T_N}(x)\vee\sqrt{T_N}(z)\\&=\sqrt{T_N}(x\wedge y\wedge z)\vee\sqrt{T_N}(z)\\\sqrt{I_N}(y\wedge z)\vee\sqrt{I_N}(x\wedge z)&=\sqrt{I_N}(y\wedge z)\vee\sqrt{I_N}(x)\vee\sqrt{I_N}(z)\\&=\sqrt{I_N}(x\wedge y\wedge z)\vee\sqrt{I_N}(z)\\\sqrt{F_N}(y\wedge z)\wedge\sqrt{F_N}(x\wedge z)&=\sqrt{F_N}(y\wedge z)\wedge\sqrt{F_N}(x)\wedge\sqrt{F_N}(z)\\&=\sqrt{F_N}(x\wedge y\wedge z)\wedge\sqrt{F_N}(z).\end{aligned}$$

Thus

$$\begin{aligned}T_N(x\wedge y)\vee\sqrt{T_N}(x\wedge y\wedge z)\vee\sqrt{T_N}(z)&<T_N(x\wedge y\wedge z)\\I_N(x\wedge y)\vee\sqrt{I_N}(x\wedge y\wedge z)\vee\sqrt{I_N}(z)&<I_N(x\wedge y\wedge z)\\F_N(x\wedge y)\wedge\sqrt{F_N}(x\wedge y\wedge z)\wedge\sqrt{F_N}(z)&>F_N(x\wedge y\wedge z).\end{aligned}$$

This implies that

$$\begin{aligned}\sqrt{T_N}(x\wedge y\wedge z)&<T_N(x\wedge y\wedge z)\\\sqrt{I_N}(x\wedge y\wedge z)&<I_N(x\wedge y\wedge z)\\\sqrt{F_N}(x\wedge y\wedge z)&>F_N(x\wedge y\wedge z),\end{aligned}$$

which is not possible. Hence N is $N2APRI$. □

EXAMPLE 6.14
Figure 7 illustrates the lattice. Let's examine the ideal $\Lambda=(\alpha]$. The only prime ideals of L containing the ideal $\Lambda=(\alpha]$ are $(\zeta]$ and $(\eta]$. Hence $\sqrt{\Lambda}=(\zeta]\cap(\eta]=(\delta]$. As Λ is a 2-absorbing primary ideal and by lemma 6.4, $\chi_\Lambda=<\chi_{T_\Lambda},\chi_{I_\Lambda},\chi_{F_\Lambda}>$ is a $N2APRI$. However, $\sqrt{\chi_\Lambda}$ is not NPI. In view of the fact that $\epsilon\wedge\digamma=\gamma\in\sqrt{\Lambda}$ but $\epsilon\notin\sqrt{\Lambda}$ and $digamma\notin\sqrt{\Lambda}$. Thus $\sqrt{\Lambda}$ is not a prime ideal of Π. Hence by Theorem 3.1, $\sqrt{\chi_\Lambda}$ is not a NPI of Π.

LEMMA 6.15
Let N be a $SVNI$ of Π. Then $\sqrt{N}=\sqrt{\sqrt{N}}$.

THEOREM 6.16
Let $N=<T_N,I_N,F_N>$ be a $SVNI$ of Π. Then $\sqrt{A}$ is NPI if and only if $\sqrt{A}$ is $NPRI$.

PROOF. It follows from Lemma 6.2, that if $\sqrt{N}$ is NPI, then $\sqrt{N}$ is $NPRI$. The converse follows from the definition of an $NPRI$ and by Lemma 6.8. □

THEOREM 6.17
Let N be a $SVNI$ of Π. Then $\sqrt{N}$ is $N2AI$ if and only if $\sqrt{N}$ is $N2APRI$.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

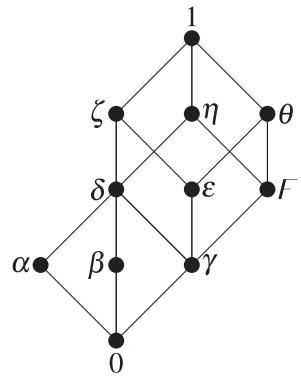


Figure 7. A representative from the family of 11-element lattices.

7 Primary neutrosophic ideals

In this section, we will introduce the concepts of primary neutrosophic ideal (*PRNI*) and 2-absorbing primary neutrosophic ideal (2*APRNI*). Then, we will discuss their relationship with the *PNI* and 2*ANI* concepts introduced in the previous section.

DEFINITION 7.1
A proper *SVNI* N of a lattice Π is called a *PRNI* of Π if for $M, K \in SVNS(\Pi)$, $M \cap K \subseteq N$ implies that either $M \subseteq N$ or $K \subseteq N$.

LEMMA 7.2
If N is a *PNI* of Π, then N is a *PRNI* of Π.

PROOF. Let $N =< T_N, I_N, F_N >$ is a *PNI* of Π. Let $M \cap K \subseteq N$ for some $M, K \in SVNS(\Pi)$. Since N is a *PNI*, either $M \subseteq N$ or $K \subseteq N$. Since $N \subseteq \sqrt{N}$, we get the result. □

THEOREM 7.3
Let Λ be a primary ideal of Π, $\Lambda \neq \Pi$. The neutrosophic subset $N =< T_N, I_N, F_N >$ of Π defined by

$$N(x) = \begin{cases} (1,1,0), & x \in \Lambda \\ (\alpha, \beta, \gamma), & x \in \Pi - \Lambda \end{cases}$$

is a *PRNI* of Π, where $\alpha + \beta + \gamma \leq 3$.

PROOF. Let $N =< T_N, I_N, F_N >$ be a *SVNI* of Π. Since $N \subseteq \sqrt{N}$, we have $N(x) \leq \sqrt{N}(x)$ for all $x \in \Pi$. Hence if $x \in \Lambda$, then $\sqrt{N}(x) = (1,1,0)$ and $x \notin \Lambda$, then $\sqrt{N}(x) = (t, m, s)$, where $t \geq \alpha$ and $m \geq \beta$, $s \leq \gamma$. Let $M, K \in SVNS(\Pi)$ such that $M \cap K \subseteq N$. Suppose that $M \nsubseteq N$ and $K \nsubseteq \sqrt{N}$. Let $x \in \Pi$ be such that $T_M(x) > T_N(x)$, $I_M(x) > I_N(x)$, and $F_M(x) < F_N(x)$. This implies that $x \notin \Lambda$, for otherwise, $T_M(x) > 1$, $I_M(x) > 1$, and $F_M(x) < 0$, which is not possible. Let $T_M(x) = k_1 > \alpha = T_N(x)$, $I_M(x) = n_1 > \beta = I_N(x)$, and $F_M(x) = l_1 < \gamma = F_N(x)$. Let $y \in \Pi$ be such that $T_K(y) > \sqrt{T_N}(y)$, $I_K(y) > \sqrt{I_N}(y)$, and $F_K(y) < \sqrt{F_N}(y)$. Clearly, $y \in \sqrt{\Lambda}$, otherwise,

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$T_K(y) > \sqrt{T_N}(y) \geq T_N(y) = 1$, $I_K(y) > \sqrt{I_N}(y) \geq I_N(y) = 1$, and $F_K(y) < \sqrt{F_N}(y) \leq F_N(y) = 0$, which is not possible. Let $T_K(y) = k_2 > \alpha = T_N(y)$, $I_K(y) = n_2 > \beta = I_N(y)$, and $F_K(y) = l_2 < \gamma = F_N(y)$. Since Λ is primary, $x \wedge y \notin \Lambda$. Hence $N(x \wedge y) = (\alpha, \beta, \gamma)$. We have

$$(T_M \cap T_K)(x \wedge y) \geq min\{T_M(x), T_K(y)\} = min\{k_1, k_2\} > \alpha = T_N(x \wedge y) \text{ and}$$
$$(I_M \cap I_K)(x \wedge y) \geq min\{I_M(x), I_K(y)\} = min\{n_1, n_2\} > \beta = I_N(x \wedge y) \text{ and}$$
$$(F_M \cap F_K)(x \wedge y) \leq max\{F_M(x), F_K(y)\} = max\{l_1, l_2\} < \gamma = F_N(x \wedge y),$$

which is not possible. Thus N is a *PRNI* of Π. □

THEOREM 7.4
If $N =< T_N, I_N, F_N >$ is a *PRNI* of Π,then the level set $A_{(t,s,r)}$ is a primary ideal of Π.

PROOF. Let $\alpha, \beta \in \Pi$ be such that $\alpha \wedge \beta \in A_{(t,s,r)}$ and $\alpha \notin A_{(t,s,r)}$. Define *SVNI*s M and K of Π as follows.

$$M(x) = \begin{cases} (t,s,r), & x \leq \alpha \\ (0,0,1), & x \nleq \alpha \end{cases}$$

and

$$K(x) = \begin{cases} (t,s,r), & x \leq \beta \\ (0,0,1), & x \nleq \beta \end{cases}$$

Then $M \cap K \subseteq N$. Also $M \nsubseteq N$ as $\alpha \notin A_{(t,s,r)}$ implies $T_N(\alpha) < t = T_M(\alpha)$, $I_N(\alpha) < s = I_M(\alpha)$, and $F_N(\alpha) > r = F_M(\alpha)$. Since N is a *PRNI*, we have $K \subseteq \sqrt{N}$. Hence $t = T_K(\beta) \leq \sqrt{T_N}(\beta)$, $s = I_K(\beta) \leq \sqrt{I_N}(\beta)$, and $t = F_K(\beta) \geq \sqrt{F_N}(\beta)$. So $\beta \in \sqrt{A_{(t,s,r)}}$. Thus $A_{(t,s,r)}$ is a primary ideal of Π. □

DEFINITION 7.5
A proper *SVNI* $N =< T_N, I_N, F_N >$ of a lattice Π is called a 2*APRNI* of Π, if whenever, $M \cap K \cap L \subseteq N$ for $M, K, L \in SVNS(\Pi)$, then either

$$M \cap K \subseteq N \text{ or } K \cap L \subseteq \sqrt{N} \text{ or } L \cap M \subseteq \sqrt{N}.$$

LEMMA 7.6
Let Λ be an ideal of Π. If χ_Λ is a 2*APRNI* of Π, then Λ is a 2-absorbing ideal of Π.

PROOF. Suppose that χ_Λ is a 2*APRNI* of Π. Let $\alpha \wedge \beta \wedge \gamma \in \Lambda$ for some $\alpha, \beta, \gamma \in \Pi$. Presume that $\alpha \wedge \beta \notin \Lambda$,$\beta \wedge \gamma \notin \sqrt{\Lambda}$, and $\gamma \wedge \alpha \notin \sqrt{\Lambda}$. Then, it is clear that $\alpha \notin \Lambda$ and $\beta, \gamma \notin \sqrt{\Lambda}$. Define *SVNI*s

$$N(x) = \begin{cases} (1,1,0), & x \in (\alpha] \\ (0,0,1), & otherwise \end{cases}$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$$M(x) = \begin{cases} (1,1,0), & x \in (\beta] \\ (0,0,1), & otherwise \end{cases}$$

$$K(x) = \begin{cases} (1,1,0), & x \in (\gamma] \\ (0,0,1), & otherwise \end{cases}$$

We note that

$$(N \cap M \cap K)(x) = \begin{cases} (1,1,0), & x \in (\alpha \wedge \beta \wedge \gamma] \\ (0,0,1), & otherwise \end{cases}$$

Then $N \cap M \cap K \subseteq \chi_\Lambda$ but $N \cap M \nsubseteq \chi_\Lambda$, $M \cap K \nsubseteq \sqrt{\chi_\Lambda}$, and $K \cap N \nsubseteq \sqrt{\chi_\Lambda}$. This contradicts the assumption that $< \chi_\Lambda$ is a $2APIFI$. □

LEMMA 7.7
If N is a $PRNI$ of Π, then N is a $2APNI$ of Π.

PROOF. Let N be a $PRNI$ of Π. Let $M \cap K \cap L \subseteq N$ for some $M, K, L \in SVNS(\Pi)$. Since N is a $PRNI$ of Π, either (1) $M \cap K \subseteq N$ or $L \subseteq \sqrt{N}$ or (2) $M \subseteq N$ or $K \cap L \subseteq \sqrt{N}$ or (3) $M \subseteq \sqrt{N}$ or $K \cap L \subseteq N$ or (4) $K \subseteq N$ or $M \cap L \subseteq \sqrt{N}$. These possibilities imply that either (i) $M \cap K \subseteq N$ or (ii) $K \cap L \subseteq \sqrt{N}$ or (iii) $M \cap L \subseteq \sqrt{N}$. Hence N is a $2APRNI$ of Π. □

LEMMA 7.8
If N is a $2ANI$ of Π, then N is a $2APRNI$ of Π.

PROOF. Let N be a $2ANI$ of Π. Let $M \cap K \cap L \subseteq N$ for some $M, K, L \in SVNS(\Pi)$. Since N is a $2ANI$ of Π, either $M \cap K \subseteq N$ or $K \cap L \subseteq N$ or $M \cap L \subseteq N$. Since $N \subseteq \sqrt{N}$, we get the result. □

DEFINITION 7.9
Let $N =< T_N, I_N, F_N >$ be an $SVNI$ of Π. If $X =< T_X, I_X, F_X >$ is the only PNI containing N, then we say that N is a X-PNI of Π.

THEOREM 7.10
Let N, M be $SVNI$s and X, Y be PNI of Π. Suppose that N is a X-PNI and M is a Y-PNI. Then $N \cap M$ is a $2APRNI$ of Π.

PROOF. Since N is a X-$PRNI$, we get $\sqrt{N} = X$. As M is a Y-$PRNI$, we get $\sqrt{M} = Y$. Let $K = N \cap M$. Then $\sqrt{K} = X \cap Y$. Now, presume that $L \cap S \cap Z \subseteq K$ for some $L, S, Z \in SVNS(\Pi)$. Assume that $L \cap S \nsubseteq \sqrt{K}$ and $S \cap Z \nsubseteq \sqrt{K}$. Then $L, S, Z \nsubseteq \sqrt{K} = X \cap Y$. By Proposition 3.1, $\sqrt{K} = X \cap Y$ is a $2ANI$ of Π. Since $L \cap S \nsubseteq \sqrt{K}$,$S \cap Z \nsubseteq \sqrt{K}$ we have $Z \cap L \subseteq \sqrt{K}$. We show that $Z \cap L \subseteq K$. Since $Z \cap L \subseteq \sqrt{K} \subseteq X$, we assume that $L \subseteq X$. As $L \nsubseteq \sqrt{K}$ and $L \cap Z \subseteq \sqrt{K} \subseteq Y$, we conclude that $L \nsubseteq Y$ and $Z \subseteq Y$. Since $Z \subseteq Y$ and $Z \nsubseteq \sqrt{K}$, we have $Z \nsubseteq X$. If $L \subseteq N$ and $Z \subseteq M$, then $L \cap Z \subseteq K$, and we are done. We may assume that $L \nsubseteq N$. Since N is an X-$PRNI$ and $L \nsubseteq N$, we have $S \cap Z \subseteq X$. Since $Z \subseteq Y$ and $S \cap Z \subseteq \sqrt{K}$, this is a contradiction. Thus, $L \subseteq N$. Since M is a Y-$PRNI$ of Π and $Z \nsubseteq M$, we get $L \cap S \subseteq Y$. Since $L \subseteq X$ and $L \cap S \subseteq Y$, we have $L \cap S \subseteq \sqrt{K}$, which is a contradiction.Thus, $Z \subseteq M$. Hence $L \cap S \subseteq K$. □

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

THEOREM 7.11
Suppose that N is a non-constant *SVNI* of Π such that $\sqrt{N}$ is a *PNI*. Then N is a 2*APRNI*.

PROOF. Suppose that for some $M, K, L \in SVNS(\Pi)$, $M \cap K \cap L \subseteq N$ and $M \cap K \nsubseteq N$. (i): We note that $M \cap K \cap L \subseteq N \subseteq \sqrt{N}$. Hence, if $M \cap K \nsubseteq \sqrt{N}$, then as $\sqrt{N}$ is *PNI* we get $L \subseteq \sqrt{N}$ and so $K \cap L \subseteq \sqrt{N}$. (ii):If $M \cap K \subseteq N$, then as $\sqrt{N}$ is *PNI*, either $M \subseteq \sqrt{N}$ or $K \subseteq \sqrt{N}$. Hence either $M \cap L \subseteq \sqrt{N}$ or $L \cap K \subseteq \sqrt{N}$. Thus, N is a 2*APRNI* of Π. □

THEOREM 7.12
Let N be a non-constant *SVNI* of a lattice Π. Then $\sqrt{N}$ is a *PNI* of Π if and only if $\sqrt{N}$ is a *PRNI* of Π.

PROOF. Let $\sqrt{N}$ be a *PNI* of Π.Let $M, K \in SVNS(\Pi)$ be such that $M \cap K \subseteq \sqrt{N}$.As $\sqrt{N}$ is a *PNI* of Π, either $M \subseteq \sqrt{N}$ or $K \subseteq \sqrt{N}$. Since $\sqrt{N} \subseteq \sqrt{\sqrt{N}}$ we conclude that $\sqrt{N}$ is a *PNI* of Π. Conversely, suppose that $\sqrt{N}$ is a *PNI* of Π. Let $M, K \in SVNS(\Pi)$ be such that $M \cap K \subseteq \sqrt{N}$.As $\sqrt{N}$ is a *PNI*, either $M \subseteq \sqrt{N}$ or $K \subseteq \sqrt{\sqrt{N}} \subseteq \sqrt{N}$. Hence $\sqrt{N}$ is a *PNI* of Π. □

THEOREM 7.13
Let N be a non-constant *SVNI* of a lattice Π. Then $\sqrt{N}$ is a 2*ANI* of Π if and only if $\sqrt{N}$ is a 2*APRNI* of Π.

PROOF. Let $\sqrt{N}$ be a 2*ANI* of Π. Let $M, K, L \in SVNS(\Pi)$ be such that $M \cap K \cap L \subseteq \sqrt{N}$. Since $\sqrt{N}$ is a 2*ANI* of Π, either $M \cap K \subseteq \sqrt{N}$ or $K \cap L \subseteq \sqrt{N}$ or $M \cap L \subseteq \sqrt{N}$. Using $\sqrt{\sqrt{N}} = \sqrt{N}$, we conclude that $\sqrt{N}$ is a 2*APRNI* of Π. Conversely, suppose that $\sqrt{N}$ be a 2*APRNI* of Π. Let $M, K, L \in SVNS(\Pi)$ be such that $M \cap K \cap L \subseteq \sqrt{N}$. As $\sqrt{N}$ is a 2*APRNI* of Π,either $M \cap K \subseteq \sqrt{N}$ or $K \cap L \subseteq \sqrt{\sqrt{N}} = \sqrt{N}$ or $M \cap L \subseteq \sqrt{\sqrt{N}} = \sqrt{N}$. Hence $\sqrt{N}$ is a 2*ANI* of Π. □

8 Single valued neutrosophic ideals in a direct product of a lattice

Let's start this section by stating, as mentioned in [20], that "the cartesian product of finitely many lattices is a lattice." One of the first questions that comes to mind here is how the partial order relation "$\leq$" will be defined on such a lattice.

DEFINITION 8.1
$\Pi_1, \Pi_2, ...\Pi_n$ are lattices, and let $\Pi = \Pi_1 \times \Pi_2 \times .... \times \Pi_n$. Then, let $i \in [1, n] \cap \mathbb{N}$ and $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n), \beta = (\beta_1, \beta_2, ..., \beta_n) \in \Pi$ for every $\alpha_i, \beta_i \in \Pi_i$. Saying $\alpha \leq \beta$ means $\alpha_i \leq \beta_i$, for every i.

Even if Π is a lattice formed by the Cartesian product, it is still ultimately a lattice. Therefore, there is no need to redefine the notions of *SVNS* and *SVNI* on such a lattice. However, the *SVNS* (or *SVNI*) we obtain from each lattice forming this lattice, through cartesian products, also constitute an *SVNS* (or *SVNI*) in Π. Let's explain how we will define this.

DEFINITION 8.2
$\Pi_1, \Pi_2, ...\Pi_n$ are lattices, and let $\Pi = \Pi_1 \times \Pi_2 \times .... \times \Pi_n$. Then, let $N_1 =< T_{N_1}, I_{N_1}, F_{N_1} >, N_2 =<$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

$T_{N_2}, I_{N_2}, F_{N_2} >, ..., N_n =< T_{N_n}, I_{N_n}, F_{N_n} >$ denote, respectively, the *SVNI* on lattices $\Pi_1, \Pi_2, ..., \Pi_n$. Also, let $N = N_1 \times N_2 \times ... \times N_n$. Thus, $N =< T_N = T_{N_1} \times T_{N_2} \times ... \times T_{N_n}, I_N = I_{N_1} \times I_{N_2} \times ... \times I_{N_n}, F_N = F_{N_1} \times F_{N_2} \times ... \times F_{N_n} >$, and for $\alpha \in \Pi$, it is defined as

$$T_N(\alpha) = T_{N_1}(\alpha_1) \times T_{N_2}(\alpha_2) \times ... \times T_{N_n}(\alpha_n) = T_{N_1}(\alpha_1) \wedge T_{N_2}(\alpha_2) \wedge ... \wedge T_{N_n}(\alpha_n)$$
$$I_N(\alpha) = I_{N_1}(\alpha_1) \times I_{N_2}(\alpha_2) \times ... \times I_{N_n}(\alpha_n) = I_{N_1}(\alpha_1) \wedge I_{N_2}(\alpha_2) \wedge ... \wedge I_{N_n}(\alpha_n)$$
$$F_N(\alpha) = F_{N_1}(\alpha_1) \times F_{N_2}(\alpha_2) \times ... \times F_{N_n}(\alpha_n) = F_{N_1}(\alpha_1) \vee F_{N_2}(\alpha_2) \vee ... \vee F_{N_n}(\alpha_n).$$

THEOREM 8.3
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. If $N : \Pi \to [0,1] \times [0,1] \times [0,1]$ is a SVNIs of Π,then there exist SVNIs N_1, N_2 of Π_1 and Π_2, respectively, such that $T = T_{N_1} \times T_{N_2}$, $I = I_{N_1} \times I_{N_2}$, and $F = F_{N_1} \times F_{N_2}$. Moreover, if N is *NPI*, then so are N_1 and N_2.

PROOF. Define $T_{N_i}, N_i, F_{N_i} : \Pi_i \to [0,1]$ by $T_{N_1}(x) = T_N(x,0)$, $I_{N_1}(x) = I_N(x,0)$, $F_{N_1}(x) = F_N(x,0)$, $T_{N_2}(y) = T_N(0,y)$, $I_{N_2}(y) = I_N(0,y)$, $F_{N_2}(y) = F_N(0,y)$. Let $x, y \in \Pi_1$. We have

$$T_N[(x,0) \wedge (y,0)] = T_N(x \wedge y, 0) = T_{N_1}(x \wedge y),$$
$$I_N[(x,0) \wedge (y,0)] = I_N(x \wedge y, 0) = I_{N_1}(x \wedge y),$$
$$F_N[(x,0) \wedge (y,0)] = F_N(x \wedge y, 0) = F_{N_1}(x \wedge y) \text{ and}$$
$$T_N[(x,0) \vee (y,0)] = T_N(x \vee y, 0) = T_{N_1}(x \vee y),$$
$$I_N[(x,0) \vee (y,0)] = I_N(x \vee y, 0) = I_{N_1}(x \vee y),$$
$$F_N[(x,0) \vee (y,0)] = F_N(x \vee y, 0) = F_{N_1}(x \vee y).$$

Hence

$$T_{N_1}(x \wedge y) \wedge T_{N_1}(x \vee y) = T_N[(x,0) \wedge (y,0)] \wedge T_N[(x,0) \vee (y,0)],$$
$$I_{N_1}(x \wedge y) \wedge I_{N_1}(x \vee y) = I_N[(x,0) \wedge (y,0)] \wedge I_N[(x,0) \vee (y,0)],$$
$$F_{N_1}(x \wedge y) \vee F_{N_1}(x \vee y) = F_N[(x,0) \wedge (y,0)] \vee F_N[(x,0) \vee (y,0)].$$

As N is a *SVNI*, we get

$$\begin{aligned} T_{N_1}(x \wedge y) \wedge T_{N_1}(x \vee y) &= T_N[(x,0) \wedge (y,0)] \wedge T_N[(x,0) \vee (y,0)] \\ &\geq T_N(x,0) \wedge T_N(y,0) \\ &= T_{N_1} \wedge T_{N_2}(y), \\ I_{N_1}(x \wedge y) \wedge I_{N_1}(x \vee y) &= I_N[(x,0) \wedge (y,0)] \wedge I_N[(x,0) \vee (y,0)] \\ &\geq I_N(x,0) \wedge I_N(y,0) \\ &= I_{N_1} \wedge I_{N_2}(y), \\ F_{N_1}(x \wedge y) \vee F_{N_1}(x \vee y) &= F_N[(x,0) \wedge (y,0)] \vee F_N[(x,0) \vee (y,0)] \\ &\leq F_N(x,0) \vee F_N(y,0) \\ &= F_{N_1}(x) \vee F_{N_1}(y). \end{aligned}$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Also

$$T_{N_1}(x \vee y) = T_N[(x,0) \vee (y,0)] = T_N(x,0) \wedge T_N(y,0) = T_{N_1}(x) \wedge T_{N_1}(y).$$
$$I_{N_1}(x \vee y) = I_N[(x,0) \vee (y,0)] = I_N(x,0) \wedge I_N(y,0) = I_{N_1}(x) \wedge I_{N_1}(y).$$
$$F_{N_1}(x \wedge y) = F_N[(x,0) \wedge (y,0)] = F_N(x,0) \vee F_N(y,0) = F_{N_1}(x) \vee F_{N_1}(y).$$

Thus N_1 is a *SVNI* of Π_1. Similarly, we can show that N_2 is a *SVNI* of Π_2. The second part follows from the definition of an *NPI*. We have

$$T_N(x,y) = T_N(x,0) \vee T_N(0,y) = T_N(x,0) \wedge T_N(0,y) = T_{N_1}(x) \wedge T_{N_2}(y),$$
$$I_N(x,y) = I_N(x,0) \vee I_N(0,y) = I_N(x,0) \wedge I_N(0,y) = I_{N_1}(x) \wedge I_{N_2}(y),$$
$$F_N(x,y) = F_N(x,0) \wedge F_N(0,y) = F_N(x,0) \vee F_N(0,y) = F_{N_1}(x) \vee F_{N_2}(y).$$

□

EXAMPLE 8.4
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_1, N_2 be *NPI*s of Π_1 and Π_2, respectively. Then $T_N = T_{N_1} \times T_{N_2}$, $I_N = I_{N_1} \times I_{N_2}$, and $F_N = F_{N_1} \times F_{N_2}$ need not be a *NPI*s of Π.

Consider the lattices Π_1 and Π_2 as shown in Fig. 8. Define $T_{N_1}, I_{N_1}, F_{N_1} : \Pi_1 \to [0,1]$ and $T_{N_2}, I_{N_2}, F_{N_2} : \Pi_2 \to [0,1]$ as follows $T_{N_1}(0) = 1, T_{N_1}(\alpha) = 1/2, T_{N_1}(\beta) = 1, T_{N_1}(1) = 3/4$, $I_{N_1}(0) = 7/10, I_{N_1}(\alpha) = 3/5, I_{N_1}(\beta) = 2/7, I_{N_1}(1) = 4/7, F_{N_1}(0) = 0, F_{N_1}(\alpha) = 1/2, F_{N_1}(\beta) = 0, F_{N_1}(1) = 2/5$, $T_{N_2}(0) = 1, T_{N_2}(1) = 0$, $I_{N_2}(0) = 1, I_{N_2}(1) = 0$, $F_{N_2}(0) = 0, F_{N_2}(1) = 1$. We note that N_1 is a *NPI* of Π_1 and N_2 that of Π_2. We consider $N : \Pi_1 \times \Pi_2 \to [0,1]$ defined by $T_N(x,y) = T_{N_1}(x) \wedge T_{N_2}(y)$, $I_N(x,y) = I_{N_1}(x) \wedge I_{N_2}(y)$, and $F(x,y) = F_{N_1}(x) \vee F_{N_2}(y)$. We have

$$T_N[(0,1) \wedge (1,0)] = T_N(0,0) = 1, T_N(0,1) = 0, T_N(1,0) = 3/4,$$
$$I_N[(0,1) \wedge (1,0)] = I_N(0,0) = 7/10, I_N(0,1) = 0, I_N(1,0) = 4/7,$$
$$F_N[(0,1) \wedge (1,0)] = F_N(0,0) = 0, F_N(0,1) = 1, F_N(1,0) = 1.$$

Thus,

$$T_N[(0,1) \wedge (1,0)] \not\leq T_N(0,1) \vee T_N(1,0),$$
$$I_N[(0,1) \wedge (1,0)] \not\leq I_N(0,1) \vee I_N(1,0),$$
$$F_N[(0,1) \wedge (1,0)] \not\geq F_N(0,1) \wedge F_N(1,0).$$

Hence N is not a *NPI* of Π.

THEOREM 8.5
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. If N is a SVNIs of Π_1 and χ_Λ is a SVNIs of Π_2. Then $N \times \chi_\Lambda$ is a *NPI* of Π iff N is a *NPI* of Π_1.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

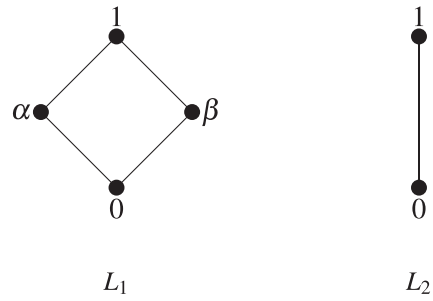


Figure 8. A representative of a family of 4- and 2-element lattices.

PROOF. Suppose that N is a *NPI* of Π_1. We have

$$\begin{aligned}[T_N \times \chi_{T_\Lambda}][(x_1,y_1)\wedge(x_2,y_2)] &= [T_N \times \chi_{T_\Lambda}](x_1\wedge x_2, y_1\wedge y_2)\\ &= T_N(x_1\wedge x_2)\wedge \chi_{T_\Lambda}(y_1\wedge y_2)\\ &= T_N(x_1\wedge x_2) \text{ as } \chi_{T_\Lambda}(y_1\wedge y_2)=1\end{aligned}$$

$$\begin{aligned}[I_N \times \chi_{I_\Lambda}][(x_1,y_1)\wedge(x_2,y_2)] &= [I_N \times \chi_{I_\Lambda}](x_1\wedge x_2, y_1\wedge y_2)\\ &= I_N(x_1\wedge x_2)\wedge \chi_{I_\Lambda}(y_1\wedge y_2)\\ &= I_N(x_1\wedge x_2) \text{ as } \chi_{I_\Lambda}(y_1\wedge y_2)=1\end{aligned}$$

and

$$\begin{aligned}[F_N \times \chi_{F_\Lambda}][(x_1,y_1)\wedge(x_2,y_2)] &= [F_N \times \chi_{F_\Lambda}](x_1\wedge x_2, y_1\wedge y_2)\\ &= F_N(x_1\wedge x_2)\vee \chi_{F_\Lambda}(y_1\wedge y_2)\\ &= F_N(x_1\wedge x_2) \text{ as } \chi_{F_\Lambda}(y_1\wedge y_2)=0.\end{aligned}$$

Since N is a *NPI*,

$$T_N(x_1\wedge x_2)=T_N(x_1)\vee T_N(x_2), I_N(x_1\wedge x_2)=I_N(x_1)\vee I_N(x_2), \text{ and } F_N(x_1\wedge x_2)=F_N(x_1)\wedge F(x_2).$$

Thus

$$\begin{aligned}[T_N \times \chi_{T_\Lambda}](x_1\wedge x_2, y_1\wedge y_2) &= [T_N(x_1)\wedge \chi_{T_\Lambda}(y_1)]\vee[T_N(x_2)\wedge \chi_{T_\Lambda}(y_2)]\\ &= [T_N \times \chi_{T_\Lambda}](x_1,y_1)\vee[T_N \times \chi_{T_\Lambda}](x_2,y_2).\end{aligned}$$

$$\begin{aligned}[I_N \times \chi_{I_\Lambda}](x_1\wedge x_2, y_1\wedge y_2) &= [I_N(x_1)\wedge \chi_{I_\Lambda}(y_1)]\vee[I_N(x_2)\wedge \chi_{I_\Lambda}(y_2)]\\ &= [I_N \times \chi_{I_\Lambda}](x_1,y_1)\vee[I_N \times \chi_{I_\Lambda}](x_2,y_2),\end{aligned}$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

and

$$[F_N \times \chi_{F_A}](x_1 \wedge x_2, y_1 \wedge y_2) = [F_N(x_1) \wedge \chi_{F_A}(y_1)] \wedge [F_N(x_2) \wedge \chi_{F_A}(y_2)]$$
$$= [F_N \times \chi_{F_A}](x_1, y_1) \wedge [F_N \times \chi_{F_A}](x_2, y_2).$$

Hence $N \times M$ is a *NPI* of Π. The converse can be similarly proved. □

THEOREM 8.6
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_1, N_2 be *SVNI*s of Π_1 and Π_2, respectively. Suppose that $N_1(0_1) = N_2(0_2) = (1, 1, 0)$, where 0_1 is the least element of Π_1 and 0_2 that of Π_2. If $N = N_1 \times N_2$ is a *N2AI* of Π,then N_1 is a N2AI of Π_1 and N_2 that of Π_2.

PROOF. Let $x, y, z \in \Pi_1$. Since N is an *N2AI* of Π, we have

$$T_N(x \wedge y \wedge z, 0_2) \leq T_N(x \wedge y, 0_2) \vee T_N(y \wedge z, 0_2) \vee T_N(x \wedge z, 0_2),$$
$$I_N(x \wedge y \wedge z, 0_2) \leq I_N(x \wedge y, 0_2) \vee I_N(y \wedge z, 0_2) \vee I_N(x \wedge z, 0_2),$$
$$F_N(x \wedge y \wedge z, 0_2) \leq F_N(x \wedge y, 0_2) \wedge F_N(y \wedge z, 0_2) \wedge F_N(x \wedge z, 0_2).$$

By the definition of N, we can write

$$T_{N_1}(x \wedge y \wedge z) \wedge T_{N_2}(0_2) \leq [T_{N_1}(x \wedge y) \wedge T_{N_2}(0_2)]$$
$$\vee [T_{N_1}(y \wedge z) \wedge T_{N_2}(0_2)] \vee [T_{N_1}(x \wedge z) \wedge T_{N_2}(0_2)],$$
$$I_{N_1}(x \wedge y \wedge z) \wedge I_{N_2}(0_2) \leq [I_{N_1}(x \wedge y) \wedge I_{N_2}(0_2)]$$
$$\vee [I_{N_1}(y \wedge z) \wedge I_{N_2}(0_2)] \vee [I_{N_1}(x \wedge z) \wedge I_{N_2}(0_2)],$$
$$F_{N_1}(x \wedge y \wedge z) \vee F_{N_2}(0_2) \geq [F_{N_1}(x \wedge y) \wedge F_{N_2}(0_2)]$$
$$\wedge [F_{N_1}(y \wedge z) \wedge F_{N_2}(0_2)] \wedge [T_{N_1}(x \wedge z) \wedge F_{N_2}(0_2)].$$

By using $A_2(0_2) = (1, 1, 0)$, we get

$$T_{N_1}(x \wedge y \wedge z) \leq T_{N_1}(x \wedge y) \vee T_{N_1}(y \wedge z) \vee T_{N_1}(x \wedge z),$$
$$I_{N_1}(x \wedge y \wedge z) \leq I_{N_1}(x \wedge y) \vee I_{N_1}(y \wedge z) \vee I_{N_1}(x \wedge z),$$
$$F_{N_1}(x \wedge y \wedge z) \geq F_{N_1}(x \wedge y) \wedge F_{N_1}(y \wedge z) \wedge F_{N_1}(x \wedge z).$$

Thus N_1 is a *N2AI* of Π_1. Similarly, we can prove that N_2 is a *N2AI* of Π_2. □

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

THEOREM 8.7
Let $\Pi = \Pi_1 \times \Pi_2 \times ... \times \Pi_k$ be a direct product of lattices $\Pi_1, \Pi_2, ..., \Pi_k$. Let N_i, $1 \leq k$ be *SVNI*s of Π_i, respectively. Suppose that for each $i = 1, 2, ..., k$, $N_i(0_i) = (1, 1, 0)$, where 0_i is the least element of Π_i. If $N = N_1 \times N_2 \times ... \times N_k$ is a $N2AI$ of Π, then N_i, is a $N2AI$ of Π_i,$i = 1, ..., k$.

The following example shows that the converse of Theorem 8.5 need not hold.

EXAMPLE 8.8
Consider the lattices Π_1, Π_2 and $\Pi = \Pi_1 \times \Pi_2$ as shown in the Fig. 7. Define $T_{N_1}, I_{N_1}, F_{N_1} : \Pi_1 \to [0, 1]$ and $T_{N_2}, I_{N_2} F_{N_2} : \Pi_2 \to [0, 1]$ as follows $T_{N_1}(0) = 1, T_{N_1}(a) = 1/6, T_{N_1}(b) = 1/4, T_{N_1}(1) = 1/4, I_{N_1}(0) = 3/4, I_{N_1}(a) = 1/2, I_{N_1}(b) = 1/3, I_{N_1}(1) = 2/3, F_{N_1}(0) = 0, F_{N_1}(a) = 5/6, F_{N_1}(b) = 3/4, F_{N_1}(1) = 3/4, T_{N_2}(0) = 1, T_{N_2}(1) = 0, I_{N_2}(0) = 1, I_{N_2}(1) = 0, F_{N_2}(0) = 0, F_{N_2}(1) = 1$. We note that N_1 is a $N2AI$ of Π_1 and N_2 that of Π_2. We consider $N : \Pi_1 \times \Pi_2 \to [0, 1]$ defined by $T_N(x, y) = T_{N_1}(x) \wedge T_{N_2}(y)$, $I_N(x, y) = I_{N_1}(x) \wedge I_{N_2}(y)$, and $F_N(x, y) = F_{N_1}(x) \vee F_{N_2}(y)$. We have $T_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] = T_N(0, 0) = 1$. $T_N[(a, 1) \wedge (1, 0)] = T_N(a, 0) = 1/6$. $T_N[(1, 0) \wedge (b, 1)] = T_N(b, 0) = 1/4$. $T_N[(a, 1) \wedge (b, 1)] = T_N(0, 1) = 0$. We get $T_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] \nleq T_N[(a, 1) \wedge (1, 0)] \vee T_N[(1, 0) \wedge (b, 1)] \vee T_N[(a, 1) \wedge (b, 1)]$. Also $I_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] = I_N(0, 0) = 3/4$. $I_N[(a, 1) \wedge (1, 0)] = I_N(a, 0) = 1/2$. $I_N[(1, 0) \wedge (b, 1)] = I_N(b, 0) = 1/3$. $I_N[(a, 1) \wedge (b, 1)] = I_N(0, 1) = 0$. We get $I_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] \ngeq I_N[(a, 1) \wedge (1, 0)] \vee I_N[(1, 0) \wedge (b, 1)] \vee I_N[(a, 1) \wedge (b, 1)]$. Also $F_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] = F_N(0, 0) = 0$. $F_N[(a, 1) \wedge (1, 0)] = F_N(a, 0) = 5/6$. $F_N[(1, 0) \wedge (b, 1)] = F_N(b, 0) = 3/4$. $F_N[(a, 1) \wedge (b, 1)] = F_N(0, 1) = 1$. Hence $F_N[(a, 1) \wedge (1, 0) \wedge (b, 1)] \ngeq F_N[(a, 1) \wedge (1, 0)] \wedge F_N[(1, 0) \wedge (b, 1)] \wedge F_N[(a, 1) \wedge (b, 1)]$. Hence N is not a $N2AI$ of Π.

THEOREM 8.9
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_1, N_2 SVNIs of Π_1 and Π_2, respectively. Suppose that (i) $N_1(0_1) = N_2(0_2) = (1, 1, 0)$, where 0_1 is the least element of Π_1 and 0_2 that of Π_2 and (ii) $N_1(1_1) = N_2(1_2) = (0, 0, 1)$, where 1_1 is the greatest element of Π_1 and 1_2 that of Π_2. If $N = N_1 \times N_2$ is a $N2AI$ of Π, then N_1 is a NPI of N_1 and N_2 that of Π_2.

PROOF. Suppose that N_1 is not a NPI of Π_1. Then there exist $\alpha, \beta \in \Pi_1$ such that

$$T_{N_1}(\alpha \wedge \beta) \nleq T_{N_1}(\alpha) \vee T_{N_1}(\beta), I_{N_1}(\alpha \wedge \beta) \nleq I_{N_1}(\alpha) \vee I_{N_1} N(\beta) \text{ and}$$
$$F_{N_1}(\alpha \wedge \beta) \ngeq F_{N_1}(\alpha) \wedge F_{N_1}(\beta).$$

Consider the elements $x = (\alpha, 1), y = (1, 0), z = (\beta, 1)$ from Π. We get the following. $T_N(x \wedge y \wedge z) = T_N(\alpha \wedge \beta, 0) = T_{N_1}(\alpha \wedge \beta) \wedge T_{N_2}(0) = T_{N_1}(\alpha \wedge \beta)$. $T_N(x \wedge y) = T_N(\alpha, 0) = T_{N_1}(a) \wedge T_{N_2}(0) = T_{N_1}(\alpha)$. $T_N(y \wedge z) = T_N(\beta, 0) = T_{N_1}(\beta) \wedge T_{N_2}(0) = T_{N_1}(\beta)$. $T_N(x \wedge z) = T_N(\alpha \wedge \beta, 1) = T_{N_1}(\alpha \wedge \beta) \wedge T_{N_2}(1) = T_{N_2}(1) = 0$. $I_N(x \wedge y \wedge z) = I_N(\alpha \wedge \beta, 0) = I_{N_1}(\alpha \wedge \beta) \wedge I_{N_2}(0) = I_{N_1}(\alpha \wedge \beta)$. $I_N(x \wedge y) = I_N(a, 0) = I_{N_1}(a) \wedge I_{N_2}(0) = I_{N_1}(\alpha)$. $I_N(y \wedge z) = I_N(\beta, 0) = I_1(\beta) \wedge I_{N_2}(0) = I_{N_1}(\beta)$. $I_N(x \wedge z) = I_N(\alpha \wedge \beta, 1) = I_{N_1}(\alpha \wedge \beta) \wedge I_{N_2}(1) = I_{N_2}(1) = 0$. $F_N(x \wedge y \wedge z) = F_N(\alpha \wedge \beta, 0) = F_{N_1}(\alpha \wedge \beta) \vee F_{N_2}(0) = F_{N_1}(\alpha \wedge \beta)$. $F_N(x \wedge y) = F_N(\alpha, 0) = F_{N_1}(\alpha) \vee F_{N_2}(0) = F_{N_1}(\alpha)$. $F_N(y \wedge z) = F_N(\beta, 0) = F_{N_1}(\beta) \vee F_{N_2}(0) = F_{N_1}(\beta)$. $F_N(x \wedge z) = F_N(\alpha \wedge \beta, 1) = F_{N_1}(\alpha \wedge \beta) \vee F_{N_2}(1) = F_{N_2}(1) = 1$. Since N is a $N2AI$, we have

$$T_N(x \wedge y \wedge z) \leq T_N(x \wedge y) \vee T_N(y \wedge z) \vee T_N(x \wedge z),$$
$$I_N(x \wedge y \wedge z) \leq I_N(x \wedge y) \vee I_N(y \wedge z) \vee I_N(x \wedge z),$$
$$F_N(x \wedge y \wedge z) \geq F_N(x \wedge y) \wedge F_N(y \wedge z) \wedge F_N(x \wedge z).$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

i.e.

$$T_{N_1}(\alpha \wedge \beta) \leq T_{N_1}(\alpha) \vee T_{N_1}(\beta) \vee 0 = T_{N_1}(\alpha) \vee T_{N_1}(\beta),$$
$$I_{N_1}(\alpha \wedge \beta) \leq I_{N_1}(\alpha) \vee I_{N_1}(\beta) \vee 0 = I_{N_1}(\alpha) \vee I_{N_1}(\beta),$$
$$F_{N_1}(\alpha \wedge \beta) \geq F_{N_1}(\alpha) \wedge F_{N_1}(\beta) \wedge 0 = F_{N_1}(\alpha) \wedge F_{N_1}(\beta),$$

a contradiction. Hence N_1 is a *NPI*. Similarly, we can show that N_2 is an *NPI*. □

THEOREM 8.10
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_1, N_2 be SVNIs of Π_1 and Π_2, respectively. If $N = N_1 \times N_2$, then N is a $N2AI$ of Π.

PROOF. Let $(\alpha, x), (\beta, y), (\gamma, z) \in \Pi$. To show that N is a $N2AI$, we need to show that

$$T_N[(\alpha,x) \wedge (\beta,y) \wedge (\gamma,z)] \leq T_N[(\alpha,x) \wedge (\beta,y)] \vee T_N[(\beta,y) \wedge (\gamma,z)] \vee T_N[(\alpha,x) \wedge (\gamma,z)],$$
$$I_N[(\alpha,x) \wedge (\beta,y) \wedge (\gamma,z)] \leq I_N[(\alpha,x) \wedge (\beta,y)] \vee I_N[(\beta,y) \wedge (\gamma,z)] \vee I_N[(\alpha,x) \wedge (\gamma,z)],$$
$$F_N[(\alpha,x) \wedge (\beta,y) \wedge (\gamma,z)] \geq F_N[(\alpha,x) \wedge (\beta,y)] \wedge F_N[(\beta,y) \wedge (\gamma,z)] \wedge F_N[(\alpha,x) \wedge (\gamma,z)].$$

i.e. show that

$$T_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) \leq T_N(\alpha \wedge \beta, x \wedge y) \vee T_N(\beta \wedge \gamma, y \wedge z) \vee T_N(\alpha \wedge \gamma, x \wedge z),$$
$$I_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) \leq I_N(\alpha \wedge \beta, x \wedge y) \vee I_N(\beta \wedge \gamma, y \wedge z) \vee I_N(\alpha \wedge \gamma, x \wedge z),$$
$$F_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) \geq F_N(\alpha \wedge \beta, x \wedge y) \wedge F_N(\beta \wedge \gamma, y \wedge z) \wedge F_N(\alpha \wedge \gamma, x \wedge z).$$

We have

$$T_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) = T_N(\alpha \wedge \beta \wedge \gamma) \wedge T_N = (x \wedge y \wedge z),$$
$$I_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) = I_N(\alpha \wedge \beta \wedge \gamma) \wedge I_N(x \wedge y \wedge z),$$
$$F_N(\alpha \wedge \beta \wedge \gamma, x \wedge y \wedge z) = F_N(\alpha \wedge \beta \wedge \gamma) \vee F_N(x \wedge y \wedge z).$$

As N_1, N_2 be *NPI*s, we can write

$$T_{N_1}(x \wedge y \wedge z) = T_{N_1}(\alpha) \vee T_{N_1}(\beta) \vee T_{N_1}(\gamma),$$
$$I_{N_1}(x \wedge y \wedge z) = I_{N_1}(\alpha) \vee I_{N_1}(\beta) \vee I_{N_1}(\gamma),$$
$$F_{N_1}(x \wedge y \wedge z) = F_{N_1}(\alpha) \wedge F_{N_1}(\beta) \wedge F_{N_1}(\gamma)$$

and

$$T_{N_2}(x \wedge y \wedge z) = T_{N_2}(x) \vee T_{N_2}(y) \vee T_{N_2}(z),$$
$$I_{N_2}(x \wedge y \wedge z) = I_{N_2}(x) \vee I_{N_2}(y) \vee I_{N_2}(z),$$
$$F_{N_2}(x \wedge y \wedge z) = F_{N_2}(x) \wedge F_{N_2}(y) \wedge F_{N_2}(z).$$

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

Also, we have

$$\begin{aligned}
T_N(\alpha\wedge\beta,x\wedge y)\vee T_N(\beta\wedge\gamma,y\wedge z)\vee T_N(\alpha\wedge\gamma,x\wedge z) &= [T_{N_1}(\alpha\wedge\beta)\wedge T_{N_2}(x\wedge y)]\\
&\vee [T_{N_1}(\beta\wedge\gamma)\wedge T_{N_2}(y\wedge z)]\\
&\vee [T_{N_1}(\alpha\wedge\gamma)\wedge T_{N_2}(x\wedge z)],\\
I_N(\alpha\wedge\beta,x\wedge y)\vee I_N(\beta\wedge\gamma,y\wedge z)\vee I_N(\alpha\wedge\gamma,x\wedge z) &= [I_{N_1}(\alpha\wedge\beta)\wedge I_{N_2}(x\wedge y)]\\
&\vee [I_{N_1}(\beta\wedge\gamma)\wedge I_{N_2}(y\wedge z)]\\
&\vee [I_{N_1}(\alpha\wedge\gamma)\wedge I_{N_2}(x\wedge z)],\\
F_N(\alpha\wedge\beta,x\wedge y)\wedge F_N(\beta\wedge\gamma,y\wedge z)\wedge F_N(\alpha\wedge\gamma,x\wedge z) &= [F_{N_1}(\alpha\wedge\beta)\vee F_{N_2}(x\wedge y)]\\
&\wedge [F_{N_1}(\beta\wedge\gamma)\vee F_{N_2}(y\wedge z)]\\
&\wedge [F_{N_1}(\alpha\wedge\gamma)\vee F_{N_2}(x\wedge z)].
\end{aligned}$$

Since N_1, N_2 be *NPI*s, we can write

$$\begin{aligned}
&\{[T_{N_1}(\alpha)\vee T_{N_1}(\beta)]\wedge[T_{N_2}(x)\vee T_{N_2}(y)]\}\\
&\vee\{[T_{N_1}(\beta)\vee T_{N_1}(\gamma)]\wedge[T_{N_2}(y)\vee T_{N_2}(z)]\}\\
&\vee\{[T_{N_1}(\alpha)\vee T_{N_1}(\gamma)]\wedge[T_{N_2}(x)\vee T_{N_2}(z)]\},,\\
&\{[I_{N_1}(\alpha)\vee I_{N_1}(\beta)]\wedge[I_{N_2}(x)\vee I_{N_2}(y)]\}\\
&\vee\{[I_{N_1}(\beta)\vee I_{N_1}(\gamma)]\wedge[I_{N_2}(y)\vee I_{N_2}(z)]\}\\
&\vee\{[I_{N_1}(\alpha)\vee I_{N_1}(\gamma)]\wedge[I_{N_2}(x)\vee I_{N_2}(z)]\},,\\
&\{[F_{N_1}(\alpha)\wedge F_{N_1}(\beta)]\wedge[F_{N_2}(x)\wedge F_{N_2}(y)]\}\\
&\wedge\{[F_{N_1}(\beta)\wedge F_{N_1}(\gamma)]\wedge[F_{N_2}(y)\wedge F_{N_2}(z)]\}\\
&\wedge\{[F_{N_1}(\alpha)\wedge F_{N_1}(\gamma)]\wedge[F_{N_2}(x)\wedge F_{N_2}(z)]\}.
\end{aligned}$$

By applying distributivity,

$$\begin{aligned}
&[T_{N_1}(\alpha)\vee T_1(\beta)\vee T_{N_1}(\gamma)]\wedge[T_{N_2}(x)\vee T_{N_2}(y)\vee T_{N_2}(z)],\\
&[I_{N_1}(\alpha)\vee I_{N_1}(\beta)\vee I_{N_1}(\gamma)]\wedge[I_{N_2}(x)\vee I_{N_2}(y)\vee I_{N_2}(z)],\\
&[F_{N_1}(\alpha)\wedge F_{N_1}(\beta)\wedge F_{N_1}(\gamma)]\wedge[F_{N_2}(x)\wedge F_{N_2}(y)\wedge F_{N_2}(z)].
\end{aligned}$$

Thus N is $N2AI$. □

THEOREM 8.11
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_i, M_j be *SVNI*s of Π_1 and Π_2, respectively. Let $K_{ij} = N_i \times M_j$. Then $\bigcap K_{ij} = \bigcap N_i \times \bigcap M_j$.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

PROOF. Let $(x, y) \in \Pi$. We have

$$\begin{aligned}\bigcap K_{ij}(x,y) &= \wedge_{ij}(N_i \times M_j)(x,y)\\ &= \wedge_{ij}(N_i(x) \wedge M_j(y))\\ &= \wedge_i N_i(x) \wedge \wedge_j M_j(y)\\ &= (\wedge_i N_i \times \wedge_j M_j)(x,y)\end{aligned}$$

Thus, $\bigcap K_{ij} = \bigcap N_i \times \bigcap M_j$. □

THEOREM 8.12
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. (i) Let N be a *SVNI* of Π_1 and χ_{Λ_1} is a SVNIs of Π_2. Then $\sqrt{N \times \chi_{\Lambda_1}} = \sqrt{N} \times \chi_{\Lambda_1}$. (ii) Let K be a *SVNI* of Π_2 and χ_{Λ_2} be a *SVNI* of Π_1. Then $\sqrt{\chi_{\Lambda_2} \times K} = \chi_{\Lambda_2} \times \sqrt{K}$.

PROOF. (i) Let S be a *NPI* of Π such that $N \times \chi_{\Lambda_1} \subseteq P$. By Theorem 8.2, $S = K \times Z$ for some *NPI* K of Π_1 and Z of Π_2. Then $N \subseteq K$ and $\chi_{\Lambda_1} \subseteq Z$. It follows that $Z = \chi_{\Lambda_1}$. Thus $S \subseteq S \times M$. This shows that $\sqrt{N \times \chi_{\Lambda_1}} = \sqrt{N} \times \chi_{\Lambda_1}$. (ii) It can be proved similarly. □

THEOREM 8.13
Let $\Pi = \Pi_1 \times \Pi_2$ be a direct product of lattices Π_1, Π_2. Let N_1 be a *SVNI* of Π_1 and χ_Λ be a *SVNI*s of Π_2. Then $N_1 \times M$ is a 2*ANPRI* of Π, if and only if N_1 is a 2*ANPRI* of Π_1.

PROOF. Suppose that $N_1 \times M$ is a 2*ANPRI* of Π. Let $N_2, N_3, N_4 \in SVNS(\Pi_1)$ be such that $N_2 \cap N_3 \cap N_4 \subseteq N_1$. Consider $N_i \times M$. Then $(N_2 \cap N_3 \cap N_4) \times M \subseteq N_1 \times M$. This implies $(N_2 \times M) \cap (N_3 \times M) \cap (N_4 \times M) \subseteq N_1 \times M$. Since $N_1 \times M$ is a 2*ANPRI* of Π, we get either

$$(N_2 \times M) \cap (N_3 \times M) \subseteq N_1 \times M$$

or

$$(N_3 \times M) \cap (N_4 \times M) \subseteq \sqrt{N_1 \times M}$$

or

$$(N_2 \times M) \cap (N_4 \times M) \subseteq \sqrt{N_1 \times M}.$$

Thus either

$$N_2 \cap N_3 \subseteq N_1$$

or

$$N_3 \cap N_4 \subseteq \sqrt{N_1}$$

or

$$N_2 \cap N_4 \subseteq \sqrt{N_1}.$$

Hence N_1 is a 2*ANPRI* of Π_1. The converse follows by following similar steps. □

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

9 Conclusion

In this study, we have introduced and explored several key concepts related to lattices, specifically focusing on neutrosophic prime ideals, prime neutrosophic ideals, neutrosophic 2-absorbing ideals, 2-absorbing neutrosophic ideals, neutrosophic primary ideals, and primary neutrosophic ideals. Subsequently, we have provided a detailed characterization of neutrosophic 2-absorbing primary ideals within a lattice framework. The study delves into the transitions among these various concepts and examines their interrelationships with the traditional ideals of a lattice. Additionally, we have extended our investigation to consider these concepts in the context of lattices formed by the Cartesian product of individual lattices. This comprehensive examination offers a deeper understanding of the intricate relationships and behaviors of these neutrosophic ideals in complex lattice structures.

Based on our findings, we have addressed several key research questions:

1) In what ways do complex neutrosophic prime ideals and prime complex neutrosophic ideals influence the structural dynamics of lattices?

2) How can the conditions for a bipolar neutrosophic 2-absorbing ideal to become a bipolar neutrosophic 2-absorbing primary ideal be systematically identified and analyzed?

3) What roles do neutrosophic hesitant fuzzy primary ideals and primary neutrosophic hesitant fuzzy ideals play in shaping the relationships between neutrosophic hesitant fuzzy and traditional lattice ideals?

4) How do the characteristics of hesitant bipolar valued neutrosophic 2-absorbing primary ideals change when considered within lattices formed by Cartesian products?

5) What are the broader theoretical and practical implications of the transitions and interrelationships among neutrosophic vague prime, primary, and 2-absorbing ideals in complex lattice structures, and how can this framework be applied to practical domains such as decision-making under uncertainty?

Abbreviations The following abbreviations are used in this manuscript:

SVNS	Single Valued Neutrosophic Set
SVNI	Single Valued Neutrosophic Ideal
NPI	Neutrosophic Prime Ideal
PNI	Prime Neutrosophic Ideal
*N*2*AI*	Neutrosophic 2-Absorbing Ideal
2*ANI*	2-Absorbing Neutrosophic Ideal
NPRI	Neutrosophic Primary Ideal
*N*2*APRI*	Neutrosophic 2-Absorbing Primary Ideal
PRNI	Primary Neutrosophic Ideal
2*APRNI*	2-Absorbing Primary Neutrosophic Ideal

Acknowledgements

This work has been supported by Yildiz Technical University Scientific Research Projects Coordination Unit under project number FBA-2024-6550 and the Scientific and Technological Research Council of Türkiye (TÜBİTAK) under the 2211–Domestic Graduate Scholarship Program conducted by the Directorate of Scientist Training and Development (BİDEB).

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

References

[1] Agarwal RP, Milles S, Ziane B *et al.* Ideals and filters on neutrosophic topologies generated by neutrosophic relations. *Axioms* 2024;**13**:292. https://doi.org/10.3390/axioms13050292.

[2] Atanassov KT. Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 1986;**20**:87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

[3] Bennoui A, Zedam L, Milles S. Several types of single-valued neutrosophic ideals and filters on a lattice. *TWMS J Appl Eng Math* 2023.

[4] Borzooei RA, Jun YB *et al.* Neutrosophic li-ideals in lattice implication algebras. *Neutrosophic Sets Syst* 2020;**31**:282–96.

[5] Cetkin V, Aygün H. An approach to neutrosophic ideals. *Univers J Math Appl* 2018;**1**:132–6.

[6] Gratzer G. *Lattice Theory: First Concept and Distributive Lattices*. Reading, MA: W.H. Freeman and Co. San Francisco, 1971.

[7] Qingqing H, Zhang X. Neutrosophic triangular norms and their derived residuated lattices. *Symmetry* 2019;**11**:817.

[8] James J, Mathew SC. Lattice valued neutrosophic sets. *Infin Stud* 2021.

[9] Jun YB, Xin XL. Implicative neutrosophic li-ideals of lattice implication algebras. *J Intell Fuzzy Syst* 2020;**38**:2141–9.

[10] Kandasamy V, Smarandache F. Neutrosophic lattices. *Infin Stud* 2014.

[11] Vasantha, Kandasamy WB, Smaradache F *et al.* Pseudo lattice graphs and their applications to fuzzy and neutrosophic models. *Infin Stud* 2014.

[12] Sahin M, Uluçay V, Olgun N *et al.* On neutrosophic soft lattices. *Afr Mat* 2017;**28**:379–88.

[13] Salama AA, Smarandache F, Alblowi SA. The characteristic function of a neutrosophic set. *Infin Stud* 2014;**3**:14–7.

[14] Singh PK. Three-way fuzzy concept lattice representation using neutrosophic set. *Int J Mach Learn Cybern* 2017;**8**:69–79.

[15] Singh PK. Complex neutrosophic concept lattice and its applications to air quality analysis. *Chaos Soliton Fract* 2018;**109**:206–13.

[16] Singh PK. Interval-valued neutrosophic graph representation of concept lattice and its (α, β, γ)-decomposition. *Arab J Sci Eng* 2018;**43**:723–40.

[17] Singh PK. Three–way bipolar neutrosophic concept lattice. In: *Fuzzy Multi-Criteria Decision-Making Using Neutrosophic Sets*. Springer, 2018, 417–32.

[18] Smarandache F. Neutrosophic set—a generalization of the intuitionistic fuzzy set. *Int J Pure Appl Math* 2005;**24**:287.

[19] Patil YS, Nimbhorkar SK. Generalizations of prime fuzzy ideals of a lattice. *J Hyperstruct* 2020;**9**:1–33.

[20] Patil YS, Nimbhorkar SK. Generalizations of prime fuzzy ideals of a lattice. *J Hyperstruct* 2020;**9**:1–33.

[21] Topal S, Smarandache F. A lattice theoretic look: a negated approach to adjectival (intersective, neutrosophic and private) phrases. In: *Collected Papers. Volume VI: On Neutrosophic Theory and Applications*. 2022, 180.

[22] Vimala J, Rajareega S, Preethi D *et al.* Lattice ordered neutrosophic soft set and its usage for a large-scale company in adopting best Ngo to deliver funds during covid-19. *J Comput Cogn Eng* 2023;**2**:210–9.

[23] Yang H, Qin K. Neutrosophic three-way concept lattice and its application in conflict analysis. *J Intell Fuzzy Syst* 2021;**41**:3219–36.

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025

[24] Zadeh LA. Fuzzy sets. *Inf Control* 1965;**8**:338–53.
[25] Zedam L, Milles S, Bennoui A. Ideals and filters on a lattice in neutrosophic setting. *Appl Appl Math* 2021;**16**.
[26] Zhang X, Bo C, Smarandache F *et al.* New inclusion relation of neutrosophic sets with applications and related lattice structure. *Int J Mach Learn Cybern.* 2018;**9**:1753–63.

Received 9 August 2025

Downloaded from https://academic.oup.com/jigpal/article/33/6/jzaf088/8342721 by Yildiz Teknik University Library (YTU) user on 28 November 2025