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A study of 1-absorbing intuitionistic fuzzy ideals of commutative semirings

E. Mehmet Ozkan *
Department of Mathematics
Yildiz Technical University
Istanbul
Turkey

Serkan Onar †
Department of Mathematical Engineering
Yildiz Technical University
Istanbul
Turkey

Ayten Ozkan §
Ilayda Kaplan ‡
Department of Mathematics
Yildiz Technical University
Istanbul
Turkey

Abstract

The primary objective of this investigation is to offer an extensive and thorough comprehension of the characteristics, attributes, and operational tendencies inherent in 1-absorbing intuitionistic fuzzy ideals as well as 1-absorbing intuitionistic fuzzy primary ideals within the context of commutative semirings. By examining their contribution to the literature, we can gain insight into their unique characteristics and explore their potential applications in various fields. Furthermore, defining the Cartesian product of these ideals

* *E-mail:* **mozkan@yildiz.edu.tr (Corrsponding Author)**
† *E-mail:* **serkan10ar@gmail.com**
§ *E-mail:* **uayten@yildiz.edu.tr**
‡ *E-mail:* **kaplanilayda53@gmail.com**

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allows for a deeper analysis of their interactions and opens up new avenues for research in this area.

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1. Introduction

Zadeh proposed the idea of a fuzzy subset of a given set in his publication [1], defining it as a mapping originating from X and extending to the interval [0,1], outlining the association between elements of the set and their degrees of membership within this range. This mapping allows for the representation of degrees of membership for elements in the set X. By assigning a value between 0 and 1, Zadeh introduced a way to capture the uncertainty and vagueness inherent in many real-world situations. In mathematics and many other applicable domains, Zadeh and other researchers developed fuzzy set theory. Chang [2], a student of Zadeh's, published the work "Fuzzy Topological Spaces" in 1968. Inspired by Chang's article, Rosenfeld wrote "Fuzzy Groups" in [3] because he believed that algebraic structures could benefit from these methods. Mukherjee et al. [4], Swamy et al. [5], and Yue [6] modified a few common ideas about rings and ideals using fuzzy set theory. Dutta et al. [7] studied fuzzy k-ideals of semirings and fuzzy prime k-ideals of semirings of nonnegative integers. Their research focused on extending the concepts of rings and ideals to include fuzzy sets, allowing for a more flexible and nuanced understanding of these mathematical structures. By introducing the notion of fuzzy prime k-ideals, they were able to explore the properties and applications of these structures in semirings of nonnegative integers and semirings of semirings. This work has contributed to the development of fuzzy set theory in the context of rings and ideals. The theory of fuzzy sets was specifically extended to the ideals of a ring by several researchers [8, 9, 10].

The study of intuitionistic fuzzy sets(IFS) has a long history. To deal with vagueness, IFS [11] has been proven to be quite helpful by Atanassov. There are instances where the evaluation of membership values is not achievable to our satisfaction due to insufficient information being given (apart from the presence of vagueness). In situations where fuzzy set theory is not appropriate, IFS theory is a better choice. The concept of intuitionistic fuzzy topological spaces with IFS has recently been studied by many authors. These studies focused on different aspects of intuitionistic

fuzzy topological spaces, such as their applications in decision-making processes and their mathematical properties. Coker [12] explored the use of IFS in modeling uncertainty in decision-making, while Es et al. [13] investigated the concept of intuitionistic fuzzy open sets and their relationship with other types of fuzzy sets. Gurcay et al. [14] examined the concept of intuitionistic fuzzy continuity and its implications for the topological structure of these spaces, while Lee et al. [15] proposed a new approach to defining intuitionistic fuzzy neighborhoods based on IFS.

Since its inception by Hur et al. [16], the notion of intuitionistic fuzzy subrings has spurred a multitude of scientific endeavors aimed at further exploration and expansion. Drawing from the foundational concept of fuzzy spaces, Marashdeh and Salleh [17] introduced the concept of intuitionistic fuzzy rings, whereas Sharma [18] delved into the realm of translations pertaining to intuitionistic fuzzy subrings. Extending beyond these initial contributions, Hameed et al. embarked on a comprehensive investigation into the interconnections among KU fuzzy algebra translations, fuzzy extensions, and fuzzy multiplications [19, 20]. Subsequently, their work delved into magnified translations of heuristic fuzzy AT ideals into AT algebra in [21]. In 2015, Jun et al. introduced the novel concept of a pseudo BH-algebra [22], prompting A.H. Nouri and H.H. Abbass to delve into various pseudo-BHalgebraic ideal types [23]. Continuing this trajectory, recent work in 2021 has centered on pseudo intuitionistic fuzzy ideals of pseudo BH-algebras [24].

Expanding upon the notion of a prime idea, Badawi [25] introduced the concept of a 2-absorbing ideal, while Groenewald [26] furthered this exploration within a non-commutative ring context, generalizing the idea of a prime ideal to form the basis of a 2-absorbing ideal. In a related context, Hashempoor et al. elaborated on the perspective of 2-absorbing fuzzy ideals within the semiring $\mathcal{P}$ (2AFIS($\mathcal{P}$)) framework in [27].

Numerous researchers have expanded the concept of 2-absorbing structures by applying it to fuzzy algebraic systems, thereby extending its theoretical framework and enabling deeper explorations of its properties within the context of fuzzy set theory and algebraic structures (see [33, 38]).

Subsequently, 1-absorbing primary ideals(1API) were initially outlined and studied by Badawi et al. in [28]. They also examined some of their significant features. After this study, they looked into 1-absorbing prime ideals of commutative rings in [29].

The aim of this article is to introduce the concept of 1-absorbing fuzzy ideals of semiring $\mathcal{P}$ (1AIFIS($\mathcal{P}$)) by using intuitionistic fuzzy ideals(IFI) and to show some results of 1-absorbing intuitionistic fuzzy ideals of semiring $\mathcal{P}$ (1AIFPIS($\mathcal{P}$)).

2. Preliminaries

A semiring is a nonempty set $\mathcal{P}$ in which operations addition and multiplication (denoted by + and Â. , respectively) have been defined so that the following conditions are satisfied:

(1) $(\mathcal{P},+)$ is a commutative monoid with identity 0,

(2) $(\mathcal{P},\cdot)$ is a commutative semigroup,

(3) $p(q+r)=pq+pr$ and $(p+q)r=pr+qr$ for all $p,q,r\in\mathcal{P}$,

(4) $0p=p0=0$ for all $p\in\mathcal{P}$.

If a semiring $\mathcal{P}$ is commutative satisfies $pq=qp$ for all $p,q\in\mathcal{P}$. In this paper, all semirings are commutative.

If $\mathcal{I}$ stands as an ideal within a commutative semiring denoted as $\mathcal{P}$, the radical of $\mathcal{I}$, denoted as $\sqrt{\mathcal{I}}$, is established as a prime ideal of $\mathcal{P}$, as indicated in reference [30]. Within the context of a commutative semiring equipped with an identity element, denoted as $\mathcal{P}$, a specific characterization emerges for a proper ideal $\mathcal{I}$ of $\mathcal{P}$ labeled as 1AIS$(\mathcal{P})$, defining it such that whenever elements p, q, and r from $\mathcal{P}$ satisfy the condition $pqr\in\mathcal{I}$, it results in either $pq\in\mathcal{I}$ or $r\in\mathcal{I}$. The designation of $\mathcal{P}$ as a 1-absorbing semiring occurs if and only if its zero ideal conforms to the specification of 1AIS$(\mathcal{P})$. Moreover, considering a proper ideal $\mathcal{I}$ of $\mathcal{P}$ labeled as 1APIS $(\mathcal{P})$, it adheres to the condition that whenever elements p, q, and r from $\mathcal{P}$ satisfy $pqr\in\mathcal{I}$, the consequence leads to either $pq\in\mathcal{I}$ or $r\in\sqrt{\mathcal{I}}$. Additionally, if $\mathcal{I}$ aligns with the criteria of 1APIS$(\mathcal{P})$, it naturally complies with 1AIS$(\mathcal{P})$. Under the assumption that $\mathcal{I}$ represents a nontrivial proper ideal within a semiring denoted as $\mathcal{P}$, certain conditions are deemed equivalent.

(1) $\mathcal{I}$ is a 1AIS$(\mathcal{P})$.

(2) If $\mathcal{I}_1\mathcal{I}_2\mathcal{I}_3\subseteq\mathcal{I}$ for some ideals $\mathcal{I}_1,\mathcal{I}_2,\mathcal{I}_3$ of $\mathcal{P}$, then $\mathcal{I}_1\mathcal{I}_2\subseteq\mathcal{I}$ or $\mathcal{I}_3\subseteq\mathcal{I}$.

Following these arguments, the definitions of intuitionistic fuzzy sets (IFS) and intuitionistic fuzzy ideals will be expounded upon. Intuitionistic fuzzy sets are delineated as entities existing within a non-empty set denoted as $\mathcal{R}$, characterized by the structure $A=\langle p,\phi(p),\psi(p)\rangle \mid p\in\mathcal{R}$, where the functions $\phi:\mathcal{R}\to[0,1]$ and $\psi:\mathcal{R}\to[0,1]$ represent the degrees denoting membership and non-membership, respectively, of each element

p belonging to set A. It is specified that $0 \le \phi(p) + \psi(p) \le 1$ holds true for all elements $p \in \mathcal{R}$, as detailed in reference [32].

Let $\mathcal{R}$ be a nonempty set and let $A = \langle \phi_A, \psi_A \rangle$ and $B = \langle \phi_B, \psi_B \rangle$ be intuitionistic fuzzy sets in $\mathcal{X}$. Then [11],

(1) $A \subset B$ iff $\phi_A \le \phi_B$ and $\psi_A \ge \psi_B$,
(2) $A = B$ iff $A \subset B$ and $B \subset A$,
(3) $A^c = \langle \psi_A, \phi_A \rangle$,
(4) $A \cap B = (\phi_A \wedge \phi_B, \psi_A \vee \psi_B)$,
(5) $A \cup B = (\phi_A \vee \phi_B, \psi_A \wedge \psi_B)$.

Let I be an IFS in a set R and $p, q \in L$ such that $\ell + \eta \le 1$. Then, the set $I_{(\ell,\eta)} = \{r \in R \mid \phi_I(r) \ge \ell \text{ and } \psi_I(r) \le \eta\}$ is called (ℓ, η) level subset of I [31]. Let $\mathcal{P}$ be a semiring. An IFS $A = \{\langle p, \phi(p), \psi(p) \rangle \mid p \in \mathcal{P}\}$ is said to be an IFIS$(\mathcal{P})$ if $\forall p, q \in \mathcal{P}$ [32]

(1) $\phi(p+q) \ge \phi(p) \wedge \phi(q)$,
(2) $\psi(p+q) \le \psi(p) \vee \psi(q)$,
(3) $\phi(pq) \ge \phi(p) \vee \phi(q)$,
(4) $\psi(pq) \le \psi(p) \wedge \psi(q)$.

Let $\Upsilon = \langle \phi_\Upsilon, \psi_\Upsilon \rangle$ be an IFIS$(\mathcal{P})$. Then, $\sqrt{\Upsilon}$ is called the radical of Υ and it is defined by $\sqrt{\Upsilon}(p) = \vee_{n \ge 1} \Upsilon(p^n)$ [35]. If Υ and Θ are IFIS$(\mathcal{P})$, then [34] $\sqrt{\Upsilon \cap \Theta} = \sqrt{\Upsilon} \cap \sqrt{\Theta}$. Let $f : \mathcal{P} \to \mathcal{Q}$ be a semiring homomorphism and let Υ be an IFIS$(\mathcal{P})$ such that Υ is an constant on $\ker f$ and Θ be an IFIS $(\mathcal{Q})$. Then [37],

(1) $\sqrt{f(\Upsilon)} = f(\sqrt{\Upsilon})$,
(2) $\sqrt{f^{-1}(\Theta)} = f^{-1}(\sqrt{\Theta})$.

A fuzzy ideal Υ of a semiring $\mathcal{P}$ is called intuitionistic fuzzy primary ideal of $\mathcal{P}$

$$\phi_\Upsilon(pq) = \phi_\Upsilon(p) \text{ and } \psi_\Upsilon(pq) = \psi_\Upsilon(p)$$

or

$$\phi_\Upsilon(pq) \le \phi_\Upsilon(q^n) \text{ and } \psi_\Upsilon(pq) \ge \psi_\Upsilon(q^n)$$

for all $p,q \in \mathcal{P}$, and $n \in \mathbb{Z}^+$ [36].

3. 1-Absorbing Intuitionistic Fuzzy Ideals of Semiring

This section will be devoted to 1AIFIS$(\mathcal{P})$ and unless otherwise indicated, $\mathcal{P}$ symbolizes a commutative semiring.

Definition 1: Let Υ be an IFS and $\mathcal{P}$ be a semiring. An IFI $\Upsilon =< \phi, \psi >$ of semiring $\mathcal{P}$ is said to be 1AIFIS$(\mathcal{P})$

$$\phi(pqr) \geq \ell \text{ implies } \phi(pq) \geq \ell \text{ or } \phi(r) \geq \ell,$$

$$\psi(pqr) \leq \eta \text{ implies } \psi(pq) \leq \eta \text{ or } \psi(r) \leq \eta$$

for all $p,q,r \in \mathcal{P}$ and $\ell, \eta \in [0,1]$.

Example 1: Let $\Upsilon =< \phi, \psi >$ be an IFIS$(\mathcal{P})$ and $\mathcal{P} = \{0, \omega, \kappa\}$ represent a set with defined addition and multiplication operations as outlined below:

$+$	0	ω	κ
0	0	ω	κ
ω	ω	0	κ
κ	κ	κ	0

$\cdot$	0	ω	κ
0	0	0	0
ω	0	0	0
κ	0	0	κ

Then $(\mathcal{P}, +, \cdot)$ forms a commutative semiring. Define fuzzy subsets ϕ and ψ of $\mathcal{P}$ by $\phi(0) = 1$, $\phi(\omega) = 0.3$, $\phi(\kappa) = 1$, $\psi(0) = 0$, $\psi(\omega) = 0.5$ and $\psi(\kappa) = 0$. Hence, $\Upsilon =< \phi, \psi >$ is a 1AIFIS$(\mathcal{P})$.

Example 2: Let $\Upsilon =< \phi, \psi >$ be an IFIS$(\mathcal{P})$ and $\mathcal{P} = \{0, \omega, \kappa, \xi\}$ represent a set with defined addition and multiplication operations as outlined below:

$+$	0	ω	κ	ξ
0	0	ω	κ	ξ
ω	ω	ω	v	ξ
κ	v	v	κ	ξ
ξ	ξ	ξ	ξ	ξ

$\cdot$	0	ω	κ	ξ
0	0	0	0	0
ω	0	0	0	0
κ	0	0	κ	κ
ξ	0	0	κ	ξ

Then $(\mathcal{P}, +, \cdot)$ forms a commutative semiring.

Define fuzzy subsets ϕ and ψ of $\mathcal{P}$ by $\phi(0) = 1$, $\phi(\omega) = 0.1$, $\phi(\kappa) = 1$, $\phi(\xi) = 0.5$, $\psi(0) = 0$, $\psi(\omega) = 0.7$, $\psi(\kappa) = 0$ and $\psi(\xi) = 0.2$. Hence, $\Upsilon = \langle \ell, \eta \rangle$ is a 1AIFIS$(\mathcal{P})$.

Example 3: Let $\Upsilon = \langle \ell, \eta \rangle$ be an IFIS$(\mathcal{P})$ defined by

$$\ell_{\Upsilon}(x) = \begin{Bmatrix} 1, & \text{if } x \in 75\mathbb{Z} \\ 0, & \text{otherwise} \end{Bmatrix} \text{ and } \eta_{\Upsilon}(x) = \begin{Bmatrix} 0, & \text{if } x \in 75\mathbb{Z} \\ 1, & \text{otherwise} \end{Bmatrix}$$

Υ is not a 1AIFIS$(\mathcal{P})$ since $3_{(1,0)}5_{(1,0)}5_{(1,0)} \in \Upsilon$, but $3_{(1,0)}5_{(1,0)} \notin \Upsilon$ and $5_{(1,0)} \notin \Upsilon$.

Definition 2: An IFIS$(\mathcal{P})$ $\Upsilon = < \phi, \psi >$ of $\mathcal{P}$ is named as a 1-absorbing intuitionistic fuzzy primary ideal of $\mathcal{P}$

$$\phi(pqr) \geq \ell \text{ implies } \phi(pq) \geq \ell \text{ or } \sqrt{\phi}(r) \geq \ell,$$

$$\psi(pqr) \leq \eta \text{ implies } \psi(pq) \leq \eta \text{ or } \sqrt{\psi}(r) \leq \eta,$$

for all $p, q, r \in \mathcal{P}$ and $\ell, \eta \in [0,1]$.

Theorem 1: *Let Υ be an intuitionistic fuzzy subset of $\mathcal{P}$. Then, Υ is a 1AIFIS $(\mathcal{P})$ if and only if $\Upsilon_{(\phi,\psi)}$ is a 1AIS$(\mathcal{P})$.*

Proof: $(\Rightarrow)$: Let Υ is a 1AIFIS$(\mathcal{P})$ and we want to show that $\Upsilon_{(\ell,\eta)}$ is a 1AIS$(\mathcal{P})$.

Take $pqr \in \Upsilon_{(\ell,\eta)}$ such that $p, q, r \in \mathcal{P}$ and $\ell, \eta \in [0,1]$. In view of the fact Υ is a 1AIFIS$(\mathcal{P})$, we get

$$\begin{aligned} pqr \in \Upsilon_{(\ell,\eta)} &\Rightarrow \phi(pqr) \geq \ell \text{ and } \psi(pqr) \leq \eta \\ &\Rightarrow \phi(pq) \geq \ell \text{ or } \phi(r) \geq \ell \text{ and} \\ &\Rightarrow \psi(pq) \leq \eta \text{ or } \psi(r) \leq \eta \\ &\Rightarrow pq \in \Upsilon_{(\ell,\eta)} \text{ or } r \in \Upsilon_{(\ell,\eta)}. \end{aligned}$$

Therefore, $\Upsilon_{(\ell,\eta)}$ is a 1AIS$(\mathcal{P})$.

$(\Leftarrow)$: Contrariwise, assume Υ be intuitionistic fuzzy subset of $\mathcal{P}$ with $\Upsilon_{(\ell,\eta)}$ is a 1AIS$(\mathcal{P})$ for every $\ell, \eta \in [0,1]$, however Υ is not a 1AIFIS $(\mathcal{P})$. That being the case, $\phi(pqr) \geq \ell$ doesn't indicate $\phi(pq) \geq \ell$ or $\phi(r) \geq \ell$, and $\psi(pqr) \leq \eta$ doesn't show $\psi(pq) \leq \eta$ or $\psi(r) \leq \eta$ for $p, q, r, \in \mathcal{P}$ and $\ell, \eta \in [0,1]$. It means that $pqr \in \Upsilon_{(\ell,\eta)}$ doesn't give $pq \in \Upsilon_{(\ell,\eta)}$ or $r \in \Upsilon_{(\ell,\eta)}$.

This is a contradiction with $\Upsilon_{(\ell,\eta)}$ is a 1AIS$(\mathcal{P})$. So, our basis assumption is not true. Hence, we reach Υ is a 1AIFIS$(\mathcal{P})$.

Theorem 2: *Let Θ be a 1AIS$(\mathcal{P})$ and $\ell, \eta \in [0,1]$. Afterward an intuitionistic fuzzy subset Υ of $\mathcal{P}$ is represented as*

$$\Theta(a)=\begin{cases}(1,0) & ; \quad a\in\Upsilon\\ (A,w) & ; \quad \text{otherwise}\end{cases}$$

Then Θ is a 1AIFIS$(\mathcal{P})$.

Proof: Take Θ be a 1AIFIS$(\mathcal{P})$ and $p,q,r\in\mathcal{P}$. Presume that $\phi(pqr)\geq\ell$ and $\psi(pqr)\leq\eta$.

Condition I: $\ell=1$ and $\eta=0$

$pqr\in\Theta_{(1,0)}$ implies that $pqr\in\Theta$. Then, in case

$$\phi(pq)<\ell=1 \text{ or } \phi(r)<\ell=1 \text{ and}$$
$$\psi(pq)>\eta=0 \text{ or } \psi(r)>\eta=0.$$

It follows that $pq\notin\Theta$ and $r\notin\Theta$. That is a contradiction in terms of Θ is a 1AIS$(\mathcal{P})$.

Condition II: $\ell=A$ and $\eta=w$

In view of this $\Theta(pqr)=(A,w)$ such that $\phi(pqr)=A$ and $\psi(pqr)=w$. Providing one of them $\Theta(pq)$ or $\Theta(r)$ is equal to (A,w). In all other respects, all of $\Theta(pq)$ and $\Theta(r)$ is equal to $(1,0)$. In that event, $pq,r\in\Theta$, though $pqr\notin\Theta$. This is a contradiction on Θ is a.

As a result, the demonstration has been brought to a conclusive end, encompassing all necessary steps and logical reasoning, thereby finalizing the verification or establishment of the stated proposition or argument.

Theorem 3: *Let* Υ *be a 1AIFPIS*$(\mathcal{P})$. *Then* $\sqrt{\Upsilon}$ *is a 1AIFIS*$(\mathcal{P})$.

Proof: Let Υ be a 1AIFPIS$(\mathcal{P})$. Assume that $\sqrt{\Upsilon}(pqr)\geq(\ell,\eta)$ for $p,q,r\in\mathcal{P}$ and $\ell,\eta\in[0,1]$. It follows that

$$\sqrt{\phi}(pqr)\geq\ell \text{ and } \sqrt{\psi}(pqr)\leq\eta.$$

It means that

$$\sqrt{\phi}(pqr)\geq\ell\Rightarrow \bigvee\{\phi(pqr)^k \mid k\in\mathbb{Z}\geq\ell\}$$
$$\Rightarrow \bigvee\{\phi(p^kq^kr^k) \mid k\in\mathbb{Z}\geq\ell\}$$

and

$$\sqrt{\psi}(pqr)\leq\eta\Rightarrow \bigwedge\{\psi(pqr)^k \mid k\in\mathbb{Z}\leq\eta\}$$
$$\Rightarrow \bigwedge\{\psi(p^kq^kr^k) \mid k\in\mathbb{Z}\leq\eta\}.$$

Presume that $r \notin \sqrt{\Upsilon}$ such that $r \notin \sqrt{\phi}$ and $r \notin \sqrt{\psi}$. Thus

$$\sqrt{\phi}(r^k) \not\geq \ell \text{ and } \sqrt{\psi}(r^k) \not\leq \eta$$

for $\ell, \eta \in [0,1]$ and $k \in \mathbb{Z}$.

In view of the fact that Υ is a 1AIFPIS$(\mathcal{P})$, we have $\phi((p^k q^k)^l) \geq \ell$ and $\psi((p^k q^k)^l) \leq \eta$ for some $l \in \mathbb{Z}$ and hence

$$\bigvee\{\phi(p^u q^u) \mid u \in \mathbb{Z} \geq \ell\}$$

$$\bigwedge\{\psi(p^u q^u) \mid u \in \mathbb{Z} \leq \eta\}.$$

It means that $pq \in \sqrt{\phi}$ and $pq \in \sqrt{\psi}$, so $pq \in \sqrt{\Upsilon}$. As a result, $\sqrt{\Upsilon}$ is a 1AIFIS$(\mathcal{P})$.

Now, we will show intersection of non-empty family of 1AIFIS$(\mathcal{P})$ is also 1AIFIS$(\mathcal{P})$.

Theorem 4: *Let Υ_i be a non-empty family of 1AIFIS$(\mathcal{P})$ for $i = 1,2,\cdots$. Then $\bigcap \Upsilon_i$ is also a 1AIFIS$(\mathcal{P})$ of $\mathcal{P}$.*

Proof: It is trivial that intersection of a non-empty family IFIS$(\mathcal{P})$ is also an IFIS$(\mathcal{P})$. Then for 1AIS$(\mathcal{P})$, let $\{\Upsilon_i : i \in J\}$ be a non-empty collection of 1AIFIS$(\mathcal{P})$ and $p, q, r \in \mathcal{P}$. Then, we get

$$\begin{aligned}
&(\bigcap_{i\in J} \Upsilon_i)(pqr) \geq (\ell, \eta) \\
\Rightarrow\ &(\bigcap_{i\in J} \phi_i)(pqr) \geq \ell \text{ and } (\bigcup_{i\in J} \psi_i)(pqr) \leq \eta \\
\Rightarrow\ &\bigwedge_{i\in J}\{\phi_i(pqr)\} \geq \ell \text{ and } \bigvee_{i\in J}\{\psi_i(pqr)\} \leq \eta \\
\Rightarrow\ &\bigwedge_{i\in J}\{\phi_i(pq)\} \geq \ell \text{ or } \bigwedge_{i\in J}\{\phi_i(r)\} \geq \ell \\
&\text{and} \\
&\bigvee_{i\in J}\{\psi_i(pq)\} \leq \eta \text{ or } \bigvee_{i\in J}\{\psi_i(r)\} \leq \eta \\
\Rightarrow\ &(\bigcap_{i\in J} \phi_i)(pq) \geq \ell \text{ or } (\bigcap_{i\in J} \phi_i)(r) \geq \ell \\
&\text{and} \\
&(\bigcup_{i\in J} \psi_i)(pq) \leq \eta \text{ or } (\bigcup_{i\in J} \psi_i)(r) \leq \eta \\
\Rightarrow\ &(\bigcap_{i\in J} \Upsilon_i)(pq) \geq (\ell, \eta) \text{ or } (\bigcap_{i\in J} \Upsilon_i)(r) \geq (\ell, \eta).
\end{aligned}$$

Thus, $\bigcap_{i\in J} \Upsilon_i$ is a 1AIFIS$(\mathcal{P})$.

Theorem 5: *Let* Υ_i *be a non-empty family of 1AIFPIS* $(\mathcal{P})$ *for* $i = 1, 2, \cdots$. *Then* $\bigcap \Upsilon_i$ *is also a 1AIFPIS* $(\mathcal{P})$.

Proof: Straightforward.

Theorem 6: *Let* $\hbar : \mathcal{P}_1 \to \mathcal{P}_2$ *be a homomorphism of semirings and* Υ *be a 1AIFIS* $(\mathcal{P}_2)$. *Then,* $\hbar^{-1}(\Upsilon)$ *is also a 1AIFIS* $(\mathcal{P}_1)$.

Proof: Let $\hbar : \mathcal{P}_1 \to \mathcal{P}_2$ be a homomorphism of semirings and Υ be a 1AIFIS $(\mathcal{P}_2)$. Now for $p, q, r \in \mathcal{P}_2$ and $\ell, \eta \in [0,1]$.

$$
\begin{aligned}
& \hbar^{-1}(\Upsilon)(pqr) \geq (\ell, \eta) \\
\Rightarrow \quad & \hbar^{-1}(\phi)(pqr) \geq \ell \text{ and } \hbar^{-1}(\psi)(pqr) \leq \eta \\
\Rightarrow \quad & \phi(\hbar(pqr)) \geq \ell \text{ and } \psi(\hbar(pqr)) \leq \eta \\
\Rightarrow \quad & \phi(\hbar(p)\hbar(q)\hbar(r)) \geq \ell \text{ and } \psi(\hbar(p)\hbar(q)\hbar(r)) \leq \eta \\
\Rightarrow \quad & \phi(\hbar(p)\hbar(q)) \geq \ell \text{ or } \phi(\hbar(r)) \geq \ell \\
& \text{and} \\
& \psi(\hbar(p)\hbar(q)) \leq \eta \text{ or } \psi(\hbar(r)) \leq \eta \\
\Rightarrow \quad & \hbar^{-1}(\phi)(pq) \geq \ell \text{ or } \hbar^{-1}(\phi)(r) \geq \ell \\
& \text{and} \\
& \hbar^{-1}(\psi)(pq) \leq \ell \text{ or } \hbar^{-1}(\psi)(r) \leq \ell \\
\Rightarrow \quad & \hbar^{-1}(\Upsilon)(pq) \geq (\ell, \eta) \text{ or } \hbar^{-1}(\Upsilon)(r) \geq (\ell, \eta).
\end{aligned}
$$

Hence, $\hbar^{-1}(\Upsilon)$ is also a 1AIFIS $(\mathcal{P}_1)$.

Theorem 7: *Let* $\hbar : \mathcal{P}_1 \to \mathcal{P}_2$ *be a homomorphism of semirings and* Θ *be a 1AIFIS* $(\mathcal{P}_1)$. *Then,* $\hbar(\Theta)$ *is also a 1AIFIS* $(\mathcal{P}_2)$.

Proof: Straightforward.

Theorem 8: *Let* Υ_1, Υ_2 *be two intuitionistic fuzzy subsets of* $\mathcal{P}$ *such that* Υ_1 *is a 1AIFIS* $(\mathcal{P})$ *and* $\Upsilon_1 \subseteq \Upsilon_2$. *If* $\Upsilon_2(pqr) \geq \Upsilon_1(pqr) \geq (\ell, \eta)$ *for* $p, q, r \in \mathcal{P}$ *and* $\ell, \eta \in [0,1]$, *then* Υ_2 *is also a 1AIFIS* $(\mathcal{P})$.

Proof: Assume that Υ_1, Υ_2 be two intuitionistic fuzzy subsets of $\mathcal{P}$ such that $\Upsilon_1 \subseteq \Upsilon_2$ and Υ_1 is a 1AIFIS $(\mathcal{P})$. Let $\Upsilon_2(pqr) \geq (\ell, \eta)$ for $p, q, r \in \mathcal{P}$ and $\ell, \eta \in [0,1]$. On the grounds that $\Upsilon_2(pqr) \geq \Upsilon_1(pqr) \geq (\ell, \eta)$ and Υ_1 is a 1AIFIS $(\mathcal{P})$, then we have

$$\Upsilon_1(pq) \geq (\ell, \eta) \text{ or } \Upsilon_1(r) \geq (\ell, \eta)$$

it follows that

$$\Upsilon_2(pq) \geq \Upsilon_1(pq) \geq (\ell, \eta) \text{ or } \Upsilon_2(r) \geq \Upsilon_1(r) \geq (\ell, \eta).$$

As a result, Υ_2 is also a 1AIFIS$(\mathcal{P})$

Now, we will give cartesian product of intuitionistic fuzzy subsets of semirings.

Definition 3: Let Υ and Θ be two intuitionistic fuzzy subsets of $\mathcal{P}$ and $\mathcal{Q}$, respectively. The cartesian product of Υ and Θ is given as the follows.

Let $\Upsilon =< \phi_1, \psi_1 >$ and $\Theta =< \phi_2, \psi_2 >$.

$$(\Upsilon \times \Theta)(p,q) = \Upsilon(p) \wedge \Theta(q) \text{ for all } (p,q) \in \mathcal{P} \times \mathcal{Q}, \textit{i.e}$$
$$(\Upsilon \times \Theta)(p,q) = (\phi_1(p) \wedge \phi_2(q), \psi_1(p) \vee \psi_2(q)).$$

Definition 4: *Let $\mathcal{P}$ a semiring and $\mathcal{I}$ be a non-empty subset of $\mathcal{P}$.*

The specific representation of the characteristic function of $\mathcal{I}$ within the framework of IFS is designated and symbolized as $\mathcal{X}_{\mathcal{I}} =< \phi_{\mathcal{X}_{\mathcal{I}}}, \psi_{\mathcal{X}_{\mathcal{I}}} >$, indicating the distinct way in which this function operates or describes the elements belonging to the given set I within the context of IFS and given as

$$\phi_{\mathcal{X}_{\mathcal{I}}} : \mathcal{P} \to [0,1]$$
$$p \to \phi_{\mathcal{X}_{\mathcal{I}}}(p) = \begin{cases} 1 \ ; & \text{if } p \in \mathcal{P} \\ 0 \ ; & \text{if } p \notin \mathcal{P} \end{cases}$$

and

$$\psi_{\mathcal{X}_{\mathcal{I}}} : \mathcal{P} \to [0,1]$$
$$p \to \psi_{\mathcal{X}_{\mathcal{I}}}(p) = \begin{cases} 0 \ ; & \text{if } p \in \mathcal{P} \\ 1 \ ; & \text{if } p \notin \mathcal{P}. \end{cases}$$

Theorem 9: *Let $\mathcal{P}$ and $\mathcal{Q}$ be two commutative semirings and $\mathcal{I}$ be a 1AIFIS $(\mathcal{P})$. Then $\mathcal{I} \times \mathcal{X}_{\mathcal{Q}}$ is a 1AIFIS$(\mathcal{P} \times \mathcal{Q})$, where $\mathcal{X}_{\mathcal{Q}}$ is the characteristic function of $\mathcal{Q}$.*

Proof: Admit that $\mathcal{P}$ and $\mathcal{Q}$ be two commutative semirings and $\mathcal{I}$, $\mathcal{X}_{\mathcal{Q}}$ be two 1AIFIS$(\mathcal{P}, \mathcal{Q})$, respectively.

Let $\mathcal{X}_{\mathcal{Q}} =< \phi_{\mathcal{X}_{\mathcal{Q}}}, \psi_{\mathcal{X}_{\mathcal{Q}}} >$.

$$(\mathcal{I}\times\mathcal{X}_{\mathcal{Q}})(p_1q_1r_1,p_2q_2r_2)\geq(\ell,\eta)$$
$$\Rightarrow(\phi\times\mathcal{X}_{\phi_{\mathcal{Q}}})(p_1q_1r_1,p_2q_2r_2)\geq\ell \text{ and } (\psi\times\mathcal{X}_{\psi_{\mathcal{Q}}})(p_1q_1r_1,p_2q_2r_2)\leq\eta$$
$$\Rightarrow\phi(p_1q_1r_1)\wedge\mathcal{X}_{\phi_{\mathcal{Q}}}(p_2q_2r_2)\geq\ell \text{ and } \psi(p_1q_1r_1)\vee\mathcal{X}_{\psi_{\mathcal{Q}}}(p_2q_2r_2)\leq\eta$$
$$\Rightarrow\phi(p_1q_1)\wedge\mathcal{X}_{\phi_{\mathcal{Q}}}(p_2q_2r_2)\geq\ell \text{ or } \phi(r_1)\wedge\mathcal{X}_{\phi_{\mathcal{Q}}}(p_2q_2r_2)\geq\ell$$
and
$$\psi(p_1q_1)\vee\mathcal{X}_{\psi_{\mathcal{Q}}}(p_2q_2r_2)\leq\eta \text{ or } \psi(r_1)\vee\mathcal{X}_{\psi_{\mathcal{Q}}}(p_2q_2r_2)\leq\eta$$

As $p_2,q_2,r_2\in\mathcal{Q}$, so $\mathcal{X}_{\phi_{\mathcal{Q}}}(p_2q_2r_2)=1$ and $\mathcal{X}_{\psi_{\mathcal{Q}}}(p_2q_2r_2)=0..$
We infer that

$$\phi(p_1q_1)\wedge\mathcal{X}_{\phi_{\mathcal{Q}}}(de)\geq\ell \text{ or } \phi(r_1)\wedge\mathcal{X}_{\phi_{\mathcal{Q}}}(de)\geq\ell$$
and
$$\psi(p_1q_1)\vee\mathcal{X}_{\psi_{\mathcal{Q}}}(de)\leq\eta \text{ or } \psi(r_1)\vee\mathcal{X}_{\psi_{\mathcal{Q}}}(de)\leq\eta$$

where $d,e\in\{p_2,q_2,r_2\}$. Therefore,

$$\mathcal{I}\times\mathcal{X}_{\mathcal{Q}}(p_1q_1,de)\geq(\ell,\eta) \text{ or } \mathcal{I}\times\mathcal{X}_{\mathcal{Q}}(r_1,de)\geq(\ell,\eta)$$

For this reason, $\mathcal{I}\times\mathcal{X}_{\mathcal{Q}}$ is a 1AIFIS$(\mathcal{P}\times\mathcal{Q})$.

Theorem 10: *Let* $\Upsilon=<\phi,\psi>$ *be a 1AIFPIS*$(\mathcal{P})$. *Then* $\sqrt{\Upsilon}$ *is an intuitionistic fuzzy prime ideal of* $\mathcal{P}$.

Proof: Let $\sqrt{\phi}(pq)\geq\ell$ and $\sqrt{\psi}(pq)\leq\eta$ for some $p,\ q\in\mathcal{P}$ and $\ell,\eta\in[0,1]$. We may assume that p and q are non-unit elements of $\mathcal{P}$. Let $n\in\mathbb{Z}^+$ and $n\geq2$ such that $\phi(pq)^n\geq\ell$ and $\psi(pq)^n\leq\eta$. Subsequently, it follows that $n=2m$ where $m\in\mathbb{Z}^+$ and $m\geq1$. This relationship is established based on the given conditions. As

$$\phi(pq)^n=\phi(p^nq^n)=\phi(p^mp^mq^n)\geq\ell$$
$$\psi(pq)^n=\psi(p^nq^n)=\psi(p^mp^mq^n)\leq\eta$$

and $\Upsilon=<\phi,\psi>$ is a 1AIFPIS$(\mathcal{P})$, we conclude that

$$\phi(p^mp^m)=\phi(p^n)\geq\ell \text{ or } \phi(q^n)\geq\ell$$
$$\psi(p^mp^m)=\psi(p^n)\leq\eta \text{ or } \psi(q^n)\leq\eta$$

Hence, $\sqrt{\phi}(p)\geq\ell$ or $\sqrt{\phi}(q)\geq\ell$ and $\sqrt{\psi}(p)\leq\eta$ or $\sqrt{\psi}(q)\leq\eta$. Thus $\sqrt{\Upsilon}$ is an intuitionistic fuzzy prime ideal of $\mathcal{P}$.

Theorem 11: *Let* $\Upsilon =< \phi, \psi >$ *be an intuitionistic fuzzy proper ideal of* $\mathcal{P}$. *If* Υ *is an intuitionistic fuzzy primary ideal of semiring* $\mathcal{P}$ *(IFPIS* $(\mathcal{P})$*), then* Υ *is a 1AIFPIS* $(\mathcal{P})$.

Proof: Let Υ be an intuitionistic fuzzy primary ideal of semiring and for all $p, q, r \in \mathcal{P}$ and $\ell, \eta \in [0,1]$. Admit that $\phi(pqr) \geq \ell$ and $\psi(pqr) \leq \eta$. Since $\Upsilon =< \phi, \psi >$ is an intuitionistic fuzzy primary ideal of semiring $\mathcal{P}$, then

$$\begin{aligned} \phi(pq) &\geq \ell \text{ or } \sqrt{\phi}(r) \geq \ell \\ \psi(pq) &\leq \eta \text{ or } \sqrt{\psi}(r) \leq \eta. \end{aligned}$$

It follows that $\Upsilon =< \phi, \psi >$ is a 1AIFPIS $(\mathcal{P})$.

Theorem 12: *Let* $\Upsilon =< \phi, \psi >$ *be an intuitionistic fuzzy proper ideal of* $\mathcal{P}$. *If* Υ *is a 1AIFPIS* $(\mathcal{P})$, *then is a 2-absorbing intuitionistic fuzzy primary ideal of semiring* $\mathcal{P}$ *(2AIFPIS* $(\mathcal{P})$*).*

Proof: Let $\mathcal{P}$ be a 1-absorbing primary ideal of semiring *R*. It follows that the characteristic function $\langle \chi_P, \chi_{P^c} \rangle$ of *P* is an intuitionistic fuzzy 1-absorbing primary ideal of R and admit that Υ is a 1AIFPIS $(\mathcal{P})$. Since every 1-absorbing primary ideal is a 2-absorbing primary ideal, then we get $\mathcal{P}$ is a 2-absorbing primary ideal. It means that $\langle \chi_P, \chi_{P^c} \rangle$ is an intuitionistic fuzzy 1-absorbing primary ideal of R and Υ is a 2AIFPIS $(\mathcal{P})$

The illustration provided demonstrates a concept where the inverse proposition of Theorem 12 does not necessarily hold or prove to be accurate, elucidating a specific case that contradicts the reverse implication stated within the theorem, thereby highlighting a situation where the theorem's opposite assertion does not hold valid in all instances.

Example 4: Let $\mathcal{P} = \mathbb{Z}$. Consider the ideal $\mathcal{I} = 2\mathbb{Z}$.

$$\Upsilon(p) = \begin{cases} (1,0), & p \in \mathcal{I} \\ (0,1), & p \notin \mathcal{I} \end{cases}$$

Then $\Upsilon =< \phi, \psi >$ is a 2AIFPIS $(\mathcal{P})$. As $\psi(2.1.1) \leq 0$, $\psi(1.1) = 1 \not\leq 0$ but $\sqrt{\psi}(2) = 0 \leq 0$. Thus $\Upsilon =< \phi, \psi >$ is a 1AIFPIS $(\mathcal{P})$.

Here is an instance that exemplifies a specific case of a 2AIFPIS $(\mathcal{P})$, which distinctly does not fall within the category of a 1AIFPIS $(\mathcal{P})$.

Example 5: Let $\mathcal{P} = \mathbb{Z}$. Consider the ideal $\mathcal{I} = 18\mathbb{Z}$.

$$\Upsilon(p) = \begin{cases} (1,0), & p \in \mathcal{I} \\ (0,1), & \text{otherwise} \end{cases}$$

Then $\Upsilon =< \phi, \psi >$ is a 2AIFPIS$(\mathcal{P})$. However $\Upsilon =< \phi, \psi >$ is a 1AIFPIS $(\mathcal{P})$. As

$$\psi(2.3.3) \leq 0, \psi(2.2) = 1 \not\leqslant 0 \text{ and } \sqrt{\psi}(3) = 1 \not\leqslant 0.$$

The schema provided can be viewed by referencing the algebraic structures derived within this work.

Remark 1: We can present the theorems provided as a schema:

$$\text{IFPIS}(\mathcal{P}) \Rightarrow \text{1AIFIS}(\mathcal{P}) \Rightarrow \text{1AIFPIS}(\mathcal{P}) \Rightarrow \text{2AIFPIS}(\mathcal{P})$$

4. Conclusion

This study defines and examines 1AIFIS$(\mathcal{P})$ from a theoretical viewpoint. In this article, we present innovative structures that were created using research on the various features of the cartesian product of IFIS$(\mathcal{P})$. The research then concentrated on the characteristics of 1AIFIS $(\mathcal{P})$ in the cartesian product. Finally, these ideals were studied under a semiring homomorphism. This research could be expanded upon by looking at various algebraic structures and conducting additional research on their properties. We address the following research questions on the basis of our findings:

1) The idea of bipolar-valued 1AIFIS$(\mathcal{P})$ will be explored.
2) The notion of hesitant 1AIFIS$(\mathcal{P})$ will be introduced.
3) The concept of rough 1AIFIS$(\mathcal{P})$ will be studied.
4) The theory of neutrosophic vague 1AIFIS$(\mathcal{P})$ will be clarified.
5) The thought of complex 1AIFIS$(\mathcal{P})$ will be defined.

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