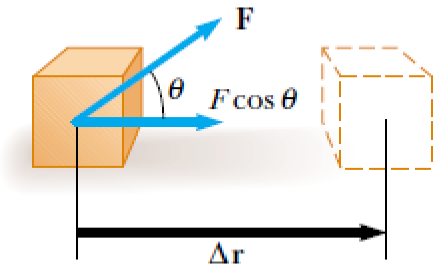
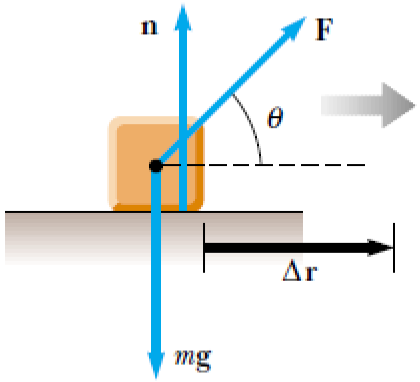
Work and Kinetic Energy



Work done by force **F**,

where $\Delta \mathbf{r}$ is the displacement vector

$$W = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos\theta$$



When an object is displaced on a frictionless, horizontal surface, the normal force **n** and the gravitational force *m***g** do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

That is, if θ=90°, then *W*=0 because cos 90°=0

SI unit of work :

1 Newton. meter (N· m)= 1 joule (J)

Work is an energy transfer.

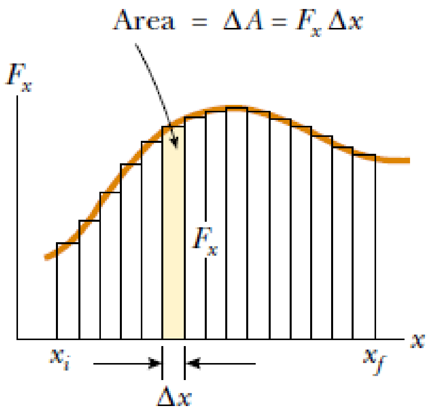
Work Done by a Varying Force

$$W = F\,\Delta r \cos\theta$$

Assume that the object is moving in x-direction,

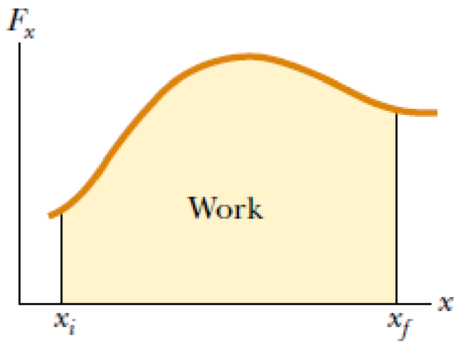
Then; $W = F_x \Delta x$

if the force is variable;



$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x\, dx$$

$$W = \int_{x_i}^{x_f} F_x\, dx$$

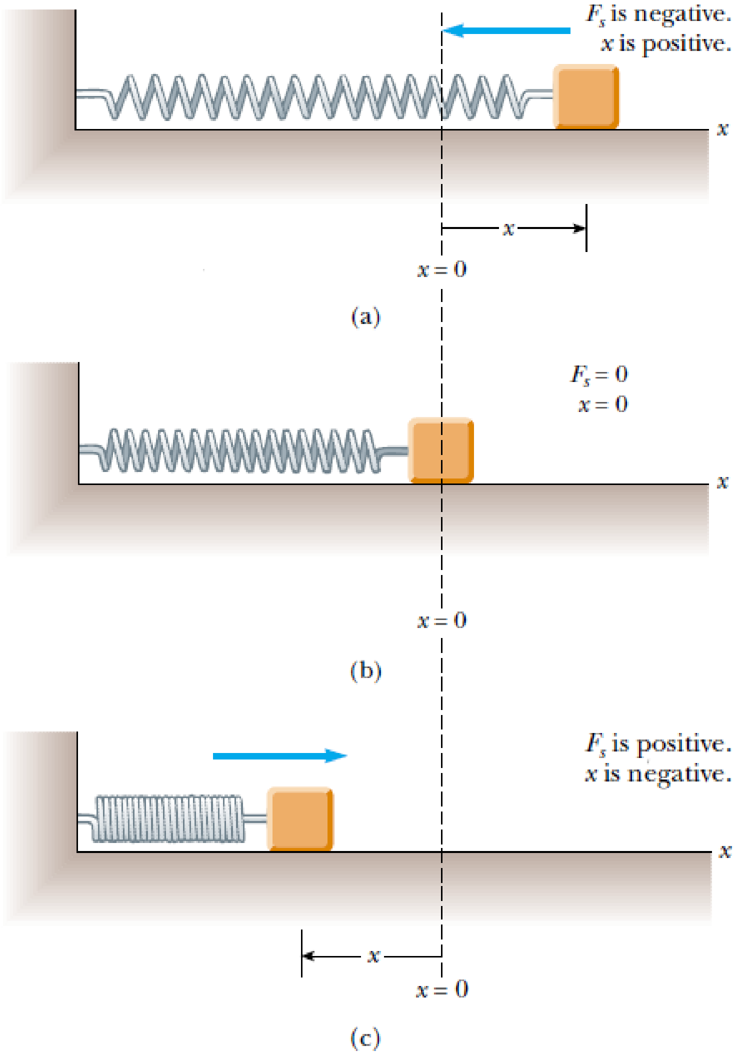


If more than one force acts on a system

the work done by the net force is the total work, or *net work,* as the particle moves from x_i to x_f

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

Work Done by a Spring

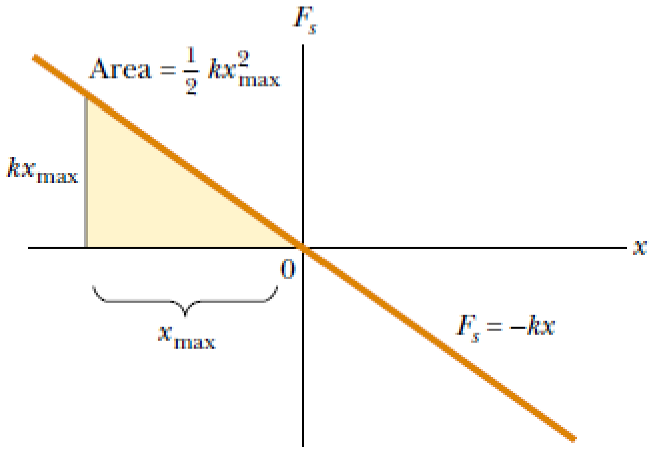


$F_s = - kx$

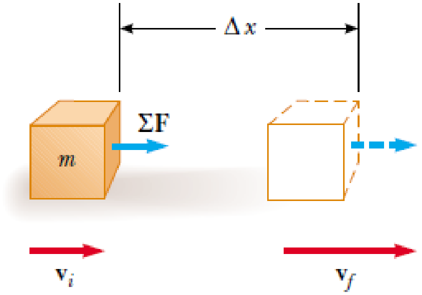
x is the position of the block relative to its equilibrium (x = 0) position and k is a positive constant called the *force constant* or the *spring constant* of the spring.

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{max}}^{0} (-kx)\, dx = \tfrac{1}{2} k x_{max}^2$$

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = \tfrac{1}{2} k x_i^2 - \tfrac{1}{2} k x_f^2$$



Kinetic Energy and the Work–Kinetic Energy Theorem



$$\sum W = \int_{x_i}^{x_f} \sum F dx$$

$$\sum W = \int_{x_i}^{x_f} ma dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{dx}\frac{dx}{dt} dx = \int_{v_i}^{v_f} mv\, dv$$

$$\sum W = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2}$$

$$K \equiv \tfrac{1}{2}mv^2$$ Kinetic Energy

$$\sum W = K_f - K_i = \Delta K$$ work–kinetic energy theorem

the work done by the net force equals the change in kinetic energy of the system.

$F_s = -kx$

x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the *force constant* or the *spring constant* of the spring.

Situations Involving Kinetic Friction

Force of friction is always in opposite direction to motion, then the work done by force of friction is;

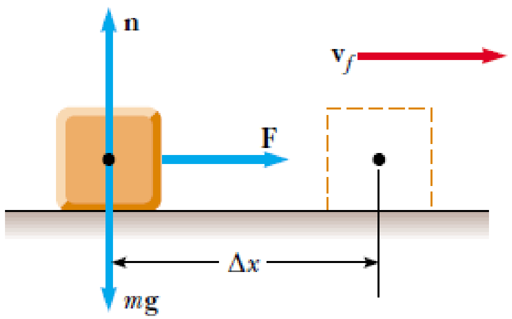
$$-f_k \Delta x = \Delta K$$

The general case involving kinetic friction

$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$K_f = K_i - f_k d + \sum W_{\text{other forces}}$$

Ex/ **a)** A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.



$$W = F\Delta x = (12\,\text{N})(3.0\,\text{m}) = 36\,\text{J}$$

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36\,\text{J})}{6.0\,\text{kg}}} = 3.5\,\text{m/s}$$

b) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$K_f = K_i - f_k d + \sum W_{\text{other forces}}$$

$$n = mg.$$

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

$$\Delta K_{\text{friction}} = -f_k d = -(8.82\,\text{N})(3.0\,\text{m}) = -26.5\,\text{J}$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - f_k d + \sum W_{\text{other forces}}$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}\left(-f_k d + \sum W_{\text{other forces}}\right)}$$

$$= \sqrt{0 + \frac{2}{6.0\,\text{kg}}(-26.5\,\text{J} + 36\,\text{J})}$$

$$= 1.8\,\text{m/s}$$

Power

average power

$$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

instantaneous power

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$dW = \mathbf{F} \cdot d\mathbf{r}.$$

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is ;

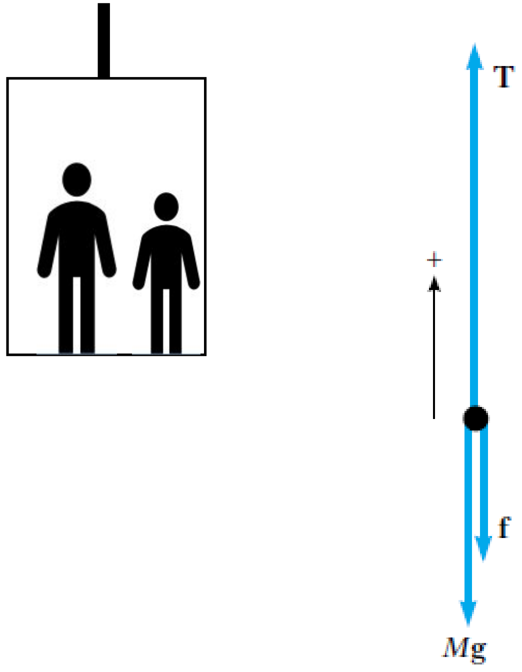
$$\mathcal{P} = \frac{dE}{dt}$$

$$1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ \mathrm{kg \cdot m^2/s^3}$$

Ex/ An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4000 N retards its motion upward, as shown in Figure.

a) What power delivered by the motor is required to lift the elevator car at a constant speed of 3.00 m/s?

b) What power must the motor deliver at the instant the speed of the elevator is *v* if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s2?



a) $a = 0.$ $\Sigma F_y = 0.$

$$\sum F_y = T - f - Mg = 0$$

M=1600+200=1800 kg

$$T = f + Mg$$
$$= 4.00 \times 10^3\ \text{N} + (1.80 \times 10^3\ \text{kg})(9.80\ \text{m/s}^2)$$
$$= 2.16 \times 10^4\ \text{N}$$

$$\mathcal{P} = \mathbf{T} \cdot \mathbf{v} = Tv$$

$$= (2.16 \times 10^4\ \text{N})(3.00\ \text{m/s}) = 6.48 \times 10^4\ \text{W}$$

b) $\sum F_y = T - f - Mg = Ma$

$$T = M(a + g) + f$$
$$= (1.80 \times 10^3\ \text{kg})(1.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2) + 4.00 \times 10^3\ \text{N}$$
$$= 2.34 \times 10^4\ \text{N}$$

$$\mathcal{P} = Tv = (2.34 \times 10^4\ \text{N})v$$