CHAPTER 2

GRAVITY SETTLERS PERFORMANCE MODELS

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1. INTRODUCTION

In the control of primary particulates, the gravity settlers, or the settling chambers, are the most primitive control devices. A gravity settler is an air pollution control device through which a flue gas containing high concentrations of particulates passes slowly. The slow motion of the flue gas within the device allows time for the particles to settle down to the bottom of the device under the action of gravity, by which the particles are separated from the gas stream and treated gas stream leaves the device.

Long used by industry for removing solid or liquid particles from gaseous streams, settling chambers have the advantages of simple construction, low initial cost, low maintenance cost, and low pressure drop. Besides, it is very simple to collect and dispose dust from the bottom. It is one of the first-developed devices used to control particulate emissions.

A settling chamber is simply an expansion chamber along the path of the duct used to transport the gaseous stream from one point to another. The characteristic of the settling chambers is low horizontal gas velocity, which allows particles of aerodynamic diameter around 50 μ to settle out. For particles smaller than this the collection efficiency of a settling chamber is significantly reduced although satisfactorily high collection efficiencies are observed for larger particles depending on the density of the particles. Current legislations, however, require cleaner air. These legislations forces the industrial facilities to obey stricter emission standards concerning particulate emissions, which, in time, relegated the settling chambers to use as pre-treatment devices prior to high-performance control devices. In contrast, the settling chambers still keep their gravity for use in some industrial processes with flue gases with very high concentrations of particulates such as some smelters and metallurgical processes.

The settling chambers still keep their position as an individual chapter in most air pollution control courses because of their simple mathematical analysis. They also form a basis for mathematical modeling of some other control devices. Thus, this chapter is reserved for the settling chambers and mathematical modeling of particulate collection performance in settling chambers.

2. TYPES AND COMPONENTS

Fundamentally, there are two types of settling chambers: the simple expansion chamber and the multiple-tray settling chamber. Two distinct designs of former one are shown in Fig. 1 and Fig. 2 shows a multi-tray settling chamber. This course note focuses on the design and performance modeling of the first type of settling chambers and the term settling chamber will be used along the text for both types.

A settling chamber consists of an inlet duct, an outlet duct, the body, and collection hoppers placed at appropriate positions on the bottom. The inlet duct must be designed so that a uniform distribution of inlet flow is accomplished. The uniform distribution of inlet stream in settling chambers is accomplished by the use of guide vanes and, most frequently, perforated plates. Perforated plates cause a very small pressure drop at the inlet to provide uniform distribution.

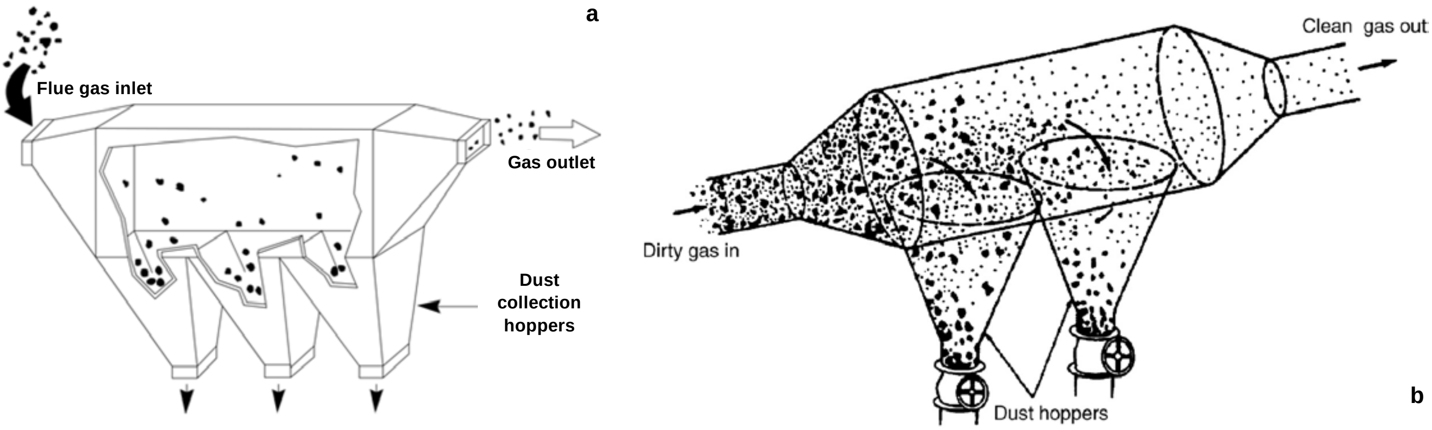

Figure 1. Two settling chamber designs with (a) rectangular cross-section[1] and (b) circular cross-section[2]

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[1] APTI 413, 1999. Student Manual for control of particulate matter emissions, 5th ed., available at USEPA web site.

[2] USEPA, 1971. Control techniques for gases and particulates.

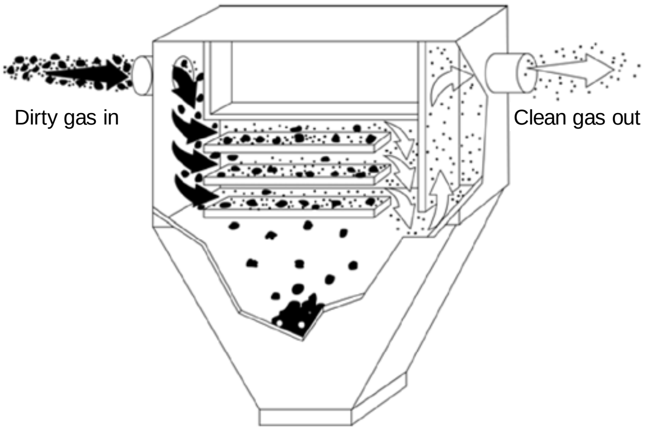

Figure 2. A multi-tray settling chamber[1]

The chamber body is the part of the chamber in which the actual collection of particles takes place. The body of the settling chamber may be rectangular or circular. For purposes of easy construction, settling chambers with rectangular cross-section are usually preferred. The body must be large enough in cross-section to slow down the horizontal gas velocity. For the purpose of allowing enough time for the particles to settle out and preventing re-entrainment of particles into the gaseous stream, the horizontal gas velocity must be as low as 1–3 $m.s^{-1}$, preferably less than 0.3 $m.s^{-1}$. However, increasing the width and the height of the chamber to increase the cross-sectional area has some disadvantages including, most importantly, the difficulty of distributing the inlet stream uniformly across the cross-sectional area.

Since the main mechanism for particulate collection in settling chambers is the gravity, which is a weak driving force for air pollution control devices, the collection efficiency per unit length of settling chambers is usually very low. Therefore, if necessity presents itself as one being forced to design a settling chamber as a pre-treatment unit, the length of the chamber body must be long enough to provide the desired collection efficiency.

The settling chamber needs an exit duct through which the clean gas flows out the chamber. The exit duct must be designed to minimize the construction costs and the pressure drop at the point of contraction.

Finally, there must be a cleaning mechanism for the chamber itself. As the chamber operates, the collected particles accumulate on the floor and must be removed manually at certain periods, or properly placed collection hoppers must be included in the design. These hoppers should be placed next to each other, and they must have inclined walls to allow the collected particles drift over the walls downward. Each hopper must be equipped with bags to collect the dust, and the bags must be replaced by new ones at certain periods.

3. PERFORMANCE MODELS

Before going into details of performance modeling in settling chambers, it is appropriate to define several modeling concepts.

The most important parameters affecting particulate collection efficiency in air pollution control devices are residence time (t_R) and the collection time (t_C). The residence time is defined as the time for a particle to travel between the inlet and the outlet of the settling chamber. Theoretically speaking, it is the average time that the flue gas remains within the control device, and is calculated as the ratio of the volume of the chamber to the actual flowrate of gaseous stream:

$$t_R = \frac{V}{Q} \qquad 1$$

Another concept in particulate collection is the collection time (t_C), which is defined as the time needed for a particle to be collected under the action of dominant driving force. These two main parameters determine whether a particle is collected in the device or not. Theoretically, for a particle to be collected the residence time must be equal to or greater than the collection time:

$$t_R \geq t_C \qquad 2$$

In air pollution control applications, it is generally assumed that a particle is collected if it hits the collection wall of the device, the floor for the settling chamber case, and it is not re-entrained into the gaseous stream after collection. In real case, however, this is not true. A particle might re-entrain after the collection depending on the flow characteristics and the properties of the particle. This situation brings two other parameters in settling chamber designs: the throughput velocity (v_g) and the pickup velocity (v_P).

The throughput velocity (v_g) is that velocity at which the gas moves through the chamber, which is also called horizontal gas velocity, while the velocity of the flue gas within the chamber at which the settled particles become re-entrained is called the pickup velocity (v_P). In order to avoid re-entrainment of collected dust, the throughput velocity must not exceed the pickup velocity. Pickup velocities for several materials are given in Table 1.

Table 1. Pickup velocities of various materials[1]

Material	Density ($kg.m^{-3}$)	Median size (μ)	Pickup velocity ($m.s^{-1}$)
Aluminum chips	2720	335	4.3
Asbestos	2200	261	5.2
Nonferrous foundry	3020	117	5.7
Lead oxide	8260	15	7.6
Limestone	2780	71	6.4
Starch	1270	64	1.8
Steel shot	6850	96	4.6
Wood chips	1180	1370	4.0
Sawdust	-	1400	6.8

If no data concerning the pickup velocity for the material is available, the best strategy is to assume the pickup velocity to be 3 $m.s^{-1}$. In this case, the throughput velocity (horizontal gas velocity) must be less than 3 $m.s^{-1}$.

3.1. Settling of particles

One should comprehend the characteristics of settling process in a settling chamber and understand the effects of driving parameters before trying to derive performance models.

In settling chamber, the particles enter through the chamber inlet and they move horizontally with the gas flow towards the outlet of the chamber. Along with this drifting effect of throughput velocity, particles also tend to move downward under the action of gravity. Therefore, a particle's motion in a settling chamber must be shown by a vector that inclines from the horizontal. The particle moves toward the chamber floor as it moves toward the outlet duct along with the gaseous stream. If the particle hits the floor until it reaches the outlet, then the particle is, theoretically, said to be collected. Otherwise, the particle leaves the chamber uncollected.

Consider two identical particles entering a settling chamber at distinct heights of h_1 and h_2, respectively, with h_2 being greater than h_1 (Fig. 3). These particles will be drifted by the gaseous flow due to which their horizontal velocity will be equal to the horizontal gas velocity. Simultaneously, these two particles will move downward to the chamber floor under the effect of gravity at a velocity of equal to their terminal velocity estimated by Stoke's law. According to h_1 and h_2 arrangement given in Fig. 3, the first particle is going to hit the chamber floor and it is going to be collected. However, the second particle will not be collected since its trajectory does show that it will not hit the floor.

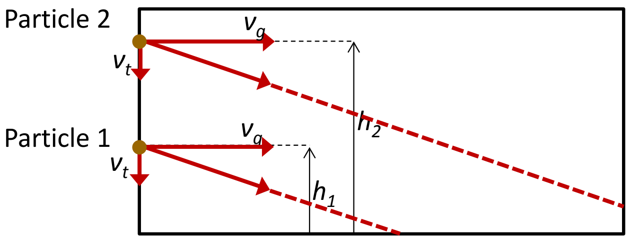

Figure 3. Two identical particles' behaviors in a settling chamber

Since the particles in Fig. 3 are identical, their terminal settling velocities are equal, and their trajectories are the same. The only difference between these two particles is the height of entrance. Thus, one can conclude that the height at which the particle enters the settling chamber determines whether the particle is collected or not. Similarly, the height (H) of the settling chamber affects its particulate collection performance negatively. The larger the chamber height is, the lower the collection efficiency is.

Now consider a particle entering the settling chamber. Fig. 4 shows the effect of horizontal gas velocity on particle's motion within the settling chamber, with blue velocity vectors being greater than the red ones in magnitude. Fig. 4 suggests that the particle may not settle down if the horizontal gas velocity is increased too much. Thus, one can conclude that the collection efficiency in a settling chamber is inversely proportional to horizontal gas velocity ($v_T = v_g$). Increased gas velocities could dramatically decrease the chamber's collection efficiency.

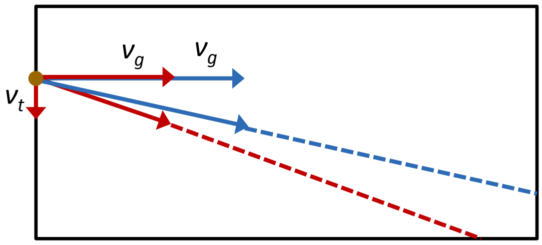

Figure 4. Effect of throughput velocity on particulate collection

Finally, consider two particles of the same material with various aerodynamic diameters entering a settling chamber at the same heights from the floor (Fig. 5). Since their diameters are different, their terminal settling velocities are also different. Of these particles, the larger one's settling velocity is greater. Thus, it follows the blue line within the chamber and s collected. The smaller particle follows the red path. Thus, it is not collected. Considering this fact, one can conclude that the collection efficiency is directly proportional to particle's settling velocity (v_t), that's, the square of particle size.

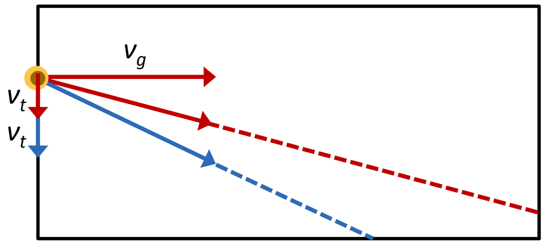

Figure 5. Effect of particle's diameter on particulate collection

The length of the settling chamber (*L*) is also effective on particulate collection performance. Consider that the chambers in Fig. 3 through Fig 5 are longer. For this case, the uncollected particles would hit the chamber floor, too, and they would be collected. Thus, the collection efficiency is also directly proportional to length of the chamber.

All of these considerations can be put together to obtain a relationship for the collection efficiency in a settling chamber as follows:

$$\eta = k\frac{L v_t}{H v_g} \qquad 3$$

Here, *L* is the length of the chamber, v_t is the terminal settling velocity of particle, *H* is the height of the chamber, v_g is the throughput velocity, and *k* is a constant relating the theoretical case with the actual case.

3.2. Plug flow model

To calculate the behavior of a settling chamber, engineers generally rely on one of two models. Either they assume that the fluid going through is totally unmixed (block flow or plug flow model) or they assume total mixing in the entire cross-section perpendicular to the flow (backmixed or mixed model). Each of these sets of assumptions leads to simple calculations. The observed behavior of the nature most often falls between these two simple cases, so that with these two models limits can be set on what nature probably does. Both models are widely used in air pollution control device calculations. Derivation of both models is shown in following paragraphs.

For a settling chamber of height *H*, width *W*, and length *L*, operated at a flowrate of *Q*, several simplifying assumptions are made and listed below concerning plug flow characteristics.

- Horizontal gas velocity, throughput velocity, is constant everywhere within the chamber and is equal to v_g
- The particles' are moving with the gas flow at horizontal velocity of equal to the gas velocity.
- Vertical components of the particles' velocities are equal to their terminal settling velocities due to gravity, v_t.
- If a particle settles to the floor, it stays there and is not re-entrained.
- No interaction between the particles takes place.
- The concentration of particles is distributed uniformly across the cross-section perpendicular to the flow.
- No mixing occurs in both horizontal and vertical directions.

With these assumptions, the horizontal gas velocity can be calculated using the continuity equation as follows:

$$v_g = \frac{Q}{WH} \qquad 4$$

Consider a particle that enters the chamber at some distance above the floor. The time required for the gas parcel that the particle enters with to leave the chamber is then

$$t = \frac{L}{v_g} \qquad 5$$

The distance that the particle settles during this period of time is calculated as follows:

$$h_t = v_t t = v_t \frac{L}{v_g} \qquad 6$$

If this distance is equal to or greater than the height at which the particle enters the chamber, the particle is said to be collected. If not, the particle leaves the chamber uncollected. Thus, it is possible to define a height h_t, the particles below which are collected, and the particles above which are not (Fig. 6). Since the concentration of the particles is distributed uniformly across the cross-section, the collection efficiency can be calculated as the ratio of this height h_t to the height *H* of the chamber:

$$\eta_{PF} = \frac{h_t}{H} \qquad 7.a$$

where η_{PF} is the collection efficiency of the chamber predicted by the plug flow assumptions. Substituting Eqn. 6 into Eqn 7.a, one can obtain the plug flow model as follows:

$$\eta_{PF} = \frac{Lv_t}{Hv_g} \qquad 7.b$$

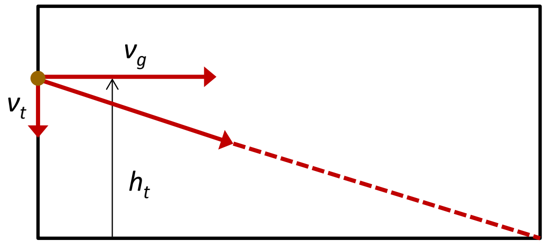

Figure 6. The height limit for particles to settle

Eqn. 7.b is known as the plug flow model for air pollution control devices. Comparing this equation with Eqn. 3 proves the arguments discussed in previous section. Here, *k* in Eqn. 3 becomes equal to unity.

The plug flow model suggests that the collection efficiency of a settling chamber is directly proportional to the product of its length and the particle's terminal settling velocity. In contrast, there is an inverse relationship between the collection efficiency and the chamber height as well as the throughput velocity.

Eqn. 7.b can be further modified by introducing Stoke's law for terminal settling velocity of the particle and Eqn. 4 for the horizontal gas velocity into the equation as follows:

$$\eta_{PF} = \frac{LWg\rho_p d_p^2}{18\mu Q} \qquad 7.c$$

where ρ_p is the particle's density, *g* is the gravitational acceleration, μ is the dynamic viscosity of the gas, and d_p is the particle's aerodynamic diameter.

Eqn. 7.c suggests that the particulate collection efficiency of a settling chamber is directly proportional to the square of particle's aerodynamic diameter. Keeping all other variables constant, Fig. 7 shows the change of collection efficiency with respect to particle size.

It is obvious from Fig. 7 that the particulate collection efficiency estimated by the plug flow model exceeds 100% after a certain particle size depending on the properties of gaseous stream and the particle. In any case, the plug flow model overestimates the efficiency especially for larger particles. Thus, the plug flow model is said to be producing unrealistic estimates of particulate collection efficiency in a settling chamber, and a more realistic approach is needed.

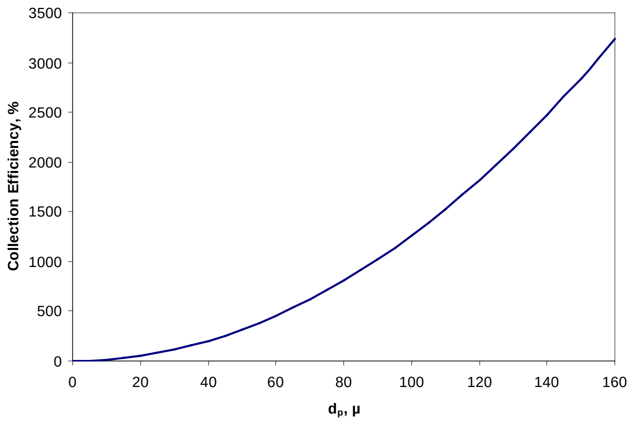

Figure 7. Change of particulate collection efficiency estimated by plug flow model of a settling chamber with respect to particle size

Example 1. *Plug flow model*

A 14-m-long settling chamber with a height of 2.5 m operates at a horizontal gas velocity of 1 m.s^{-1}. The density of limestone particles that are to be removed in the settling chamber is 2780 kg.m^{-3}. Assuming plug flow characteristics and a flue gas dynamic viscosity of 1.8x10-5 kg.m^{-1}.s^{-1}, calculate the collection efficiency for particles of 50 *μ*.

Solution.

The collection efficiency of the settling chamber for a particle size of 50-*μ* can be after calculating the terminal settling velocity, V_t, of the particle:

$$V_t = \frac{9.81\frac{m}{s^2}*2000\frac{kg}{m^3}*\left(50*10^{-6}\,m\right)^2}{18*\left(1.8*10^{-5}\frac{kg}{m.s}\right)} \approx 0.151\frac{m}{s}$$

The collection efficiency for a particle size of 50-*μ* is then

$$\eta = \frac{14m*0.151\frac{m}{s}}{2.5m*1\frac{m}{s}} \approx 0.846 = 84.6\%$$

This means that 84.6% of the particles that enter the settling chamber will be removed from gas stream and the rest 15.4% will leave the chamber without being collected. For instance, if the concentration of 50-*μ* particles in the flue gas is 3000 mg.m^{-3}, then the concentration at the exit of the chamber will be 3000 * 0.154 = 462 mg.m^{-3}, and the rest 3000 - 462 = 2538 mg.m^{-3}.

3.3. Mixed model

The plug flow model is shown to produce unrealistic estimates for collection efficiency in settling chambers. Thus, necessity arises to derive a model that simulates the behavior of settling chambers better. That's where the mixed model comes in. The mixed model assumes, in contrast to the plug flow model, that the cross-section of the chamber perpendicular to the gaseous flow is totally mixed with no mixing in the horizontal direction. Keeping all other assumptions of plug flow model valid for the mixed model, too, the system must be investigated at incremental levels of the chamber length.

Consider an incremental length *dx* of the chamber within which total mixing takes place, and the particle concentration is uniformly distributed in this incremental space (Fig. 8). Since the plug flow characteristics still prevail in the horizontal direction, the time required for the gas parcel to travel through this incremental length of the chamber is written based on Eqn. 5 as follows:

$$dt = \frac{dx}{v_g} \qquad 8$$

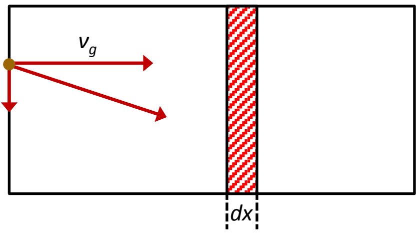

Figure 8. Incremental mixing in settling chambers

Similar to the plug flow model, a certain height h_t above the chamber floor can be defined, the particles below which settles out and the particles above which do not. This height can be calculated similarly as follows:

$$h_t = v_t dt = \frac{v_t}{v_g} dx \qquad 9$$

The collection efficiency within this incremental length is then

$$\eta_t = \frac{h_t}{H} = \frac{v_t}{Hv_g} dx \qquad 10$$

and the change of concentration within the incremental length is proportional to the concentration since total mixing occurs in the incremental space:

$$dc = -c\eta_t \qquad 11.a$$

A minus sign is placed in the equation since the concentration decreases as the particles settle out. Eqn. 10 and Eqn. 11.a can be combined and rearranged to obtain

$$\frac{dc}{c} = -\frac{v_t}{Hv_g} dx \qquad 11.b$$

defined by two boundary conditions at the inlet and outlet of the chamber. The concentration of particles at the inlet (*x=0*) is C_0 while the concentration at the outlet (*x=L*) is C_e. With these boundary conditions, Eqn. 11.b can be integrated to obtain Eqn. 12 as follows:

$$\int_{C=C_0}^{C=C_e} \frac{dc}{c} = -\int_{x=0}^{x=L} \frac{v_t}{Hv_g} dx \qquad 11.c$$

$$C_e = C_0 \exp\left[-\frac{Lv_t}{Hv_g}\right] \qquad 12$$

Eqn. 12 can then be used to derive the mixed model for particulate collection efficiency in settling chambers as follows:

$$\eta_M = 1 - \exp\left[-\frac{Lv_t}{Hv_g}\right] \qquad 13$$

One should note the similarity of mixed model equation with the plug flow model equation (Eqn. 7.b). The mixed model equation can be expressed in terms of plug flow model as follows:

$$\eta_M = 1 - \exp[-\eta_{PF}] \qquad 14$$

The mixed model estimated values for particulate collection efficiency for various particle sizes are plotted and shown in Fig. 9. It is obvious that the model predicts increasing collection efficiencies with increasing particle size. However, all the predicted values are lower than the plug flow model estimates. Besides, the predicted values increase asymptotically to the 100% line and never exceed it. Thus, one can conclude that mixed model is a better predictor of collection performance in settling chambers.

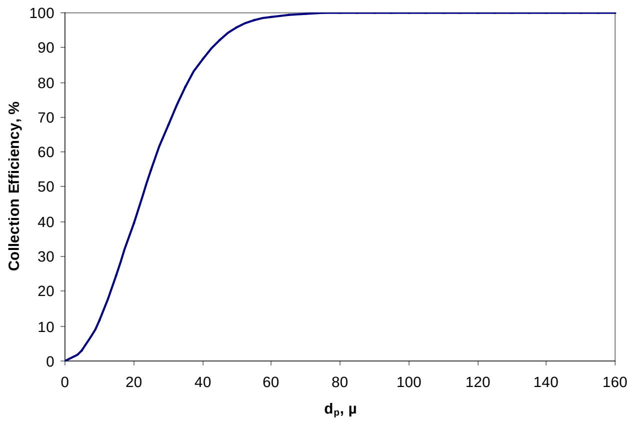

Figure 9. Change of particulate collection efficiency estimated by mixed model of a settling chamber with respect to particle size

Example 2. *Mixed model*

Repeat the previous example. However, assume mixed flow characteristics this time. Compare the change of collection performances estimated assuming plug flow and mixed models.

Solution.

The terminal settling velocity for the 50-μ particles was calculated as 0.151 m.s^{-1} in the previous example. The collection efficiency by the mixed model is then

$$\eta_M = 1 - \exp\left[-\frac{14m * 0.151\frac{m}{s}}{2.5m * 1\frac{m}{s}}\right] \approx 0.572 = 57.2\%$$

For the 50-μ particles, the collection efficiency estimated by the mixed model is somewhat lower than that calculated by the plug flow model. The main difference between the values estimated by two models lies in the assumptions made in their derivations. For the plug flow model, the assumption was that no vertical mixing takes place within the settling chamber, that's, a particle's vertical component of displacement is only to the gravitational effects. In reality, however, vertical mixing occurs within the collection device and particles in the gas stream are forced to move upward and downward due to the mixing effect, which reduces the collection efficiency.

4. EXAMPLES FOR PROBLEM SECTION

Following examples are reserved for the problem section PS3 after the lecture. All of these questions are going to be solved in class, and their solutions are not given here. Please take notes during the problem section.

Example 3. *Plug flow model*

A 14-m-long settling chamber with a height of 2.5 m operates at a horizontal gas velocity of 1 m.s^{-1}. The density of particles that are to be removed in the settling chamber is 2000 kg.m^{-3}. Assuming plug flow characteristics and a flue gas dynamic viscosity of 1.8x10^{-5} kg.m^{-1}.s^{-1}, show the change of collection efficiency with the change in particle diameters from 10-μ to 100-μ with an interval of 10-μ.

Example 4. *Mixed model*

Repeat previous question assuming mixed flow characteristics and compare the collection efficiencies estimated by both models.

Example 5. *Length of a settling chamber*

A flue gas with a particulate matter concentration of 6500 mg.m^{-3} needs to be treated before emission to the atmosphere to meet the emission standard of 100 mg.m^{-3}. The facility managers consider building a settling chamber to treat the flue gas. The area restrictions within the facility limit the height of the settling chamber to 2 m. Assuming an average particle size of 35 microns with a density of 1800 kg.m^{-3}, estimate the required length of the settling chamber to meet the emission standard. Assume a dynamic viscosity of 1.8x10^{-5} kg.m^{-1}.s^{-1} for the flue gas.

Example 6. *Width of settling chamber*

A flue gas with a particulate matter concentration of 3200 mg.m^{-3} is passed through a settling chamber to collect valuable dust with 50-μ aerodynamic diameter and 2000 kg.m^{-3} particle density in order to meet the emission standard of 100 mg.m^{-3}. The length of the chamber is 23 m, the height is 1 m, and the width is 2.5 m. The flue gas flowrate is 9000 m^3.h^{-1}.

In the region, however, more strict emission standards are required and the authorities sets a new emission standard for particulate matter emission as 50 mg.m^{-3}. Unfortunately, the land area of the facility is limited and the option of increasing the length of the chamber is not viable. As a result, the facility managers decide to modify the chamber by changing its width. Calculate the new width of the chamber required to meet the new emission standard. Assume a flue gas dynamic viscosity of 1.8×10^{-5} kg.m^{-1}.s^{-1}.

Example 7. *Overall collection efficiency*

A flue gas contains 6400 mg.m^{-3} of particulate matter that obeys log-normal distribution with a mass mean diameter of 65 μ and a standard deviation of 0.8. The particle density is 2000 kg.m^{-3} and the flue gas has a dynamic viscosity of 1.8×10^{-5} kg.m^{-1}.s^{-1}. The flue gas with a flowrate of 25200 m^{3}.h^{-1} enters a settling chamber with dimensions 2 x 4 x 30 m (HxWxL). Calculate the particulate concentration and size distribution at the exit of the chamber. Divide the whole particle size range into 10 sub-ranges.

Example 8. *Maldistribution*

Consider two identical settling chambers are operating parallel at a total flowrate of 50400 m^{3}.h^{-1}. If the flow of flue gas is distributed evenly between the chambers, the overall particulate collection efficiency of the system is 85.4%. Calculate the overall particulate collection efficiency if the flue gas is unintentionally distributed unevenly such that one chamber gets 75% of the gas flow and the other receives the rest.

5. CHAPTER PROBLEMS

5.1. Identify the primary force responsible for the particle collection in settling chambers.

- **a.** electrostatic
- **b.** impaction
- **c.** centrifugal
- **d.** gravity
- **e.** Brownian diffusion

5.2. Settling chambers are normally effective for removing particles in which of the following particle size ranges

- **a.** less than 10 microns
- **b.** between 10 and 50 microns
- **c.** greater than 70 microns
- **d.** between 30 and 40 microns
- **e.** submicron particles only

5.3. Increasing the gas volumetric feed rate to an existing settling chamber would be expected to result in

- **a.** a decrease in collection efficiency
- **b.** an increase in collection efficiency
- **c.** no change in collection efficiency
- **d.** impossible to say
- **e.** an increase in collection efficiency followed by a sharp decrease

5.4. The particles in a flue gas obey log-normal distribution with a mass mean diameter of 50 µ and a standard deviation of 0.95. The particle concentration in the flue gas is 7500 mg.m^{-3} and the particle density is 2000 kg.m^{-3}. These particles will be removed from the gas stream via a settling chamber prior to emission to meet the emission standard. The dimensioning of the settling chamber is 1.5x4x25 m (HxWxL) and the flowrate is 350 m^{3}.min^{-1}. Calculate the particle collection efficiency of the chamber as well as particle concentration and size distribution at the exit of the chamber. Assume that the flue gas has a dynamic viscosity of 1.8x10^{-5} kg.m^{-1}.s^{-1}.

5.5. A settling chamber with dimensions 2x4x35 m (HxWxL) operates at a particle collection efficiency of η. Calculate the efficiency if the height of the chamber is reduced to 1.5 m. Comment on your results.

5.6. Two identical settling chambers are operating with an overall particle collection efficiency of 75% at a total flue gas flowrate of Q, the flow being distributed evenly between the two. What would be the collection efficiency if the flow unintentionally distributed unevenly such that one of the units receives 65% of the total flow.

5.7. Design a settling chamber to collect the starch particles from the dryer flue gas in a starch manufacturing plant. The flue gas contains 5700 mg.m^{-3} of starch particles and the emission standard is 100 mg.m^{-3}. Make necessary assumptions.