YILDIZ TEKNİK ÜNİVERSİTESİ

ELEKTRİK MÜHENDİSLİĞİ BÖLÜMÜ

DEVRE TEORİSİ

Ders Notu

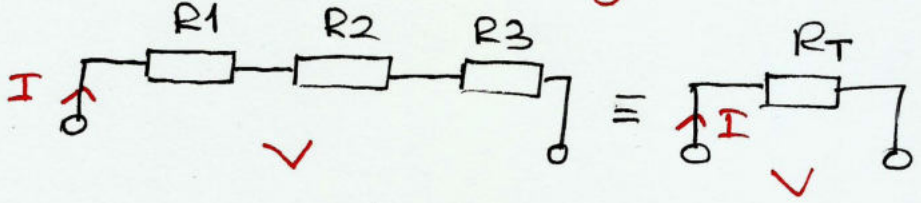
- **R,L,C ELEMANLARININ SERİ VE PARALEL BAĞLANMASI**
- **YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ**

Doç. Dr. Recep YUMURTACI

R, L, C Elemanlarının Seri ve Paralel Bağlanması

Direnç (R)

* Dirençlerin Seri Bağlanması



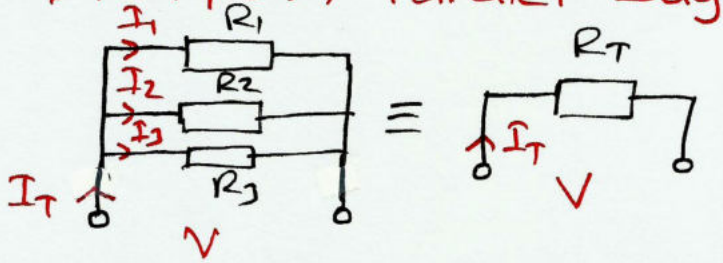
$$V = V_1 + V_2 + V_3$$

$$R_T \cdot I = (R_1 + R_2 + R_3)\, I \Rightarrow \boxed{R_T = R_1 + R_2 + R_3}$$

"Özel Durum:

n adet R direnci seri bağlı ise $\boxed{R_T = n \cdot R}$ olur.

* Dirençlerin Paralel Bağlanması

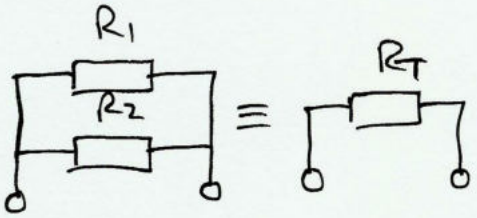


$$I_T = I_1 + I_2 + I_3$$

$$\frac{1}{R_T} \cdot V = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \cdot V \Rightarrow \boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

"Özel Durum:

1) n adet R direnci paralel bağlı ise $\boxed{R_T = \frac{R}{n}}$ olur.

2) 

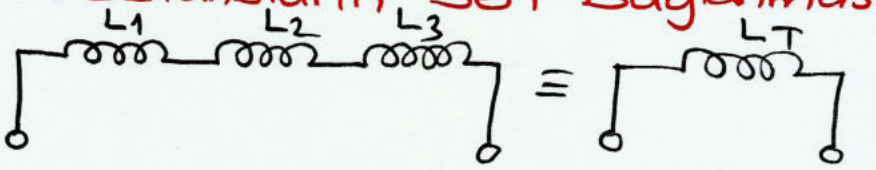
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_1 + R_2}{R_1 \cdot R_2}$$

$(R_2)\ (R_1)$

$$\frac{1}{R_T} = \frac{R_1 + R_2}{R_1 \cdot R_2} \Rightarrow \boxed{R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

Endüktans (L)

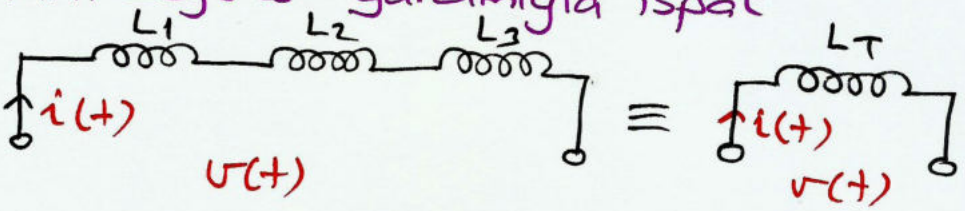
* Endüktansların Seri Bağlanması



$$L_T = L_1 + L_2 + L_3$$

İspatı:

a) Ani değerler yardımıyla ispat



$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

$$L_T \cdot \frac{di(t)}{dt} = (L_1 + L_2 + L_3) \cdot \frac{di(t)}{dt} \Rightarrow L_T = L_1 + L_2 + L_3$$

b) Endüktif reaktanslar yardımıyla ispat

$$X_{L_T} = X_{L_1} + X_{L_2} + X_{L_3}$$

$$\omega \cdot L_T = \omega \cdot (L_1 + L_2 + L_3) \Rightarrow L_T = L_1 + L_2 + L_3$$

Özel durum:

n adet L endüktansı seri bağlı ise $L_T = n \cdot L$ olur.

Örnekler:

1) 10mH 30mH 70mH ≡ 110mH

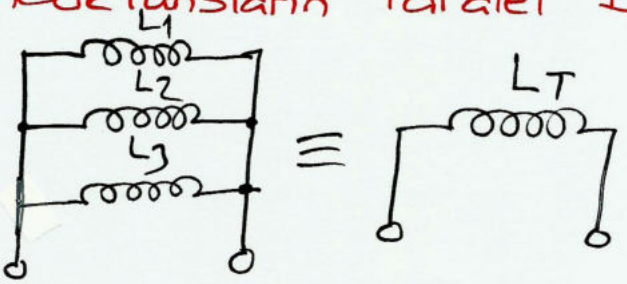
$L_T = L_1 + L_2 + L_3$

$L_T = 10 + 30 + 70 = 110mH$

2) 3mH 3mH ... 3mH (20 adet)

$L_T = n \cdot L = 20 \cdot 3 = 60mH$

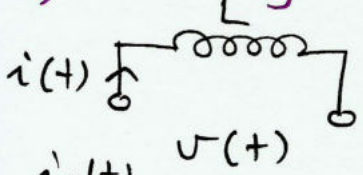
* Endüktansların Paralel Bağlanması



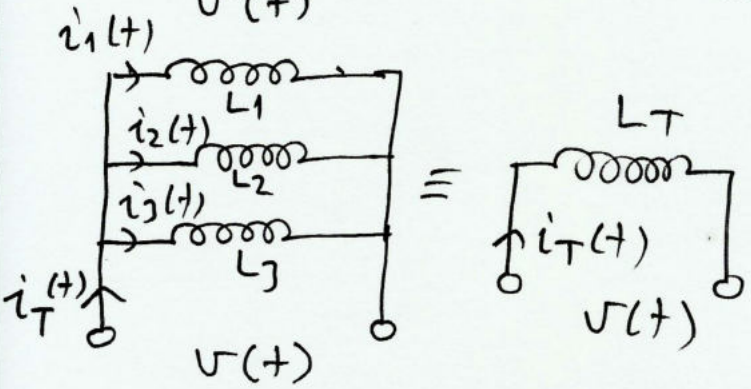
$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

ispat:

a) Anı değerler yardımıyla ispat



$$v(t) = L \cdot \frac{di(t)}{dt} \Rightarrow i(t) = \frac{1}{L}\int v(t)\,dt$$



$$i_T(t) = i_1(t) + i_2(t) + i_3(t)$$

$$\frac{1}{L_T}\int v(t)\,dt = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\cdot \int v(t)\,dt$$

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

b) Endüktif reaktanslar yardımıyla ispat

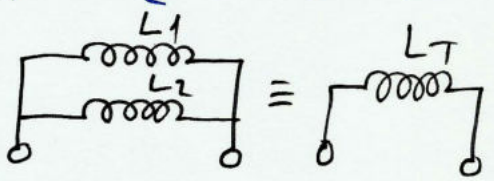
$$\frac{1}{X_{L_T}} = \frac{1}{X_{L_1}} + \frac{1}{X_{L_2}} + \frac{1}{X_{L_3}} \Rightarrow \frac{1}{\omega}\cdot\frac{1}{L_T} = \frac{1}{\omega}\cdot\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)$$

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

"Özel Durum:

* n adet L endüktansı paralel bağlı ise $\boxed{L_T = \frac{L}{n}}$ olur.

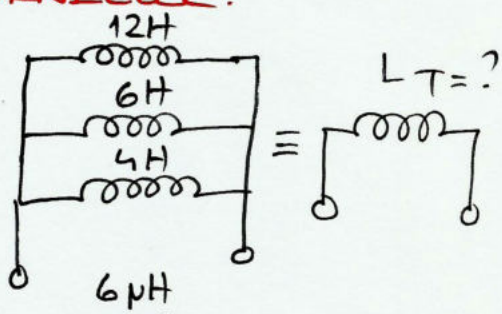
* L_1 ve L_2 endüktansları paralel bağlı ise



$$\boxed{L_T = \frac{L_1 \cdot L_2}{L_1 + L_2}}$$

"ÖRNEKLER:

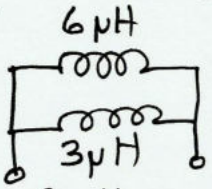
1)



$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$
$$(1)\ (2)\ (3)$$

$$\frac{1}{L_T} = \frac{6}{12} = \frac{1}{2} \Rightarrow \boxed{L_T = 2H}$$

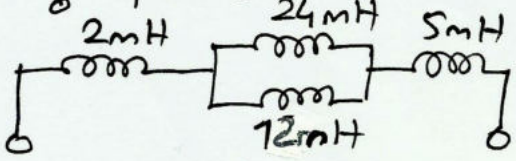
2)



$\Rightarrow L_T = ?$

$$L_T = \frac{L_1 \cdot L_2}{L_1 + L_2} = \frac{6 \cdot 3}{6+3} = \frac{18}{9} = 2\mu H$$

3)

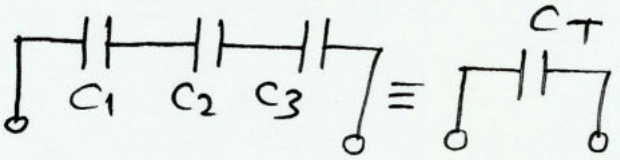


$$L_T = 2 + \left(\frac{24 \cdot 12}{24+12}\right) + 5 = (2 + 8 + 5)\,mH$$

$$L_T = 15\,mH$$

Kondansatör (C)

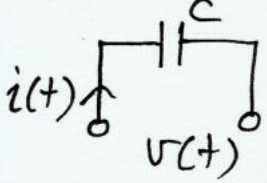
* Kondansatörlerin Seri Bağlanması



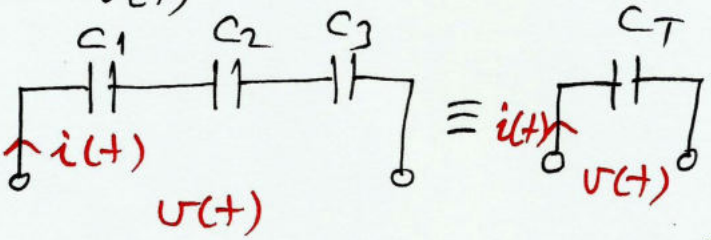
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

ispat:

a) Ani değerler yardımıyla ispat:



$$i(t) = C \cdot \frac{dv(t)}{dt} \Rightarrow v(t) = \frac{1}{C}\int i(t)\,dt$$



$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

$$\frac{1}{C_T}\cdot\int i(t)\,dt = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\cdot\int i(t)\,dt$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

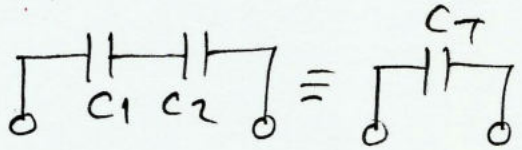
b) Kapasitif reaktanslar yardımıyla ispat:

$$\frac{1}{X_{C_T}} = \frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} \Rightarrow \frac{1}{\omega}\cdot\frac{1}{C_T} = \frac{1}{\omega}\cdot\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Özel Durum:

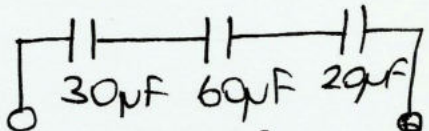
* n adet C kondansatörü seri bağlı ise $C_T = \frac{C}{n}$ olur.
* C_1 ve C_2 kondansatörleri seri bağlı ise



$$C_T = \frac{C_1 \cdot C_2}{C_1 + C_2}$$ olur.

Örnekler:

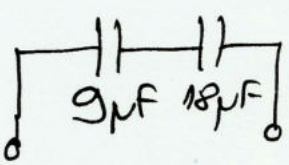
1)



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \underset{(2)}{\frac{1}{30}} + \underset{(1)}{\frac{1}{60}} + \underset{(3)}{\frac{1}{20}}$$

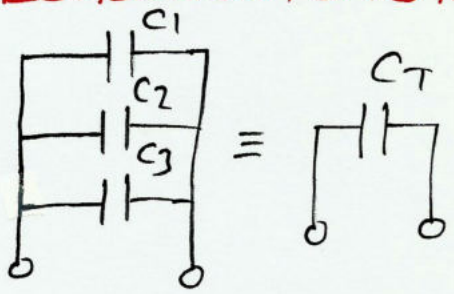
$$\frac{1}{C_T} = \frac{6}{60} = \frac{1}{10}\,\mu f \Rightarrow C_T = 10\,\mu f$$

2)



$$C_T = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{9 \cdot 18}{9 + 18} = \frac{\overset{1}{\cancel{9}} \cdot 18}{\underset{3}{\cancel{27}}} = 6\,\mu f$$

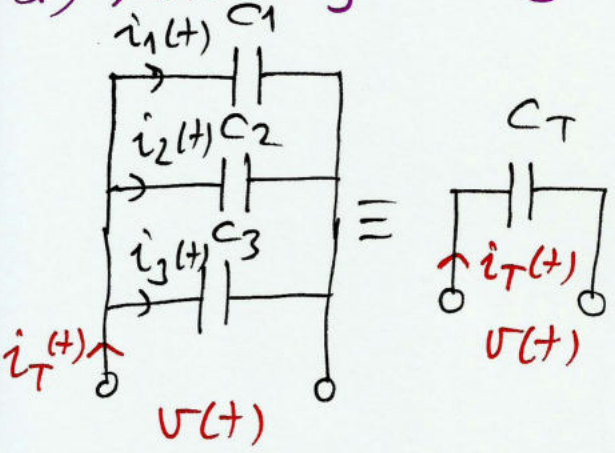
* Kondansatörlerin Paralel Bağlanması



$$C_T = C_1 + C_2 + C_3$$

ispat:

a) Ani değerler yardımıyla ispat



$$i_T(t) = i_1(t) + i_2(t) + i_3(t)$$

$$C_T \cdot \frac{dv(t)}{dt} = (C_1 + C_2 + C_3) \cdot \frac{dv(t)}{dt}$$

$$C_T = C_1 + C_2 + C_3$$

b) Kapasitif reaktanslar yardımıyla ispat

$$\frac{1}{X_{C_T}} = \frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} \Rightarrow \frac{1}{\frac{1}{\omega C_T}} = \frac{1}{\frac{1}{\omega C_1}} + \frac{1}{\frac{1}{\omega C_2}} + \frac{1}{\frac{1}{\omega C_3}}$$

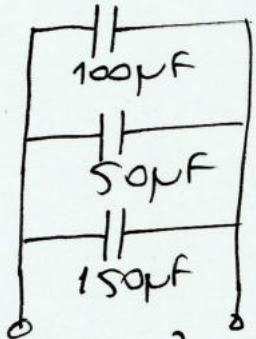
$$\omega C_T = \omega (C_1 + C_2 + C_3) \Rightarrow C_T = C_1 + C_2 + C_3$$

Özel durum:

n adet C kondansatörü paralel bağlı ise $C_T = n \cdot C$ olur.

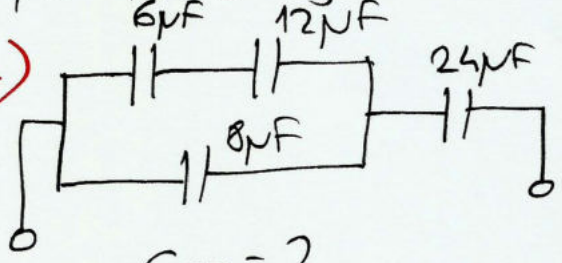
Örnekler:

1)



$$C_T = C_1 + C_2 + C_3 = 100 + 50 + 150 = 300\mu F$$

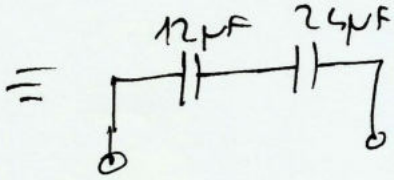
2)



$$C_{es1} = \frac{6 \cdot 12}{6+12} = 4\mu F$$

$$C_{es2} = C_{es1} + 8 = 4 + 8 = 12\mu F$$

$C_T = ?$



$$C_T = \frac{12 \cdot 24}{12+24} = \frac{\cancel{12} \cdot 24}{\cancel{36}_3}$$

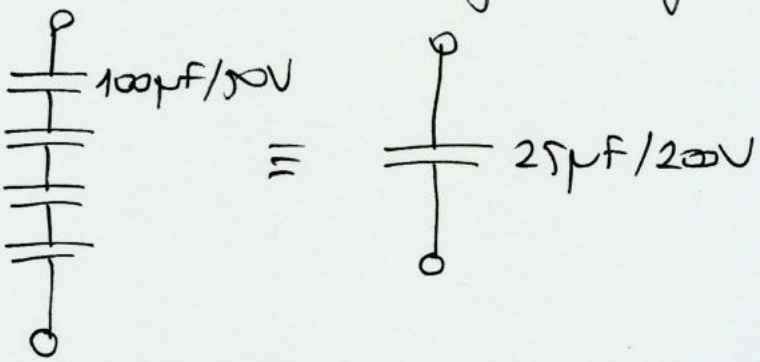
$$C_T = 8\mu F$$

Örnek:

Malzeme depomuzda çok sayıda 100μF 50V'luk kondansatörler bulunmaktadır. 150μF 200V'luk kondansatör elde etmek için nasıl bir bağlantı önerirsiniz?

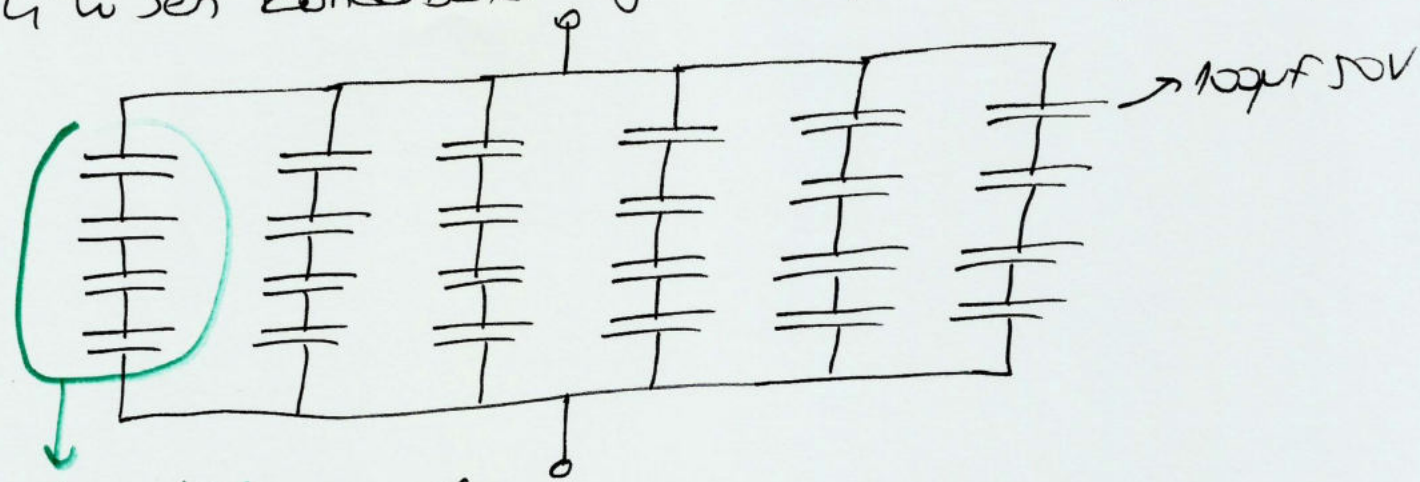
Çözüm:

200V'a dayanabilecek yani çalışma gerilimi 200V olan kondansatör elde etmek için 200/50 = 4 adet 100μF 50V'luk kondansatörü seri bağlamalıyız.

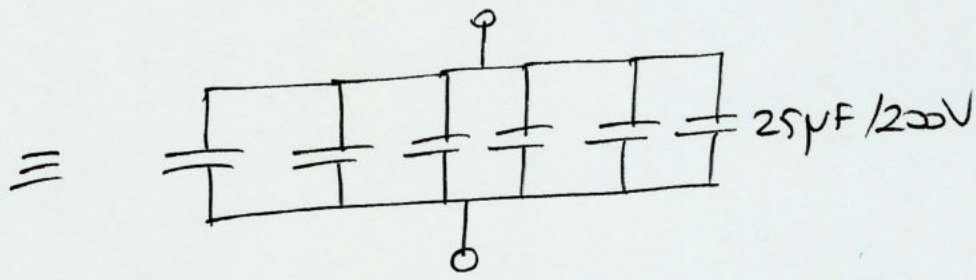


4 adet 100μF kondansatör seri bağlanınca eşdeğer kapasite $C_{eş} = \frac{C}{4} = \frac{100}{4} = 25\mu F$ olur.

İstenen eşdeğer kapasite 150μF olduğu için $\frac{150}{25} = 6$ adet 4'lü seri kondansatör grubunu paralel bağlamalıyız.



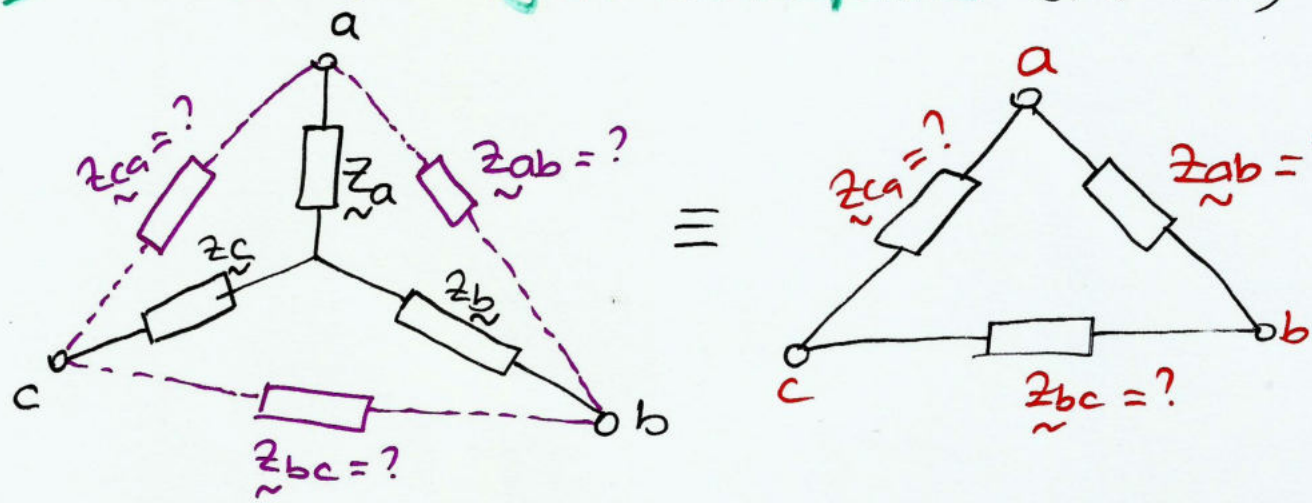
$C_{eş} = \frac{100}{4} = 25\mu F$ (200V)



$C_{eş} = 6.25 = 150\mu F / 200V$

YILDIZ ⇄ ÜÇGEN DÖNÜŞÜMLERİ

a) Yıldız → Üçgen Dönüşümü (λ → Δ)



* Burada yapılan işlem; λ bağlı $\underset{\sim}{Z}_a$, $\underset{\sim}{Z}_b$ ve $\underset{\sim}{Z}_c$ empedanslarının a, b, c düğümlerinden "aynı akımları" ve "aynı güçleri" çekecek Δ bağlı eşdeğerlerinin hesaplanmasıdır. Bu işleme λ → Δ dönüşümü denir.

$$\underset{\sim}{Z}_{ab} = \frac{\underset{\sim}{Z}_a \cdot \underset{\sim}{Z}_b + \underset{\sim}{Z}_b \cdot \underset{\sim}{Z}_c + \underset{\sim}{Z}_c \cdot \underset{\sim}{Z}_a}{\underset{\sim}{Z}_c}$$

$$\underset{\sim}{Z}_{bc} = \frac{\underset{\sim}{Z}_a \cdot \underset{\sim}{Z}_b + \underset{\sim}{Z}_b \cdot \underset{\sim}{Z}_c + \underset{\sim}{Z}_c \cdot \underset{\sim}{Z}_a}{\underset{\sim}{Z}_a}$$

$$\underset{\sim}{Z}_{ca} = \frac{\underset{\sim}{Z}_a \cdot \underset{\sim}{Z}_b + \underset{\sim}{Z}_b \cdot \underset{\sim}{Z}_c + \underset{\sim}{Z}_c \cdot \underset{\sim}{Z}_a}{\underset{\sim}{Z}_b}$$

Özel Durum:

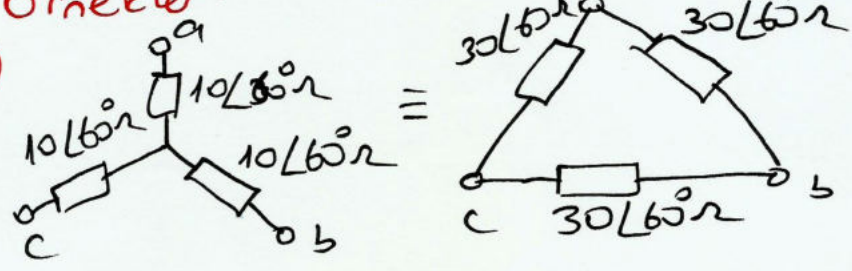
Üç fazlı sistemde λ bağlı empedanslar "dengeli" yani birbirine eşit ise

$\underset{\sim}{Z}_a = \underset{\sim}{Z}_b = \underset{\sim}{Z}_c = \underset{\sim}{Z}_\lambda$ ise $\underset{\sim}{Z}_{ab} = \underset{\sim}{Z}_{bc} = \underset{\sim}{Z}_{ca} = \underset{\sim}{Z}_\Delta = \frac{3 \cdot \underset{\sim}{Z}_\lambda \cdot \underset{\sim}{Z}_\lambda}{\underset{\sim}{Z}_\lambda}$

$$\underset{\sim}{Z}_\Delta = 3 \cdot \underset{\sim}{Z}_\lambda$$ olur.

Örnekler

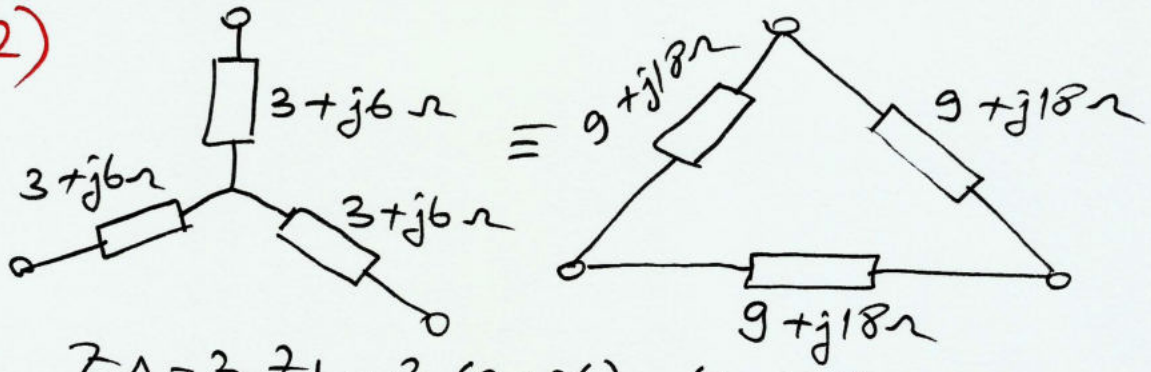
1)



$\underset{\sim}{Z}_\Delta = 3\underset{\sim}{Z}_\lambda = 3 \cdot 10\angle 60^\circ\Omega$

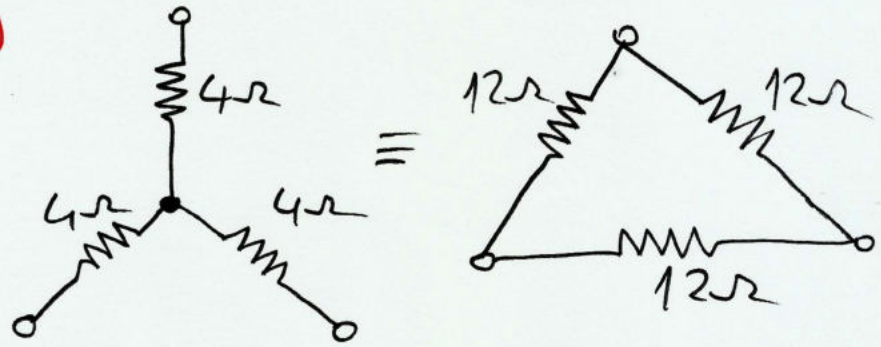
$$\underset{\sim}{Z}_\Delta = 30\angle 60^\circ\Omega$$

2)



$\underset{\sim}{Z}_\Delta = 3 . \underset{\sim}{Z}_\curlyvee = 3 . (3+j6) = (9+j18)\,\Omega$

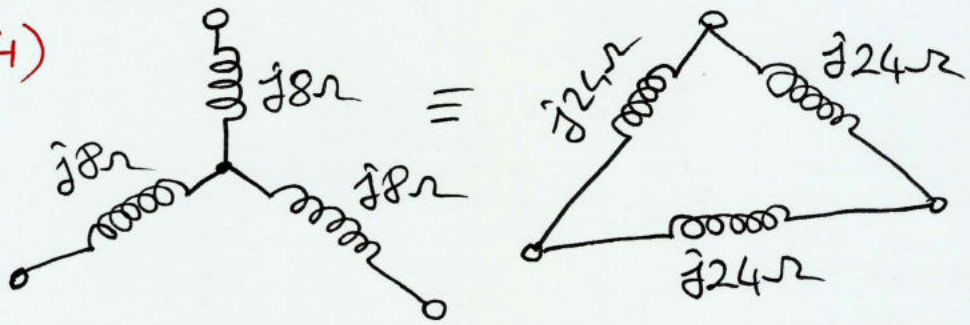
3)



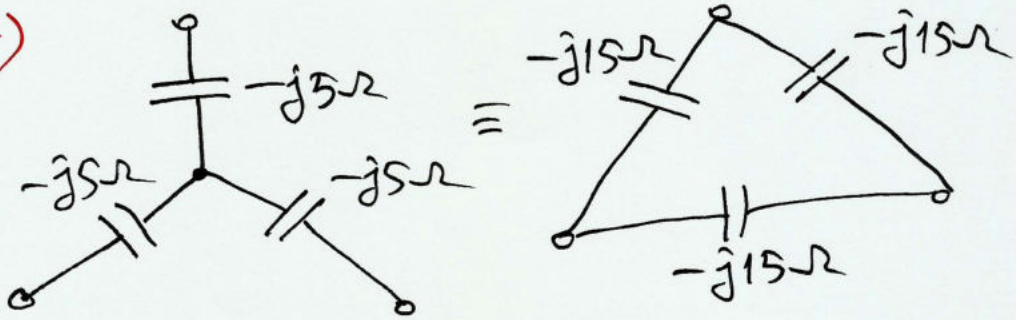
$R_\Delta = 3 R_\curlyvee = 3 . 4 = 12\,\Omega$

$\underset{\sim}{Z}_\Delta = 3 . \underset{\sim}{Z}_\curlyvee = 3 . j8 = j24\,\Omega$

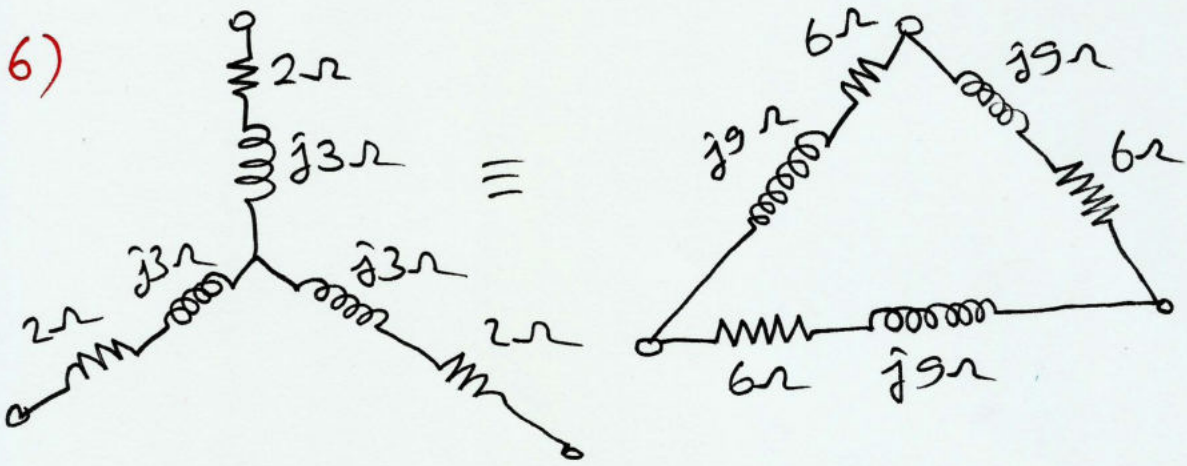
4)



5)

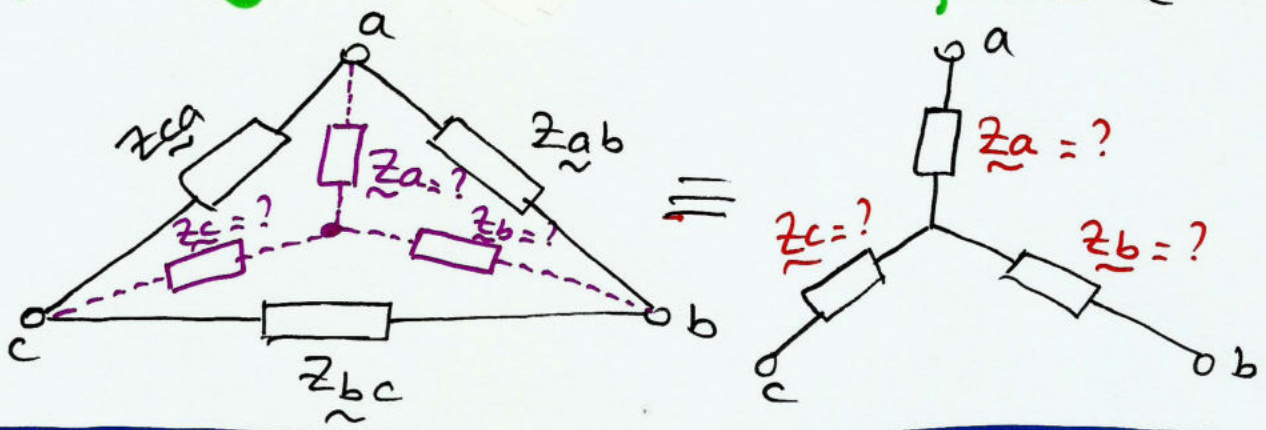


6)



$\underset{\sim}{Z}_\Delta = 3 . \underset{\sim}{Z}_\curlyvee = 3 (2+j3) = (6+j9)\,\Omega$

b) Üçgen → Yıldız Dönüşümü (Δ → Y)



$$Z_a = \frac{Z_{ab} \cdot Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_b = \frac{Z_{ab} \cdot Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_c = \frac{Z_{ca} \cdot Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

Özel Durum :

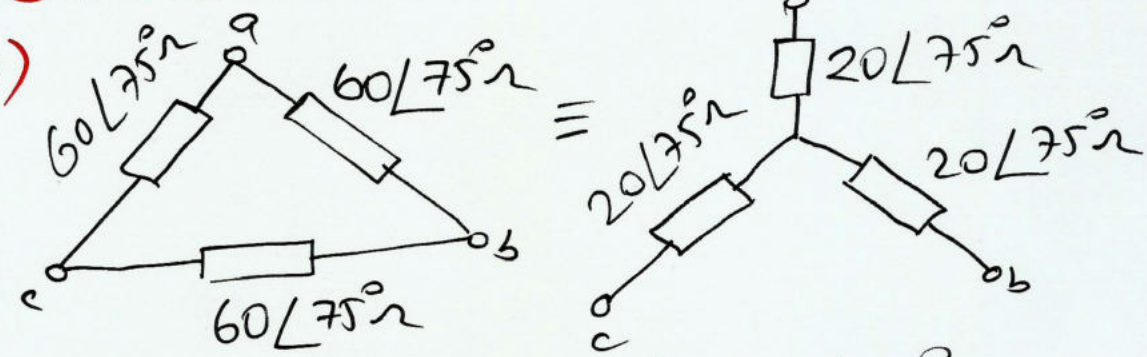
Δ bağlı 3 fazlı empedanslar dengeli ise (eşit ise)

$Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta$ olsun.

$$Z_Y = Z_a = Z_b = Z_c = \frac{Z_\Delta \cdot Z_\Delta}{3 \cdot Z_\Delta} \Rightarrow \boxed{Z_Y = \frac{Z_\Delta}{3}}$$ olur.

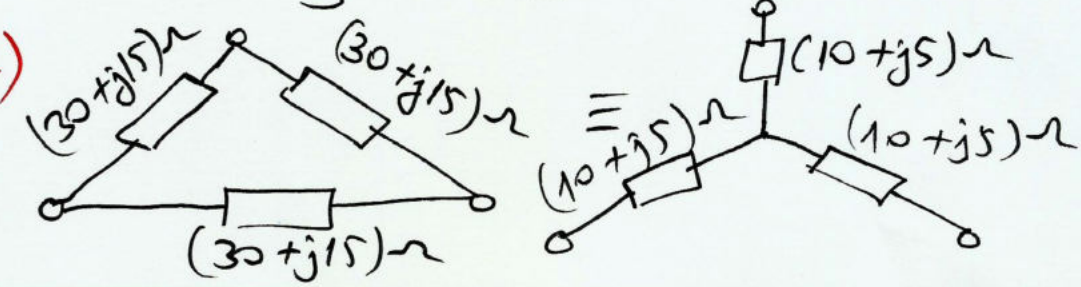
Örnekler:

1)



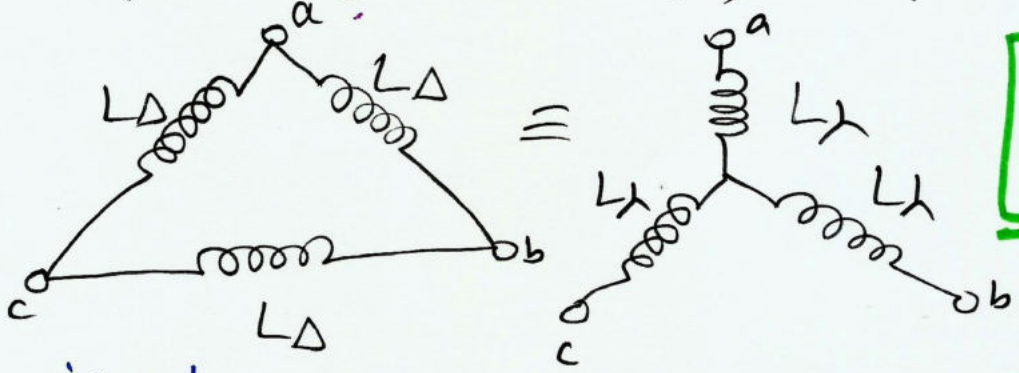
$$Z_Y = \frac{Z_\Delta}{3} = \frac{60\angle 75^\circ}{3} = 20\angle 75^\circ\Omega$$

2)



Endüktanslarda Δ → λ Dönüşümü

Δ bağlı endüktansların (L_Δ) yerine, a, b ve c düğümlerinden aynı akımları ve aynı güçleri çekecek λ bağlı eşdeğerlerinin (L_λ) hesaplanması



$$L_\lambda = \frac{L_\Delta}{3}$$

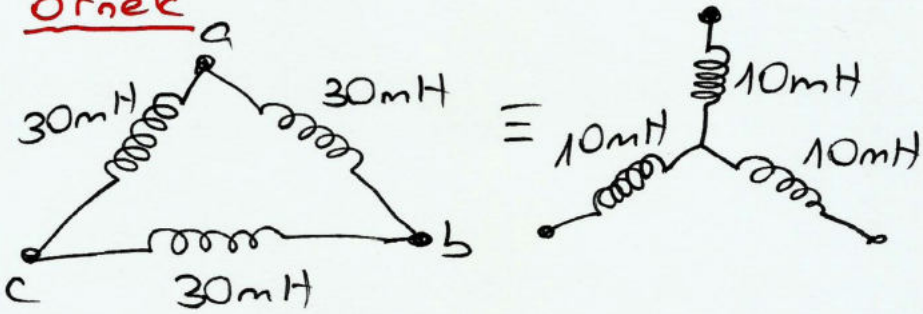
İspat:

$$X_{L_\lambda} = \frac{X_{L_\Delta}}{3} \Rightarrow \omega L_\lambda = \frac{\omega L_\Delta}{3} \Rightarrow L_\lambda = \frac{L_\Delta}{3}$$

veya reaktif güçlerin eşitliğinden;

$$\frac{3.U^2}{X_{L_\Delta}} = 3.\frac{V^2}{X_{L_\lambda}} \Rightarrow \frac{3.U^2}{\omega L_\Delta} = \frac{U^2}{\omega L_\lambda} \Rightarrow L_\lambda = \frac{L_\Delta}{3}$$

Örnek



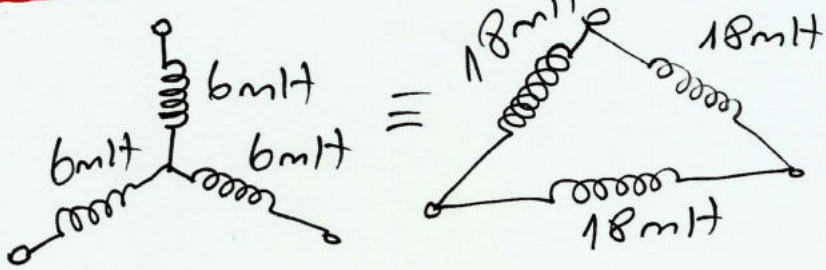
$$L_\lambda = \frac{L_\Delta}{3} = \frac{30}{3} = 10mH$$

Endüktanslarda λ → Δ Dönüşümü

Yukarıda Δ → λ dönüşümü için $L_\lambda = \frac{L_\Delta}{3}$ bulmuştuk

$$L_\lambda = \frac{L_\Delta}{3} \Rightarrow L_\Delta = 3.L_\lambda$$

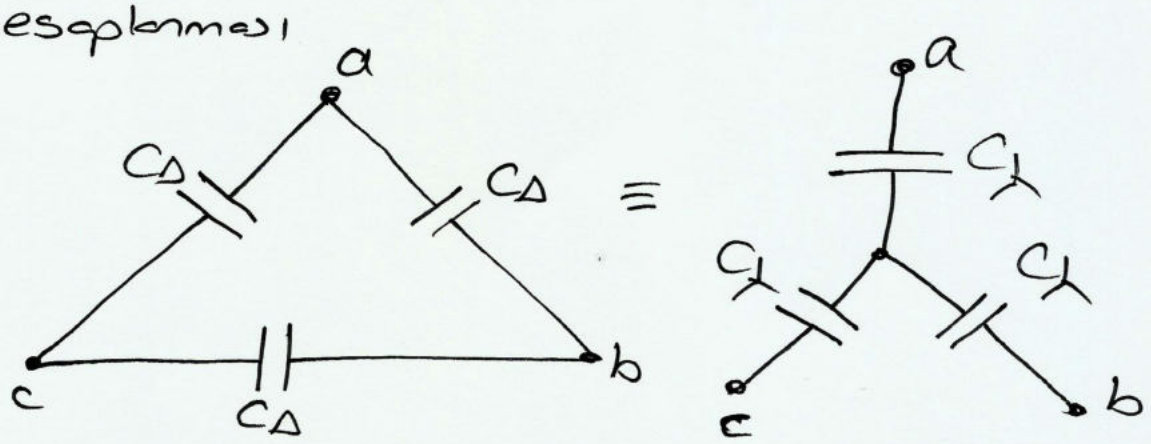
Örnek



$$L_\Delta = 3.L_\lambda = 3.6 = 18mH$$

Kondansatörlerde Δ → λ Dönüşümü

Δ bağlı kondansatörlerin a, b, c düğümlerinden aynı akımları ve aynı güçleri çekecek λ bağlı eşdeğerlerinin hesaplanması



$$C_\lambda = 3 . C_\Delta$$

İspat :

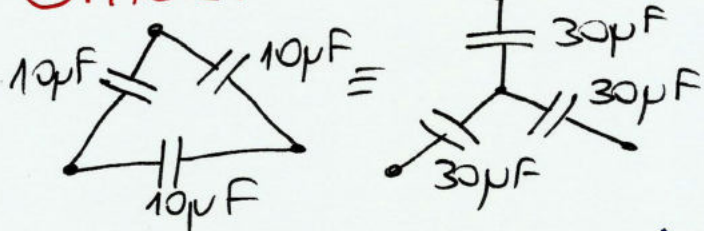
$$X_{C_\lambda} = \frac{X_{C_\Delta}}{3} \Rightarrow \frac{1}{\omega C_\lambda} = \frac{\frac{1}{\omega C_\Delta}}{3} \Rightarrow \frac{1}{\omega C_\lambda} = \frac{1}{\omega 3 C_\Delta}$$

$$C_\lambda = 3 . C_\Delta$$

veya reaktif güçlerin eşitliğinden; U^2

$$\frac{3.U^2}{X_{C_\Delta}} = 3.\frac{V^2}{X_{C_\lambda}} \Rightarrow \frac{3.U^2}{\frac{1}{\omega C_\Delta}} = \frac{U^2}{\frac{1}{\omega C_\lambda}} \Rightarrow C_\lambda = 3 . C_\Delta$$

Örnek:

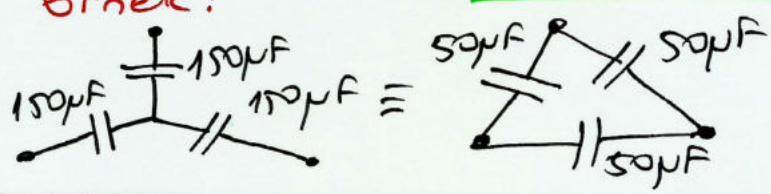


$$C_\lambda = 3 . C_\Delta = 3 . 10 = 30\mu F$$

Kondansatörlerde λ → Δ Dönüşümü

$$C_\lambda = 3 . C_\Delta \Rightarrow C_\Delta = \frac{C_\lambda}{3}$$

Örnek:



$$C_\Delta = \frac{C_\lambda}{3} = \frac{150}{3}$$

$$C_\Delta = 50\mu F$$