$$(\sin(x)-\cos(x))(\sin(x)+\cos(x)-2)$$

A similar MATLAB script can be written as the following

```
syms x real
A=[sin(x) 1 cos(x);1 1 1;cos(x) 1 sin(x)];
simplify(det(A))
```

H. 7. **Problem:** Find the solution of following equation.

$$\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix} = 0$$

Solution:

$$\begin{vmatrix} a+x & x & x \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = x\begin{vmatrix} -a & b \\ -a & 0 \end{vmatrix} + c\begin{vmatrix} a+x & x \\ -a & b \end{vmatrix}$$

$$abx + c(ba + bx + ax) = 0$$

$$abx + abc + cbx + acx = 0$$

$$x(ab + cb + ac) = -abc$$

$$x = \frac{-abc}{ab + cb + ac}$$

This equation can be solved using MATLAB through small script. A sample script is given below:

```
syms x a b c real;
A=x*ones(3)+diag([a b c]);
solve(det(A),x)
```

H 8. **Problem:** For two matrices (A and B) given below, show that $|A+B| \neq |A| + |B|$

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

Solution:

$$A + B = \begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}$$

$$|A+B| = 14, |A| = 3, \text{and } |B| = 7$$

$$14 \neq 3 + 7$$

9. **Problem:** If a given matrix A is both regular and idempotent matrix, compute the determinant of A.
Solution: If A is a regular matrix, $|A| \neq 0$. If A is an idempotent matrix, $A^2 = A$.

$$A^2 = A = A \cdot A$$

$$|A| = |A| \cdot |A|$$

$$|A|(1 - |A|) = 0$$

$$|A| \neq 0 \quad |A| = 1$$

10. **Problem:** if $A_{n\times n}$ is an inverse symmetric and n is an odd number, show that $|A| = 0$.
Solution: If A is an inverse symmetric, $A^t = -A$.

$$|A| = |-A^t| = (-1)^n |A^t|$$

$$|A| = -|A|$$

$$2|A| = 0 \quad |A| = 0$$

11. **Problem:**

$$\begin{vmatrix} x & 2y-z & z \\ y & 2z-x & x \\ z & 2x-y & y \end{vmatrix} = ?$$

Solution:

$$\begin{vmatrix} x & 2y & z \\ y & 2z & x \\ z & 2x & y \end{vmatrix} = 2\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = 2\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & z & x \\ z & x & y \end{vmatrix}$$

$$= 2(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ y & z & x \\ z & x & y \end{vmatrix} = 2(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 0 & z-y & x-y \\ 0 & x-z & y-z \end{vmatrix}$$

$$2(x+y+z)(xy+yz+zx-x^2-y^2-z^2)$$

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12. **Problem:** Find the solution of following equation.

$$\begin{vmatrix} a-1 & 0 & 1 \\ -1 & a+2 & 0 \\ 2 & 0 & a+1 \end{vmatrix} = 0$$

Solution:

$$(a+2)\begin{vmatrix} a-1 & 1 \\ 2 & a+1 \end{vmatrix} = (a+2)(a^2-1-2)$$

$$a_1 = -2, \quad a_2 = \sqrt{3}, \text{ and } \quad a_3 = -\sqrt{3}$$

13. **Problem:**

$$\begin{vmatrix} x & x & x & x \\ y & y & y & -y \\ z & z & -z & -z \\ t & -t & -t & -t \end{vmatrix} = ?$$

H

Solution:

$$xyzt \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix} = xyzt \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{vmatrix}$$

$$= 8xyzt \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = 8xyzt$$

14. **Problem:** Find the solution of the equation given below.

$$\begin{vmatrix} 2 & y & -1 \\ 8 & 2 & 7 \\ 4 & 1 & 0 \end{vmatrix} = 0$$

Solution:

$$\begin{vmatrix} 2 & y & -1 \\ 8 & 2 & 7 \\ 4 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 2 & y & -1 \\ 22 & 2+7y & 0 \\ 4 & 1 & 0 \end{vmatrix} = -1 \begin{vmatrix} 22 & 2+7y \\ 4 & 1 \end{vmatrix} = 28y - 14 = 0$$

$$y = \frac{1}{2}$$

15. **Problem:** Find the a values that hold the following equation.

$$\begin{vmatrix} 1-a & 1 & 0 \\ 2 & 0 & a \\ -1 & a & 4 \end{vmatrix} = a^3 - 7a$$

Solution: $x = 2$ $x = 4$

16. **Problem:** Let A and B given two matrices below

$$A = \begin{bmatrix} \cos(x) & \sin(x) \\ \sin(x) & \cos(x) \end{bmatrix} \quad \text{and} B = \begin{bmatrix} \cos(y) & \sin(y) \\ \sin(y) & \cos(y) \end{bmatrix}$$

Show that $\det(AB) = \cos 2x \cos 2y$

Solution:

17. **Problem:** Compute the determinant of the matrix given below

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 1 & 1 & 0 & -1 \\ 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Solution:

$$\det(A) = \begin{vmatrix} 1 & 3 & -3 & 0 \\ 1 & 1 & -1 & -1 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & -3 & 0 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 6\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 12$$

18. **Problem:**

$$\begin{vmatrix} 1 & \cos(x)+i\sin(x) & \cos(x)-i\sin(x) \\ 1 & \cos(y)+i\sin(y) & \cos(y)-i\sin(y) \\ 1 & \cos(z)+i\sin(z) & \cos(z)-i\sin(z) \end{vmatrix} =?$$

Solution:

$$\begin{vmatrix} 1 & 2\cos(x) & \cos(x)-i\sin(x) \\ 1 & 2\cos(y) & \cos(y)-i\sin(y) \\ 1 & 2\cos(z) & \cos(z)-i\sin(z) \end{vmatrix} = -2i\begin{vmatrix} 1 & \cos(x) & \sin(x) \\ 1 & \cos(y) & \sin(y) \\ 1 & \cos(z) & \sin(z) \end{vmatrix} = -2i\begin{vmatrix} 1 & \cos(x) & \sin(x) \\ 0 & \cos(y)-\cos(x) & \sin(y)-\sin(x) \\ 0 & \cos(z)-\cos(x) & \sin(z)-\sin(x) \end{vmatrix}$$

$$-2i\begin{vmatrix} -2\sin(\frac{y-x}{2})\sin(\frac{y+x}{2}) & 2\sin(\frac{y-x}{2})\cos(\frac{y+x}{2}) \\ -2\sin(\frac{z-x}{2})\sin(\frac{z+x}{2}) & 2\sin(\frac{z-x}{2})\cos(\frac{z+x}{2}) \end{vmatrix} = 8i\sin(\frac{y-x}{2})\sin(\frac{z-x}{2})\sin(\frac{y-z}{2})$$

19. **Problem:** Show that $\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & b^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = 1+a^2+b^2+c^2+d^2$

Solution:

$$= abcd\begin{vmatrix} a+\frac{1}{a} & b & c & d \\ a & a+\frac{1}{b} & c & d \\ a & b & c+\frac{1}{c} & d \\ a & b & c & d+\frac{1}{d} \end{vmatrix}$$

$$= abcd\begin{vmatrix} \frac{1}{a} & 0 & 0 & \frac{-1}{d} \\ 0 & \frac{1}{b} & 0 & \frac{-1}{d} \\ 0 & 0 & \frac{1}{c} & \frac{-1}{d} \\ a & b & c & d+\frac{1}{d} \end{vmatrix}$$

$$= bcd\begin{vmatrix} \frac{1}{b} & 0 & \frac{-1}{d} \\ 0 & \frac{1}{c} & \frac{-1}{d} \\ b & c & d+\frac{1}{d} \end{vmatrix} + abc\begin{vmatrix} 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \\ a & b & c \end{vmatrix} = a^2+b^2+c^2+d^2+1$$

20. **Problem:**

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 0 & 4 \\ 5 & 1 & 1 & -2 \\ 1 & 0 & -4 & 1 \end{vmatrix} = ?$$

Solution: One can get the following by adding first column to the last one with suitable weight 2.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 0 & 0 \\ 5 & 1 & 1 & 8 \\ 1 & 0 & -4 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & 8 \\ 0 & -4 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 1 & -10 \\ 1 & 1 & 8 \\ 0 & -4 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & -10 \\ -4 & 3 \end{vmatrix} = 74$$

21. **Problem:**

$$\begin{vmatrix} 1 & 1 & -1 & 0 & 2 \\ 1 & -1 & 2 & 0 & 3 \\ 0 & 4 & 3 & 4 & 3 \\ 2 & -1 & -1 & 2 & 0 \\ 0 & 0 & 4 & 2 & 1 \end{vmatrix} = ?$$

Solution:

$$= \begin{vmatrix} 1 & 1 & -1 & 0 & 2 \\ 0 & -2 & 3 & 0 & 1 \\ 0 & 4 & 3 & 4 & 1 \\ 0 & -3 & 1 & 2 & -4 \\ 0 & 0 & 4 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} -2 & 3 & 0 & 1 \\ 4 & 3 & 4 & 3 \\ -3 & 1 & 2 & -4 \\ 0 & 4 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 10 & -6 & 4 & 3 \\ -11 & 13 & 2 & -4 \\ 2 & 1 & 2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} 10 & -6 & 4 \\ -11 & 13 & 2 \\ 2 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 22 & -6 & 16 \\ -37 & 13 & -24 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 22 & 16 \\ -37 & -24 \end{vmatrix} = 64$$

22. **Problem:**

$$\begin{vmatrix} 2 & 0 & 3 & 6 \\ 7 & 1 & 8 & 2 \\ -5 & 1 & 2 & 0 \\ 1 & 0 & 3 & 1 \end{vmatrix} = ?$$

Solution:

$$\begin{vmatrix} 2 & 0 & -3 & 4 \\ 7 & 1 & -13 & -5 \\ -5 & 1 & 17 & 5 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -\begin{vmatrix} 0 & -3 & 4 \\ 1 & -13 & -5 \\ 1 & 17 & 5 \end{vmatrix} = -\begin{vmatrix} 0 & -3 & 4 \\ 0 & -30 & -10 \\ 1 & 17 & 5 \end{vmatrix}$$

$$= -\begin{vmatrix} -3 & 4 \\ -30 & -10 \end{vmatrix} = -150$$

14) Find the roots of

$$|A| = \begin{vmatrix} 0 & x+2 & x-1 \\ x-5 & 7 & x+1 \\ x+7 & -5 & x+3 \end{vmatrix} = 0$$

$$= (x+2)\begin{vmatrix} 1 & x+2 & x-1 \\ 1 & 7 & x+1 \\ 1 & -5 & x+3 \end{vmatrix} = x+2\begin{vmatrix} 1 & x+2 & x-1 \\ 0 & 5-x & 2 \\ 0 & -7-x & 4 \end{vmatrix}$$

$$= (x+2)\left((5-x)4 + 2(7+x)\right) = (x+2)(34-2x)$$

$$x_1 = -2 \quad x_2 = 17$$

15) Show that

$$\begin{vmatrix} a+b & a & a \cdots a \\ a & a+b & \cdots a \\ \cdots & & \\ a & a \cdots & a+b \end{vmatrix} = b^{n-1}(na+b) = (na+b)\begin{vmatrix} 1 & 1 & 1 \cdots 1 \\ a & a+b & a \cdots a \\ \vdots & & \\ a & \cdots & a+b \end{vmatrix}$$

$$= (na+b)\begin{vmatrix} 1 & 0 & 0 \cdots 0 \\ a & b & 0 \cdots 0 \\ \vdots & & \\ a & \cdots & b \end{vmatrix} = b^{n-1}(na+b)$$

16) Show that

$$\begin{vmatrix} na_1+b_1 & na_2+b_2 & na_3+b_3 \\ nb_1+c_1 & nb_2+c_2 & nb_3+c_3 \\ nc_1+a_1 & nc_2+a_2 & nc_3+a_3 \end{vmatrix} = (n+1)(n^2-n+1)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} na_1 & na_2 & na_3 \\ nb_1+c_1 & nb_2+c_2 & nb_3+c_3 \\ nc_1+a_1 & nc_2+a_2 & nc_3+a_3 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 & b_3 \\ nb_1+c_1 & nb_2+c_2 & nb_3+c_3 \\ nc_1+a_1 & nc_2+a_2 & nc_3+a_3 \end{vmatrix}$$

$$= n^3 A + A \;//$$