Q3.(35 pts)

For the following series, determine whether it converges or diverges. Show your justifications and state the name of the convergence test you use.

$$\sum_{n=1}^{\infty} \frac{2n^2+3^n}{-3+2(5^n)}$$

First,

$$\lim_{n\to\infty} \frac{\dfrac{2n^2+3^n}{-3+2(5^n)}}{\dfrac{3^n}{5^n}} = \lim_{n\to\infty} \frac{2\dfrac{n^2}{3^n}+1}{-3\dfrac{1}{5^n}+2} \longrightarrow \frac{1}{2} < \infty$$

where

$$\lim_{n \to \infty} \frac{n^2}{3^n} = \lim_{n \to \infty} \frac{2n}{3^n . \ln 3} = \lim_{n \to \infty} \frac{2}{3^n . \ln 3 . \ln 3} = 0$$

Because $\sum_{n=1}^{\infty} \frac{3^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ is convergent,

the given series is convergent by the limit comparison test.

ALTERNATIVE SOLUTION

3) $$\sum_{n=1}^{\infty} \frac{2n^2+3^n}{-3+2(5^n)}$$

Direct comparison test

For each $n \geq 1$ $$\frac{2n^2+3^n}{-3+2(5^n)} < \frac{2n^2+3^n}{-5^n+2(5^n)} = \frac{2n^2+3^n}{5^n}$$

$(-3 > -5^n$ dır$)$

$$\lim_{n\to\infty} \frac{\dfrac{2(n+1)^2+3^{n+1}}{5^{n+1}}}{\dfrac{2n^2+3^n}{5^n}} = \frac{1}{5}\lim_{n\to\infty}\frac{2(n+1)^2+3^{n+1}}{2n^2+3^n}$$

$$= \frac{1}{5}\lim_{n\to\infty}\frac{4(n+1)+(\ln 3)\,3^{n+1}}{4n+3^n\ln 3} = \frac{1}{5}\lim_{n\to\infty}\frac{4+(\ln 3)^2\,3^{n+1}}{4+3^n(\ln 3)^2}$$

$$= \lim_{n\to\infty}\frac{1}{5}\,\frac{3^{n+1}\cdot(\ln 3)^3}{3^n\cdot(\ln 3)^3} = \frac{3}{5} < 1$$

So ,it is convergent

Thus,the given series is convergent.

3) $\sum_{n=1}^{\infty} \frac{2n^2+3^n}{-3+2(5^n)}$

Direct comparison test

For each $n \geq 1$ $\frac{2n^2+3^n}{-3+2(5^n)} < \frac{2n^2+3^n}{-5^n+2(5^n)} = \frac{2n^2+3^n}{5^n}$

$(-3 > -5^n$; for $n \geq 1)$

$\sum_{n=1}^{\infty} \frac{n^2}{5^n}$; $\lim_{n\to\infty} \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2} = \frac{1}{5} < 1$ covergent

$\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$; Geometric series $\left(r = \frac{3}{5} < 1\right)$ covergent

So, $\sum_{n=1}^{\infty} \frac{2n^2+3^n}{5^n}$ is convergent

Thus the given series is convergent