

Q3.(35 pts)

For the following series, determine whether it converges or diverges. Show your justifications and state the name of the convergence test you use.

$$\sum_{n=1}^{\infty} \frac{2n^2 + 3^n}{-3 + 2(5^n)}$$

First,

$$\lim_{n \rightarrow \infty} \frac{\frac{2n^2 + 3^n}{-3 + 2(5^n)}}{\frac{3^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \frac{n^2}{3^n} + 1}{-3 \frac{1}{5^n} + 2} \rightarrow \frac{1}{2} < \infty$$

where

$$\lim_{n \rightarrow \infty} \frac{n^2}{3^n} = \lim_{n \rightarrow \infty} \frac{2n}{3^n \cdot \ln 3} = \lim_{n \rightarrow \infty} \frac{2}{3^n \cdot \ln 3 \cdot \ln 3} = 0$$

Because $\sum_{n=1}^{\infty} \frac{3^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ is convergent,

the given series is convergent by the limit comparison test.

ALTERNATIVE SOLUTION

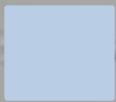
$$3) \sum_{n=1}^{\infty} \frac{2n^2 + 3^n}{-3 + 2(5^n)}$$

Direct comparison test



For each

$n \geq 1$



$$\frac{2n^2 + 3^n}{-3 + 2(5^n)} < \frac{2n^2 + 3^n}{-5^n + 2(5^n)} = \frac{2n^2 + 3^n}{5^n}$$

$$(-3) > -5^{-1} \text{ for } n=1$$

$$\lim_{n \rightarrow \infty} \frac{\frac{2(n+1)^2 + 3^{n+1}}{5^{n+1}}}{\frac{2n^2 + 3^n}{5^n}} = \frac{1}{5} \lim_{n \rightarrow \infty} \frac{2(n+1)^2 + 3^{n+1}}{2n^2 + 3^n}$$

$$= \frac{1}{5} \lim_{n \rightarrow \infty} \frac{4(n+1) + (n+1)3^{n+1}}{4n + 3^n \ln 3} = \frac{1}{5} \lim_{n \rightarrow \infty} \frac{4 + (n+1)^2 3^{n+1}}{4 + 3^n (\ln 3)^2}$$

$$\lim_{n \rightarrow \infty} \frac{1}{5} \frac{3^{n+1} \cdot (n+1)^2}{3^n \cdot (\ln 3)^2} = \frac{3}{5} < 1$$

So, it is convergent

Thus, the given series is convergent.

$$3) \sum_{n=1}^{\infty} \frac{2n^2 + 3^n}{-3 + 2(5^n)}$$

Direct comparison test

For each $n \geq 1$

$$\frac{2n^2 + 3^n}{-3 + 2(5^n)} < \frac{2n^2 + 3^n}{-5^n + 2(5^n)} = \frac{2n^2 + 3^n}{5^n}$$

for $(-3 > -5^n ; n \geq 1)$

$$\sum_{n=1}^{\infty} \frac{n^2}{5^n} ; \lim_{n \rightarrow \infty} \frac{(n+1)^2}{5^{-(n+1)}} \cdot \frac{5^n}{n^2} = \frac{1}{5} < 1$$

convergent

$$\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n ;$$

Geometric series

$$\left(r = \frac{3}{5} < 1\right)$$

convergent

So,

$$\sum_{n=1}^{\infty} \frac{2n^2 + 3^n}{5^n}$$

[Redacted]

is convergent

[Redacted]

Thus the given series is convergent