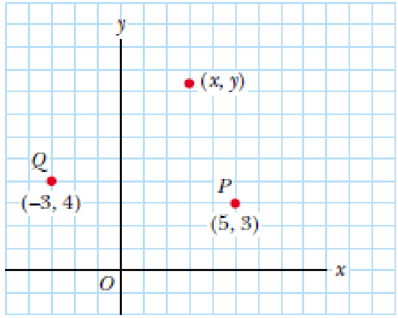
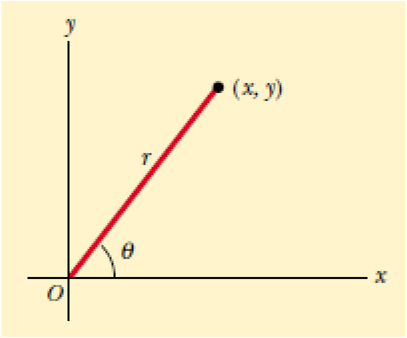
Vectors

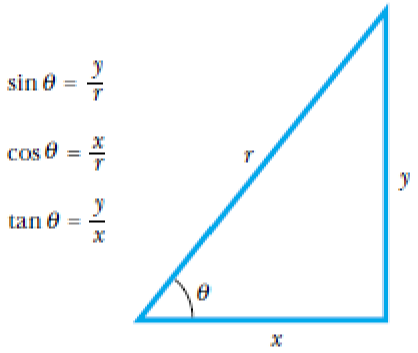
Coordinate Systems

1) *Cartesian coordinates* are also called *rectangular coordinates (x, y, z)*.



2) *Plane polar coordinates* (*r*, θ)





$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

Vector and Scalar Quantities

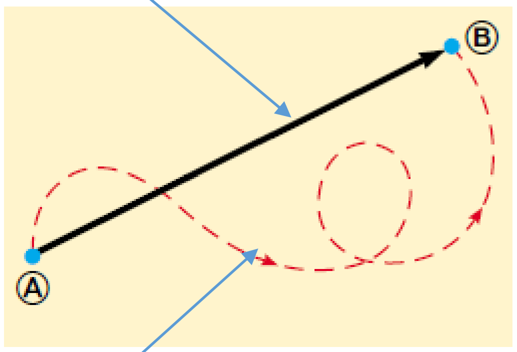
A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

Temperature, time, mass, distance and speed are some examples of *scalar quantities.*

A ***vector quantity*** is completely specified by a number and appropriate units plus a direction.

Force, displacement, velocity are some examples of *vector quantities.*

Displacement



distance

generally an arrow is written over the symbol for the vector: $\vec{\mathbf{A}}$.

but in most texts a bold letter represents a vector: $\mathbf{A}$

magnitude of the vector is A or $|\mathbf{A}|$

The magnitude of a vector is *always* a positive number and has physical units.

Some Properties of Vectors

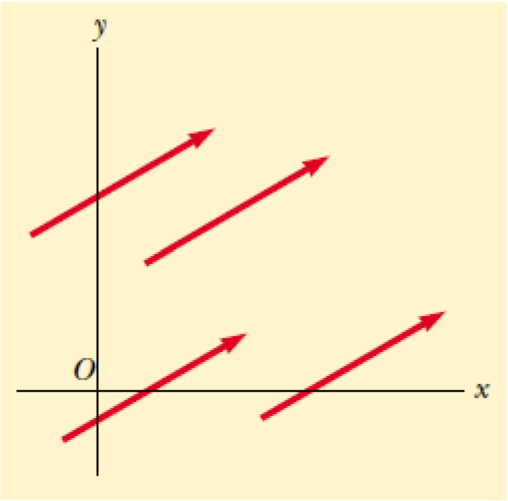
Equality of Two Vectors

Two vectors A and B are equal if they have the same magnitude and point in the same direction.

A = **B**

only if *A* = *B* and

if A and B point in the same direction along parallel lines.

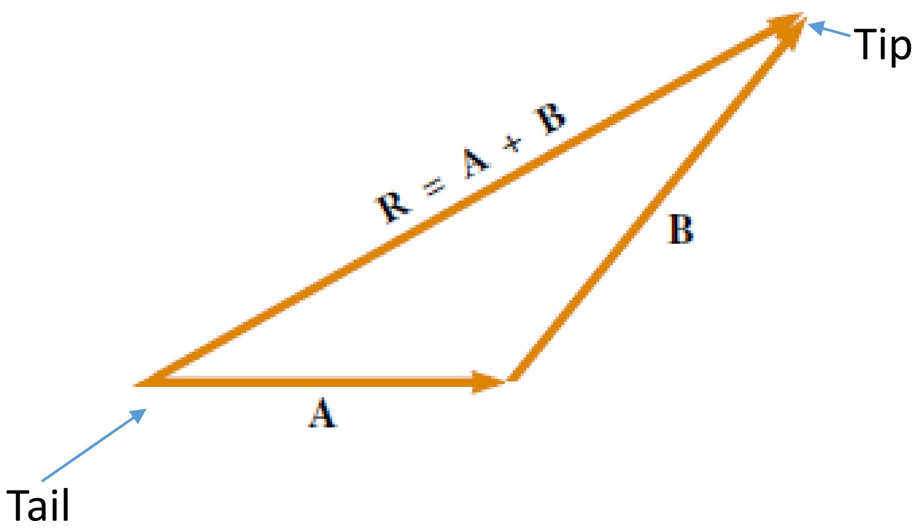


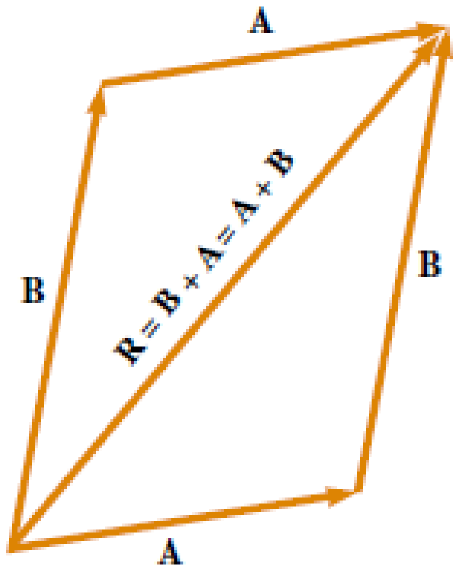
these four vectors are equal

Adding Vectors

Sum of two vectors is the resultant vector

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$





commutative law of addition

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

associative law of addition

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

Negative of a Vector;

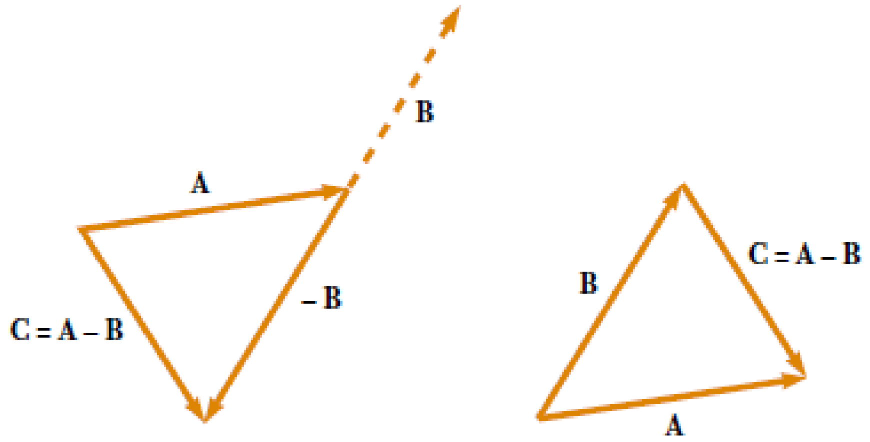
The negative of the vector A is defined as the vector that when added to A gives zero for the vector sum.

A + (-**A**) = 0

The vectors A and -A have the same magnitude but point in opposite directions.

Subtracting Vectors;

A – **B** = **A** + (-**B**)



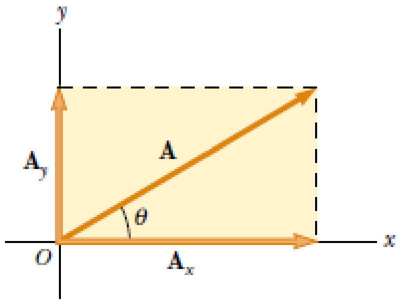
Multiplying a Vector by a Scalar:

When vector **A** is multiplied by a positive scalar *m*, then the product *m***A** is a vector with the magnitude *mA and* in the same direction as **A.**

If vector **A** is multiplied by a negative scalar -*m*, then the product -*m***A** is directed opposite **A**.

For example, the vector 3**A** is three times as long as **A** and points in the same direction as **A.**

Components of a Vector and Unit Vectors



$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y.$$

A_x and **A_y** are components of vector **A**

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \left(\frac{A_y}{A_x}\right)$$

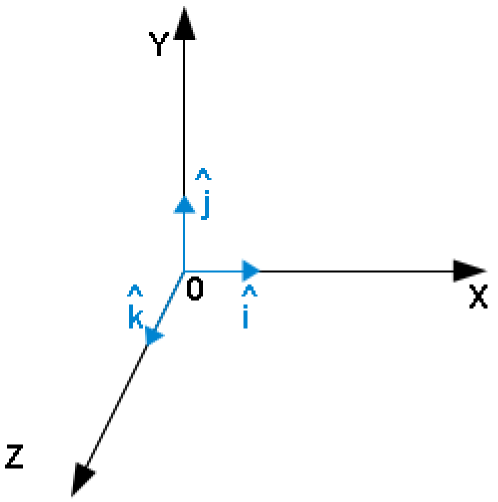
Unit Vectors

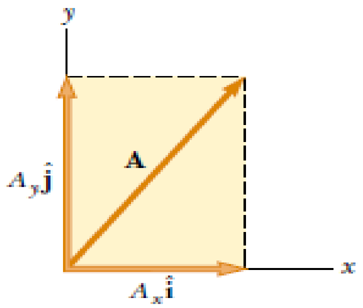
A unit vector is a vector that has a magnitude of 1 unit.

A unit vector has no dimension

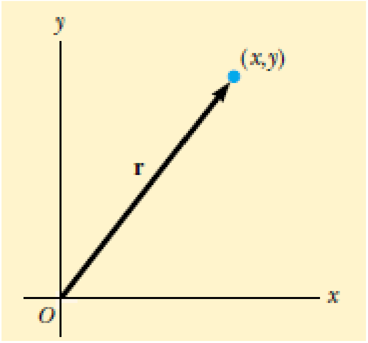
A unit vector shows the direction of a vector.

$\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors for x, y and z directions.





$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$



$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}.$$

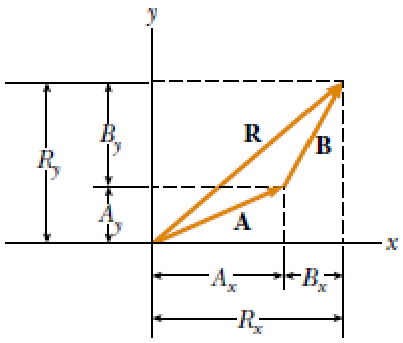
$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

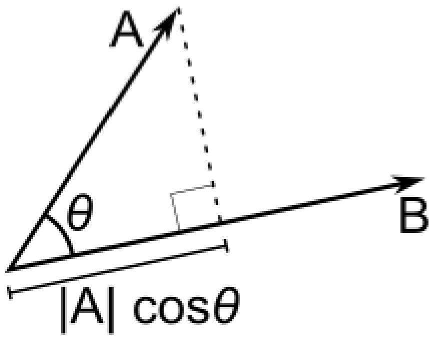
$$\mathbf{R} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}}$$



The Scalar Product of Two Vectors (dot product)

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta$$ result is scalar



$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$$

i.i= (1) (1) cos0^o=1

i.j= (1) (1) cos90^o=0

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$
$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Ex/ $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ $\quad \mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}.$

a)
$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = 4\end{aligned}$$

b)
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

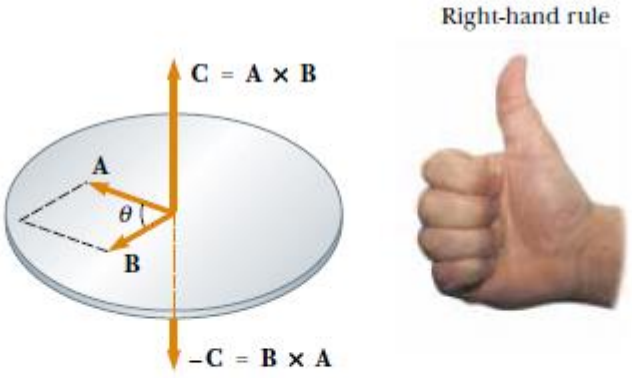
$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = 60.2^\circ$$

The Vector Product of Two Vectors (cross product)

$\mathbf{C} = \mathbf{A} \times \mathbf{B}$ result is a vector

$C \equiv AB \sin\theta$ (magnitude of the product)



$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

ixi= (1) (1) sin0^{o}=0

ixj= (1) (1) sin90^{o}=**k**

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

determinant form

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} - (A_xB_z - A_zB_x)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}$$

Ex/ $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

$$\mathbf{A} \times \mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$
$$= 2\hat{\mathbf{i}} \times 2\hat{\mathbf{j}} + 3\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) = 4\hat{\mathbf{k}} + 3\hat{\mathbf{k}} = 7\hat{\mathbf{k}}$$

$$\mathbf{B} \times \mathbf{A} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$
$$= -\hat{\mathbf{i}} \times 3\hat{\mathbf{j}} + 2\hat{\mathbf{j}} \times 2\hat{\mathbf{i}} = -3\hat{\mathbf{k}} - 4\hat{\mathbf{k}} = -7\hat{\mathbf{k}}$$