Solutions of Application Questions 3

a. $y_h = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\frac{2x}{3}} + c_4 e^{-\frac{2x}{3}} + c_5 e^{5x} + c_6 e^{-6x}$

b. $y_h = c_1 + c_2 x + c_3 e^{-2x} + c_4 x e^{-2x} + c_5 \cos 3x + c_6 \sin 3x$

c. $y_h = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x + c_5 \cos x + c_6 \sin x$

d. $y_h = (c_1 + c_2 x + c_3 x^2)e^{2x} + c_4 e^{-2x} + e^{2x}(c_5 \cos 3x + c_6 \sin 3x)$

e. $(c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x$

f. $y_h = e^{-x}\left((c_1 + c_2 x)\cos 3x + (c_3 + c_4 x)\sin 3x\right) + c_5 e^{2x} + c_6 e^{-2x}$

g. $y_h = e^{2x}\left[(c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x\right]$

h. $y_h = c_1 e^{\sqrt{2}x} + (c_2 + c_3 x)\cos 2x + (c_4 + c_5 x)\sin 2x + c_6 e^{\sqrt{3}x}$

i. $y_h = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x + c_5 \cos 3x + c_6 \sin 3x$

j. $y_h = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x + e^{x}(c_5 \cos x + c_6 \sin x)$

k. $y_h = c_1 e^{2x} + (c_2 + c_3 x)e^{-x} + (c_4 + c_5 x + c_6 x^2)e^{-3x}$

l. $y_h = (c_1 + c_2 x + c_3 x^2)\cos \frac{3x}{2} + (c_4 + c_5 x + c_6 x^2)\sin \frac{3x}{2}$

m. $y_h = e^{-3x}\left[(c_1 + c_2 x + c_3 x^2)\cos 5x + (c_4 + c_5 x + c_6 x^2)\sin 5x\right]$

n. $y_h = e^{x}(c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x) + c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x + (c_5 + c_6 x)e^{-\sqrt{2}x}$

(1) a. $r^3 - 3r^2 - r + 3 = 0$

$r^2(r-3) - (r-3) = 0 \rightarrow (r^2-1)(r-3) = 0 \Rightarrow \begin{matrix} r_1 = 1 \\ r_2 = -1 \\ r_3 = 3 \end{matrix} \Bigg\} y_h = c_1 e^{x} + c_2 e^{-x} + c_3 e^{3x}$

b. $r^3 + 2r^2 - 4r - 8 \;\big|\; r - 2$

$-r^3 - 2r^2$

$4r^2 - 4r - 8$

$-4r^2 - 8r$

$4r - 8$

$r^2 + 4r + 4$

$(r-2)(r+2)^2 = 0$

$r_1 = 2, \quad r_2 = r_3 = -2$

$y_h = c_1 e^{2x} + (c_2 + c_3 x)e^{-2x}$

c. $r^3 - 3r^2 + 3r - 1 = 0$

$(r^3 - 1) - 3r(r-1) = 0 \Rightarrow (r-1)(\underbrace{r^2 + r + 1}_{r^2 - 2r + 1} - 3r) = 0 \Rightarrow (r-1)^3 = 0 \Rightarrow r_1 = r_2 = r_3 = 1$

$y_h = (c_1 + c_2 x + c_3 x^2) e^x$

d. $r^5 + 9r^3 = 0 \Rightarrow r^3(r^2 + 9) = 0 \Rightarrow r_1 = r_2 = r_3 = 0$

$r_{4,5} = \mp 3i$

$y_h = (c_1 + c_2 x + c_3 x^2) + c_4 \cos 3x + c_5 \sin 3x$

e. $r^4 - 4r^3 + 29r^2 = 0$

$r^2(r^2 - 4r + 29) = 0$

$\Delta = 16 - 4.1.29 = -100$

$r_1 = r_2 = 0$

$r_{3,4} = \frac{4 \mp \sqrt{-100}}{2} = 2 \mp 5i$

$y_h = c_1 + c_2 x + e^{2x}(c_3 \cos 5x + c_4 \sin 5x)$

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$y_h = (c_1 + c_2 x) e^x + c_3 e^{-x}$

g. $r^3 - 5r^2 + 6r = 0 \Rightarrow r(r^2 - 5r + 6) = 0 \Rightarrow r_1 = 0,\ r_2 = 3,\ r_3 = 2$

$-3 \quad -2$

$y_h = c_1 + c_2 e^{3x} + c_3 e^{2x}$

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