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A Solution Proposal to the Interval Fractional Transportation Problem

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Abstract: In real world applications, frequently may be faced up with the fractional transportation problem that these cost and preference parameters of the fractional objective may not be known in precise manner which are changed in the interval each other. In this study, fractional transportation problem with interval coefficient is transformed to a classical transportation problem by expanding the order 1^{st} Taylor polynomial series with multi variables.

Keywords: Transportation Problem, Linear Fractional Programming, Interval Coefficients.

1. Introduction

In the modelling the real world problems like financial and, corporate planning, production planning, marketing and media selection, university planning and student admission, health care and hospital planning, air force maintenance units, bank branches, etc. frequently. We are faced up with a decision to optimise profit/cost, inventory/sales, etc. respect to some constraints (Lai and Hwang 1996). Since linear fractional programming problem approach offers more efficient method than linear programming Problem (LPP), many researches has been working on Linear Fractional Programming Problem (LFPP) intensively. In the literature [1-4], different approaches appear in solving different models of the LFPP.

When some of studies present solution methods (Lai and Hwang, 1996), (Charnes and Cooper,1962), (Zionts,1968), (Chakraborty and Gupta,2002), others have concentrated on applications (Musteanu and Rado,1960), (Gilmore and Gomory,1963) (Sengupta et all., 2001). While the traditional (classical) transportation problem considers transportation of the product from source to destination with a linear objective function on the other hand several other approaches exist for the linear transportation problem with a single or multi objective function, Kanti Swarup studied the optimal (maximum) ratio of the Linear function subject to a set of linear constraints and non negativity conditions on the variables [5], Dorina Moante has also presented a solution to a three dimensional problem with an objective function which is the ratio of the linear functions [7], Guzel, N. and Sivri, M have presented a solution to Multi objective a linear fractional programming problem by using Taylor series expansion[8], C.S. Ramakrishnan presented an optimal and near optimal initial solution in the balanced and unbalanced transportation problem by using Vogel approximation Method [6], Sivri M et all, propose an optimal or near optimal initial solution and optimality condition(by using [5]) for transportation problem with the linear fractional objective [11].

The aim of this paper is to introduce two solution procedures for the fractional transportation problem with interval coefficients. And an illustrative example is given to explain these procedures.

2. The Structure of the Fractional Transportation Problem

Consider a fractional transportation problem with m supply and n demand, in that $a_i > 0$ units are supplied by supply i *th* and $b_j > 0$ units are required by demand j *th*. There is a fractional objective function that unit shipping

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cost $c_{ij}$and unit preferring route d_{ij} for transportation. Let x_{ij} denote the number of units to be transported from supply i *th* to demand j *th*. The mathematical model of the fractional transportation problem with interval coefficients that coefficients of a fractional objective function are interval of the real numbers in this work is stated as follows:

$$Z = \min \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij}^{L}, c_{ij}^{R}\right] x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left[d_{ij}^{L}, d_{ij}^{R}\right] x_{ij}} \tag{2.1}$$

subject to

$$\begin{array}{l} \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, ..., m \\ \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, ..., n \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ x_{ij} \geq 0, \forall i, j \end{array}$$

where $\left[c_{ij}^{L}, c_{ij}^{R}\right]$ is an interval representing the uncertain cost and $\left[d_{ij}^{L}, d_{ij}^{R}\right]$ is an interval representing uncertain preference of route for the transportation problem. That is, c_{ij}^{L},d_{ij}^{L} c_{ij}^{R},$d_{ij}^{R}$are the lower and upper bounds for the unit shipping cost $c_{ij}$and unit preferring route d_{ij} to be transported from supply i *th* to demand j *th*, respectively.

3. Solving the Problem

Proposal 2.1

Since the problem 2.1 is a minimization problem, in the fractional objective function these given intervals$\left[c_{ij}^{L}, c_{ij}^{R}\right]$and $\left[d_{ij}^{L}, d_{ij}^{R}\right]$ can be stated as follows:

$\left[c_{ij}^{L}, c_{ij}^{R}\right] = c_{ij}^{L} + \theta_{ij}(c_{ij}^{R} - c_{ij}^{L}), \quad 0 \leq \theta_{ij} \leq 1 \quad$ for $i = 1, ..., m\,; j = 1, ..., n$ GrindEQ$_2$

$\left[d_{ij}^{L}, d_{ij}^{R}\right] = d_{ij}^{L} + \lambda_{ij}(d_{ij}^{R} - d_{ij}^{L}) \quad 0 \leq \lambda_{ij} \leq 1 \quad$ for $i = 1, ..., m\,; j = 1, ..., n$ GrindEQ$_3$

The problem 2.1 may be restated as follows:

$$Z = Min\left\{\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{L} + \theta_{ij}(c_{ij}^{R} - c_{ij}^{L})x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{L} + \lambda_{ij}(d_{ij}^{R} - d_{ij}^{L})x_{ij}}\right\}$$

$$\begin{array}{l} s.t \\ \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, ..., m \\ \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, ..., n \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ x_{ij} \geq 0, \forall i, j \\ \theta_{ij}, \lambda_{ij} \in [0, 1]\,, \ \forall i, j \end{array} \tag{3.1}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{L} + \theta_{ij}(c_{ij}^{R} - c_{ij}^{L})x_{ij}$$

$$\geq 0, \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{L} + \lambda_{ij}(d_{ij}^{R} - d_{ij}^{L})x_{ij} > 0.$$

When the first Taylor polynomial for the objective function of problem 3.1 about $X^{(k)} = (x_{ij}^{(k)}, \theta_{ij}^{(k)}, \lambda_{ij}^{(k)})$, the last form of 3.1 can be constructed as follows:

$$\begin{aligned} Min\, Z^{(k)} = & \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial x_{ij}}\right|_{X^{(0)}} (x_{ij} - x_{ij}^{(k)}) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \theta_{ij}}\right|_{X^{(0)}} (\theta_{ij} - \theta_{ij}^{(k)}) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \lambda_{ij}}\right|_{X^{(0)}} (\lambda_{ij} - \lambda_{ij}^{(k)}) \end{aligned}$$

$$\begin{array}{l} s.t \\ \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, ..., m \\ \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, ..., n \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ x_{ij} \geq 0, \forall i, j \\ \theta_{ij}, \lambda_{ij} \in [0, 1]\,, \ \forall i, j \end{array} \tag{3.2}$$

An initial basic feasible solution $X^{(0)} = (x_{ij}^{(0)}, \theta_{ij}^{(0)}, \lambda_{ij}^{(0)})$ is obtained with the North-West Corner Rule of the classical transportation problem or obtained by using the other known methods for transportation problem where $\theta_{ij}^{(0)}$,$\lambda_{ij}^{(0)}$ are constant $\left(0 \leq \theta_{ij}^{(0)} \leq 1,\ 0 \leq \lambda_{ij}^{(0)} \leq 1\right)$ for $i = 1, ..., m\,; j = 1, ..., n$.

If problem 3.2 for $X^{(k)} = X^{(0)}$ is solved, then the solution $X^{2.1} = (x_{ij}^{(1)}, \theta_{ij}^{2.1}, \lambda_{ij}^{2.1})$is obtained. And the objective function of 3.2 is rearranged for the new point $X^{2.1}$. In other words, the fractional objective function of problem 3.1 is again expanded to its first Taylor polynomial at the new point $X^{2.1}$. The solution of problem 3.2 with rearranged objective function is closer to the optimal value. In order to have a better solution, the solutions

$$\left\{X^{(k)} = (x_{ij}^{(k)}, \theta_{ij}^{(k)}, \lambda_{ij}^{(k)})\right\}, (k = 0, 1, 2, 3, ...)$$

of problem 3.2 can be continued until $(x_{ij}^{(k)}, \theta_{ij}^{(k)}, \lambda_{ij}^{(k)}) = (x_{ij}^{(k+1)}, \theta_{ij}^{(k+1)}, \lambda_{ij}^{(k+1)})$. A last obtained solution $(x_{ij}^{(k)}, \theta_{ij}^{(k)}, \lambda_{ij}^{(k)})$ is a solution of problem 2.1 [18].

Proposal ??

Let us consider the problem 2.1 as follows:

$$\begin{aligned} MinZ_1^R &= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{R} x_{ij}, MinZ_1^C \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n} (\frac{c_{ij}^{L} + c_{ij}^{R}}{2}) x_{ij}, \end{aligned}$$

$$\begin{aligned} MaxZ_2^L &= \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{L} x_{ij}, MaxZ_2^C \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n} (\frac{d_{ij}^{L} + d_{ij}^{R}}{2}) x_{ij} \end{aligned}$$

$$\text{s.t.} \begin{array}{l} \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, ..., m \\ \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, ..., n \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ x_{ij} \geq 0, \forall i, j \end{array} \quad \text{GrindEQ}_6$$

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In order to propose a solution to problem **??**, it is maximized of the sum of the linear membership functions of Z_1^R, Z_1^C, Z_2^L and Z_2^C under the constraints of **??** where, $\mu(Z_1^R)$, $\mu(Z_1^C), \mu(Z_2^L)$ and $\mu(Z_2^C)$ are the membership functions of Z_1^R, Z_1^C, Z_2^L and Z_2^C , respectively. By rearranging problem **??**, we have

$$Max\ (\mu(Z_1^R) + \mu(Z_1^C) + \mu(Z_2^L) + \mu(Z_2^C))$$

$$\text{Subject to } \begin{array}{l} \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, ..., m \\ \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, ..., n \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ x_{ij} \geq 0,\ \forall i, j \end{array} \quad \text{GrindEQ}_7$$

4. An Example

We consider a fractional interval transportation problem with interval shipping costs and preferring routes, crisp supplies and demands. The problem has the following fractional transportation problem form:

$\tilde{c}_{11} = [1, 3]$ $\tilde{d}_{11} = [3, 4]$	$\tilde{c}_{12} = [2, 5]$ $\tilde{d}_{12} = [3, 5]$	$\tilde{c}_{13} = [0, 3]$ $\tilde{d}_{13} = [4, 6]$	$a_1 = 30$
$\tilde{c}_{21} = [1, 2]$ $\tilde{d}_{21} = [5, 7]$	$\tilde{c}_{22} = [3, 4]$ $\tilde{d}_{22} = [1, 2]$	$\tilde{c}_{23} = [1, 3]$ $\tilde{d}_{23} = [5, 7]$	$a_2 = 20$
$b_1 = 20$	$b_2 = 10$	$b_3 = 20$	

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, the given problem is a balanced transportation problem.

The given fractional transportation problem can be written as the following interval fractional linear programming problem:

$$\tilde{Z} = \min \begin{cases} ([1,3]x_{11} + [2,5]x_{12} + [0,3]x_{13} \\ +[1,2]x_{21} + [3,4]x_{22} + [1,3]x_{23}) \\ /([3,4]x_{11} + [3,5]x_{12} + [4,6]x_{13} \\ +[5,7]x_{21} + [1,2]x_{22} + [5,7]x_{23}) \end{cases}$$

s.t.

$$\begin{array}{l} x_{11} + x_{12} + x_{13} = 30 \\ x_{21} + x_{22} + x_{23} = 20 \\ x_{11} + x_{21} = 20 \\ x_{12} + x_{22} = 10 \\ x_{13} + x_{32} = 20 \\ x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0. \end{array} \tag{3.3}$$

Using **??** and **??**, the given interval fractional transportation problem can be written as follows:

$$\begin{array}{l} \theta_{11}, \theta_{21}, \theta_{22}, \theta_{23}, \lambda_{11}, \lambda_{12}, \\ \lambda_{13}, \lambda_{21}, \lambda_{22}, \lambda_{23} \in [0, 1] \\ x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0. \end{array} \tag{3.4}$$

In order to achieve our goal, initial feasible solution values can be chosen as

$$\begin{array}{l} x_{11} = 20,\ x_{12} = 10,\ x_{13} = 0, \\ x_{21} = 0,\ x_{22} = 0,\ x_{23} = 20, \\ \theta_{11}, \theta_{21}, \theta_{22}, \theta_{23}, \lambda_{11}, \\ \lambda_{12}, \lambda_{13}, \lambda_{21}, \lambda_{22}, \lambda_{23} = 1 \end{array}$$

respect to the North West Corner Rule. And the objective function is:

$$\begin{aligned} z_1 &= (1 + 2\theta_{11})x_{11} + (2 + 3\theta_{12})x_{12} \\ &\quad +(0 + 3\theta_{13})x_{13} + (1 + \theta_{21})x_{21} \\ &\quad +(3 + \theta_{22})x_{22} + (1 + 2\theta_{23})x_{23}z_2 \\ &= (3 + \lambda_{11})x_{11} + (3 + 2\lambda_{12})x_{12} \\ &\quad +(4 + 2\lambda_{13})x_{13} + (5 + 2\lambda_{21}) \\ &\quad x_{21} + (1 + \lambda_{22})x_{22} + (5 + 2\lambda_{23})x_{23} \end{aligned}$$

$Z = \min\left(\frac{z_1}{z_2}\right)$width=0.12in, height=0.19in, keepaspectratio=false]image1.eps
width=0.12in, height=0.19in, keepaspectratio=false]image2.eps

For the above solution, $z_1 = 40$ and $z_2 = 230$ and the new objective function is

After that, the objective function 3.5 is solved subject to constraints of **??**, and the below solution is obtained:

$$\begin{array}{r} x_{11} = 0,\ x_{12} = 10,\ x_{13} = 20,\ x_{21} = 20, \\ x_{22} = 0,\ x_{23} = 0, \theta_{11} = 0,\ \theta_{12} = 0, \\ \theta_{13} = 0,\ \theta_{21} = 0,\ \theta_{22} = 0,\ \theta_{23} = 0, \lambda_{11} = 0,\ \lambda_{12} = 1, \\ \lambda_{13} = 1,\ \lambda_{21} = 1,\ \lambda_{22} = 0,\ \lambda_{23} = 0. \end{array}$$

For the above solution,$z_1 = 40$ and $z_2 = 310$and the new objective function is:

Consequently, the objective function 3.6 is solved subject to constraints of **??**, and the below solution is obtained:

$$\begin{array}{r} x_{11} = 0, x_{12} = 10, x_{13} = 20, x_{21} = 20, x_{22} = 0, x_{23} = 0, \\ \theta_{11} = 0,\ \theta_{12} = 0,\ \theta_{13} = 0, \\ \theta_{21} = 0,\ \theta_{22} = 0,\ \theta_{23} = 0, \lambda_{11} = 0,\ \lambda_{12} = 1,\ \lambda_{13} = 1, \\ \lambda_{21} = 1,\ \lambda_{22} = 0,\ \lambda_{23} = 0, \end{array}$$

Since the solution is the same as the solution obtained from a previous step, the solution is the better solution of the given original problem.

Now, If the same problem is solved respect to proposal **??**, the given original problem is written as the following multiple objective transportation problem:

$$\begin{array}{l} Min\ Z_1^R = 3x_{11} + 5x_{12} + 3x_{13} \\ \qquad +2x_{21} + 4x_{22} + 3x_{23} \\ \\ Min\ Z_1^C = 2x_{11} + 3.5x_{12} + 1.5x_{13} \\ \qquad +1.5x_{21} + 3.5x_{22} + 2x_{23} \\ \\ Max\ Z_2^L = 3x_{11} + 3x_{12} + 4x_{13} \\ \qquad +5x_{21} + x_{22} + 5x_{23} \\ \\ Max\ Z_2^C = 3.5x_{11} + 4x_{12} + 5x_{13} \\ \qquad +6x_{21} + 1.5x_{22} + 6x_{23} \end{array} \tag{3.7}$$

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$$z = \min\{\frac{(1+2\theta_{11})x_{11}+(2+3\theta_{12})x_{12}+(0+3\theta_{13})x_{13}+(1+\theta_{21})x_{21}+(3+\theta_{22})x_{22}+(1+2\theta_{23})x_{23}}{(3+\lambda_{11})x_{11}+(3+2\lambda_{12})x_{12}+(4+2\lambda_{13})x_{13}+(5+2\lambda_{21})x_{21}+(1+\lambda_{22})x_{22}+(5+2\lambda_{23})x_{23}}\}$$

$$\begin{aligned} x_{11}+x_{12}+x_{13} &= 30\\ x_{21}+x_{22}+x_{23} &= 20\\ x_{11}+x_{21} &= 20\\ x_{12}+x_{22} &= 10\\ x_{13}+x_{32} &= 20 \end{aligned}$$

$$\begin{aligned} Z^{(2)} \cong \min \tfrac{1}{230^2}\{&70(x_{11})+260(x_{12}-10)-160(x_{13}-20)+30(x_{21}-20)+650x_{22}-50x_{23}\\ &+0.\theta_{11}+6900(\theta_{12})+13800(\theta_{13})+4600(\theta_{21})+0(\theta_{22})+0(\theta_{23})\\ &-0(\lambda_{11}-1)-800(\lambda_{12}-1)-1600(\lambda_{13})-1600(\lambda_{21})-0\lambda_{22}-0(\lambda_{23}-1)\} \end{aligned} \tag{3.5}$$

$$\begin{aligned} Z^{(3)} \cong \min \tfrac{1}{310^2}\{&190(x_{11})+420(x_{12}-10)-240(x_{13}-20)+30(x_{21}-20)+890x_{22}+110x_{23}\\ &+0.\theta_{11}+1200(\theta_{12})+2400(\theta_{13})+800(\theta_{21})+0(\theta_{22})+0(\theta_{23})\\ &-0(\lambda_{11})-800(\lambda_{12}-1)-1600(\lambda_{13}-1)-1600(\lambda_{21}-1)-0\lambda_{22}-0(\lambda_{23})\} \end{aligned} \tag{3.6}$$

Subject to

$$\begin{aligned} x_{11}+x_{12}+x_{13} &= 30\\ x_{21}+x_{22}+x_{23} &= 20\\ x_{11}+x_{21} &= 20\\ x_{12}+x_{22} &= 10\\ x_{13}+x_{32} &= 20\\ x_{11},x_{12},x_{13},x_{21},x_{22},x_{23} &\geq 0. \end{aligned} \tag{3.8}$$

Maximum and minimization values of the objective functions 3.7 under the constrains 3.8 are $150 \leq Z_1^R \leq 170{,}95 \leq Z_1^C \leq 115{,}160 \leq Z_2^L \leq 210{,}195 \leq Z_2^C \leq 260$. Using the fuzzy approach their membership functions are

$$\begin{aligned} \mu(Z_1^R) &= -\frac{1}{20}(3x_{11}+5x_{12}+3x_{13}\\ &\quad +2x_{21}+4x_{22}+3x_{23}-170),\\ \mu(Z_1^C) &= -\frac{1}{20}(2x_{11}+3.5x_{12}+1.5x_{13}\\ &\quad +1.5x_{21}+3.5x_{22}+2x_{23}-115),\\ \mu(Z_2^L) &= \frac{1}{50}(3x_{11}+3x_{12}+4x_{13}\\ &\quad +5x_{21}+x_{22}+5x_{23}-160)\\ \mu(Z_2^C) &= \frac{1}{65}(3.5x_{11}+4x_{12}+5x_{13}\\ &\quad +6x_{21}+1.5x_{22}+6x_{23}-195). \end{aligned} \tag{3.9}$$

Then,

$$\begin{aligned} &Max(-.13615x_{11}-.3035x_{12}\\ &-0.0681x_{13}+0.01731x_{21}\\ &-.3319230769x_{22}-0.05769x_{23}+8.05)\\ &x_{11}+x_{12}+x_{13} = 30\\ &x_{21}+x_{22}+x_{23} = 20\\ &x_{11}+x_{21} = 20\\ &x_{12}+x_{22} = 10\\ &x_{13}+x_{32} = 20\\ &x_{ij} \geq 0,\ i=1,2; j=1,2,3. \end{aligned} \tag{3.10}$$

The solution of problem 3.10 is $x_{11} = 0$, $x_{12} = 10$, $x_{13} = 20$, $x_{21} = 20$, $x_{22} = 0$, $x_{23} = 0$. Thus, the solution obtained with proposal 2.1 is equal to the solution obtained with proposal **??**.

5. Conclusion

In this paper, two solution procedures are proposed for the interval fractional transportation problem. One of them is based on Taylor series approximation [8,11], the other one is based on interval arithmetic [10]. Also an illustrative example is given for explaining these approaches. The results obtained by these approaches are the same.

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