FINAL EXAM

1) Consider a group of people who have different probabilities of getting sick. Individuals of type H have a probability of getting sick as pH = 0.15. These individuals form 1/5 of the population. Individual of type L, the remaining 4/5 of the population, have a probability of getting sick as pL =0.1. Treatment costs and compensation for sickness is 20000TL. For each type the maximal insurance premium that they are willing to pay is 2 times of their expected losses. Assume that there is asymmetric information: insurance firms cannot distinguish between patients. The same insurance contract must be offered to all individuals. The contract is to earn an expected zero profit. Is the equilibrium insurance premium efficient? Explain. [The answer is at the end]

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2) In Turkey the probability of your home to be vulnerable to earthquake is a% where a is the last 2 digits of your ID#. Resilience tests determine whether your house will be damaged with 99% accuracy. If the resilience test of your home tells that you your home will not be damaged, what is the probability that it will be damaged?

In similar examples that we solved in the class, the answer was far lower than the accuracy of the test, 99%. Is your answer also significantly less than 99% (say, less than 90%). Explain why. [The answer is at the end]

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3) Consider the growth model of Solow. Assume that the production technology is Cobb-Douglas:

$$Y = K^b L^{1-b}.$$

Mathematically prove that the growth rate of the GDP per capita in the long-run is zero. Explain why it is zero in words.

Answer: Growth(Y/L)=Growth(A) as we discussed in the class. But A=1 in the question so Growth(A)=0. Conclude Growth(Y/L)=0. Explanation? If the technology does not grow, then the GDP per capita cannot grow in the long-run.

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4) Consider an economy where consumers can be represented by three individuals whose incomes are

$$\{Y_1, Y_2, Y_3\} = \{a, 2a, 3a\}.$$

Assume that the income tax rate is t so private consumption by consume $i = 1,2,3$ is

$$C_i = (1-t) \times Y_i$$

Tax revenues are used to finance a public good G:

$$G = t \times (Y_1 + Y_2 + Y_3).$$

The utilities of consumers from private and public goods are $U_i = (a \ln C_i) + 100(\ln G)$ for $i = 1,2,3$.

FAKE EXAM

1) Consider a group of drivers that have different probabilities of having an accident. Individuals of type H have a probability of having an accident of pH = 0.1. These individuals form 3/4 of the population. Individual of type L, the remaining 1/4 of the population, have a probability of having an accident of pL =0.05. Any individual who has an accident suffers an income loss of 8000TL. For each type the maximal insurance premium that they are willing to pay is 1/3 above their expected losses. Assume that there is asymmetric information: insurance firms cannot distinguish between the types of drivers. The same insurance contract must be offered to all individuals. If the contract is to earn an expected profit of zero, what insurance premium must be charged? Is this competitive outcome efficient?

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2) Suppose that the prevalence of European ancestry in Turkey is 30%. 23andMe is a private company that offers genetic tests which determine ancestry with 95% accuracy. If your 23andMe test tells that you have a European lineage, what is the probability that your grandparents come from Europe?

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3) Consider the growth model of Solow. Assume that the production technology is Cobb-Douglas:

$$Y = K^b (AL)^{1-b}.$$

The law of motion for capital is

$$\frac{dK}{dt} = (s \times Y) - (\Delta \times K).$$

Assume that the growth rate of L is l and the growth rate of A is a. What is GDP per capita Y/L in the long-run?

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4) Consider an economy where consumers can be represented by three individuals whose incomes are

$$\{Y_1, Y_2, Y_3\} = \{3,6,15\}$$

Assume that the income tax rate is t so private consumption by consumer $i = 1,2,3$ is

$$C_i = (1-t) \times Y_i$$

Tax revenues are used to finance a public good G:

$$G = t \times (Y_1 + Y_2 + Y_3).$$

The utilities of consumers from private and public goods are

$$U_1 = C_1 G^{1/2}$$

Calculate the most preferred tax rates for individual $i =$ 1,2,3. Suppose that two political parties A and B compete in the elections. Each political party proposes a tax rate to maximize its votes. Consumers vote for their most preferred tax. What is the election winning tax rate? Explain. Answer: $U_i = a\ln((1-t)\times Y_i) + 100\ln(t\times(Y_1+Y_2+Y_3))$. So $\frac{dU_i}{dt} = -\frac{a}{1-t} + \frac{100}{t} = 0$. The solution is $t = \frac{100}{100+a}$. There is no conflict of interest. Everyone votes for the same policy. That is the same explanation for the fake exam question.	$U_2 = C_2 G^{1/2}$ $U_3 = C_2 G^{1/2}$. Calculate the most preferred tax rates for individual $i =$ 1,2,3. Suppose that two political parties A and B compete in the elections. Each political party proposes a tax rate to maximize its votes. Consumers vote for their most preferred tax. What is the election winning tax rate? Explain.

Bir de 1. ve 2. soruların cevaplarını detaylı şekilde görelim. Sağ tarafta Avesis'e yüklediğim sizin zaten elinizde olan cevapları görebilirsiniz.

Answer of Q1: For this question, observe that the insurance companies can offer a premium with only two possible outcomes: 1) both H and L types would purchase the policy, 2) only H types would purchase the policy. (Remark: Why is there no possibility of a policy where only L would purchase it? Because, if L is in the premium would be cheap which ensures that H is also in) Let us consider the first scenario: $\pi = p - Expected\ losses = 0.$ Note that $Expected\ losses = \left(\frac{4}{5}\times 20K\times\frac{1}{10}\right) + \left(\frac{1}{5}\times 20K\times\frac{1.5}{10}\right) = 2{,}2K.$ So the policy which everyone buys costs 2,2K under the condition of zero profit. But does everyone actually buy it? H types would pay at most $2\times 20K\times\frac{1.5}{10} = 6 > 2{,}2.$ This tells us that H types would buy. L types would pay at most $2\times 20K\times\frac{1}{10} = 4 > 2{,}2.$ So L types would also buy. The result is that "a policy that everyone buys in competitive market exists". Second scenario: Only H types buy the policy. $\pi = p - Expected\ losses = 0.$ Note that $Expected\ losses = \left(20K\times\frac{1.5}{10}\right) = 3K.$	Answer of Q1: (Bu cevap önceden paylaşılmıştı) For this question, observe that the insurance companies can offer a premium with only two possible outcomes: 1) both H and L types would purchase the policy, 2) only H types would purchase the policy. (Remark: Why is there no possibility of a policy where only L would purchase it? Because, if L is in the premium would be cheap which ensures that H is also in) Let us consider the first scenario: $\pi = p - Expected\ losses = 0.$ Note that $Expected\ losses = \left(\frac{3}{4}\times 8K\times\frac{1}{10}\right) + \left(\frac{1}{4}\times 8K\times\frac{1}{20}\right) = 0{,}7K.$ So the policy which everyone buys costs 0,7K under the condition of zero profit. But does everyone actually buy it? H types would pay at most $8K\times\left(1+\frac{1}{3}\right)\times\frac{1}{10} > 800 > 700.$ This tells us that H types would buy. L types would pay at most $8K\times\left(1+\frac{1}{3}\right)\times\frac{1}{20} < 700.$ So L types would not buy. The result is that "a policy that everyone buys in competitive market does not exist". Second scenario: Only H types buy the policy. $\pi = p - Expected\ losses = 0.$ Note that $Expected\ losses = \left(8K\times\frac{1}{10}\right) = 0{,}8K.$

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<td>But L types would also buy this policy. Why? Because they are willing to pay at most
$$2 \times 20K \times \frac{1}{10} = 4 > 2{,}2.$$
The result is “only H buys” an is impossible scenario.
a) The outcome is efficient. Because L types are insured just as H are. In other words, there is no beneficial trade that do not take place at the equilibrium. This is efficient.</td>
<td>The equilibrium is $p = 0{,}8K$ and H types would buy this policy. Why? Because they are willing to pay at most
$$8K \times \left(1 + \frac{1}{3}\right) \times \frac{1}{10} > 800.$$
a) The outcome is not efficient. Because L types are not insured even though they are willing to pay more than their expected losses. In other words, there is beneficial trade between L types and the insurance companies that do not take place at the equilibrium. This is inefficient.</td>
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<td>Answer of Q2:

To solve this question we should use the Bayes’ Rule:
$$P(A|B) = P(B|A) \times \frac{P(A)}{P(B)}.$$
In this question, we are asked $P(A|B) = P(No\ Damage|+test)$. Event A is “No damage” and event B is a positive test result. Therefore,
$$P(A) = 1 - (a/100).$$
$$P(B|A) = 0{,}99.$$
$$P(B) = P(B|A)P(A) + P(B|not\ A)P(not\ A)$$
Suppose $a = 50$.
$$= 0{,}99 \times 0{,}5 + 0{,}01 \times (1 - 0{,}5) = 0{,}5$$
As a consequence,
$$P(A|B) = 0{,}99 \times \frac{0{,}5}{0{,}5} = 0{,}99.$$

[The explanation below was also given in the class for the example of European ancestry. The explanation is still the same.]
So the result is very close to the accuracy rate. Explanation? The event A is a very common event. So the accuracy rate perfectly reflects the conditional probability. If the event A were a rare event such as being infected by the Corona virus, then the result would be different than the accuracy rate.</td>
<td>Answer of Q2: (Bu cevap önceden paylaşılmıştı)

To solve this question we should use the Bayes’ Rule:
$$P(A|B) = P(B|A) \times \frac{P(A)}{P(B)}.$$
In this question, we are asked $P(A|B) = P(European\ anc.|+test)$. Event A is “European lineage” and event B is a positive test result. Therefore,
$$P(A) = 0{,}3.$$
$$P(B|A) = 0{,}95.$$
$$P(B) = P(B|A)P(A) + P(B|not\ A)P(not\ A)$$
$$= 0{,}95 \times 0{,}3 + 0{,}05 \times (1 - 0{,}3) = 0{,}32$$
As a consequence,
$$P(A|B) = 0{,}95 \times \frac{0{,}3}{0{,}32} \approx 0{,}89.$$</td>
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