END4400 – System Dynamics

Week 3 – 23/3/2021

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Outline

- Structure and Behavior of Dynamic Systems
  - Principles of Systemic Feedback Approach
  - Stocks and Flows
- Basic Stock – Flow Dynamics
- Simple Positive and Negative Feedback Loops, Doubling Time/Half Life

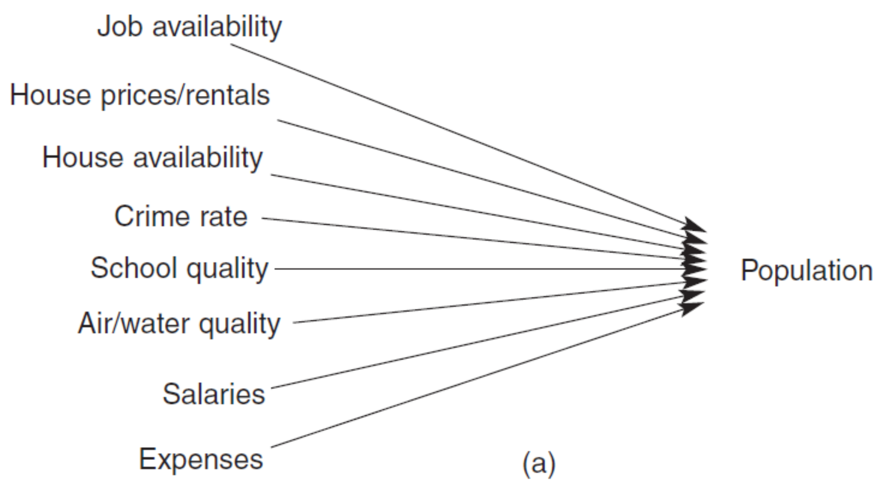
Principles of Systemic Feedback Approach

- **Principle 3: Internal Structure as the Main Cause of Dynamic Behavior**
- structure → totality of the relationships that exist between system variables
- the interaction between feedback loops and variables drive the (undesired) behavior
  - "the structure causes the behavior of the system"
  - understand the causes of an undesirable behavior and try to improve it

Principles of Systemic Feedback Approach

- e.g., population dynamics of a city → a strong early growth followed by stagnation
  - a growth-and-decline type behavior
- a static and exogenous model:

Population = f(Job availability, Salaries, Expenses, House prices, House availability, Crime rate, School quality, Air/water quality. . .)



(a)

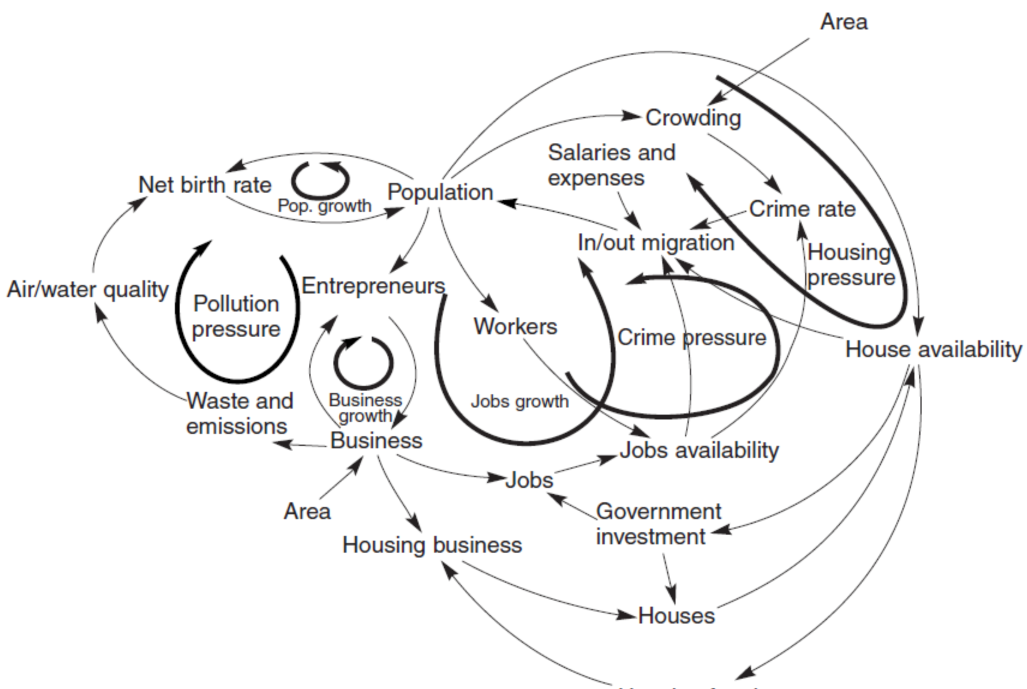
Principles of Systemic Feedback Approach

Population = f(Job availability, Salaries, Expenses, House prices, House availability, Crime rate, School quality, Air/water quality. . .)

- strict assumptions:
  - all inputs are independent (are not influenced by each other or by the output - *population*)
- obviously not a causal model
- however, it may perfectly predict the population for a given set of inputs!
- cannot understand the reasons behind the behavior of the population

Principles of Systemic Feedback Approach

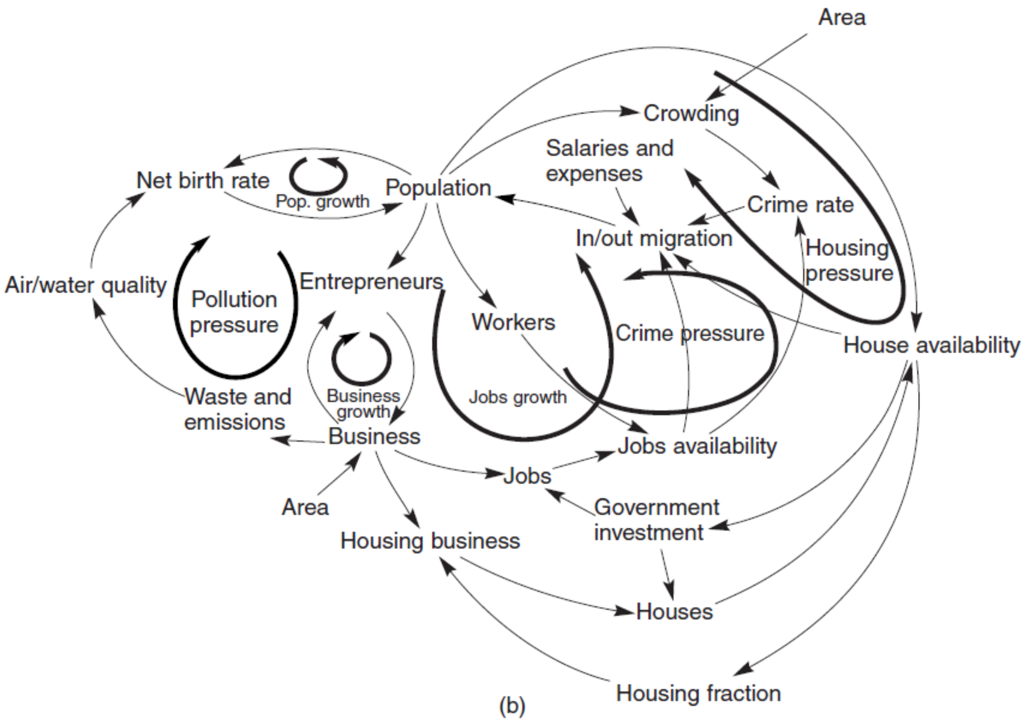
- dynamic feedback model
- no clear distinction between inputs and outputs
  - “jobs availability”



(b)

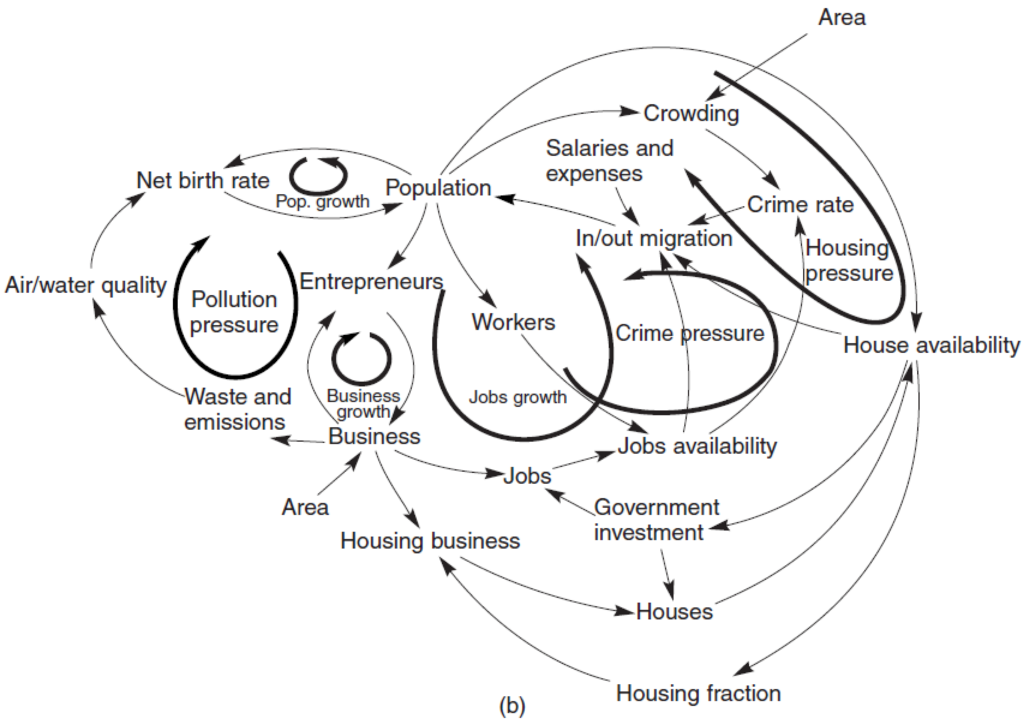
- population → entrepreneurs → business → jobs → jobs availability → in/out migration → population
- “jobs growth” is responsible for the initial increase in the population

Principles of Systemic Feedback Approach



- population → entrepreneurs → business → waste and emissions → air/water quality → net birth rate → population
- "pollution pressure" is one of the loops responsible for the decline after growth

Principles of Systemic Feedback Approach



- endogenous theory about the problem
- dynamics are internally determined

Structure and Behavior of Dynamic Systems

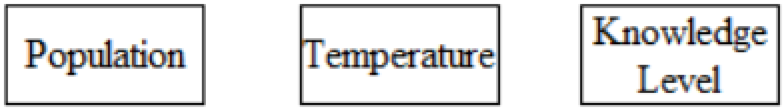
Stocks and Flows

Stocks and Flows

- In system dynamics models, it is essential to distinguish between two types of variables; *stocks and flows.*
- Stocks represent accumulations over time.
  - values → "levels" of accumulations
  - stocks are also called "states" and "state variables"
  - they (collectively) represent the state of the system at time t
  - they are altered by their "inflows" and "outflows"

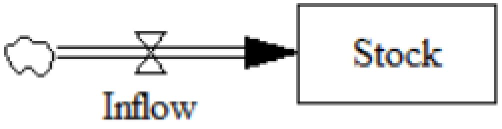
Stocks

- examples for stocks:
  - population
  - inventory of goods
  - temperature
  - blood glucose level
  - knowledge level
  - workforce
- can be physical (e.g., population) or information (e.g., knowledge level) entities
- stocks are represented with a rectangle in stock-flow diagrams (similar to a "container")

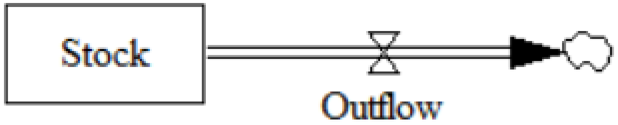


Flows

- they flow in and out of stocks (and thus, chance the values of stocks in this way)
- they represent "the rate of change" of stocks
  - "how stocks change over time?"
  - the values of stocks are changed by their inflows and outflows
- flows are represented by an arrow and a valve (in a SF diag)
  - arrow → direction of the flow (inflow/outflow)
  - valve → "regulator" of the flow
- inflow(s) are pipes pointing into the stock



- outflow(s) are pipes pointing out of the stock

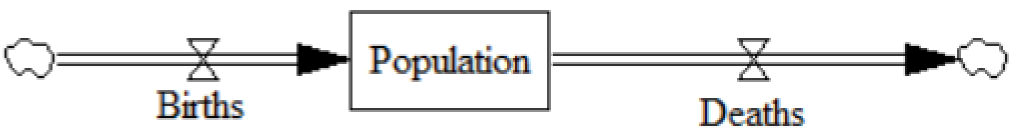


Flows

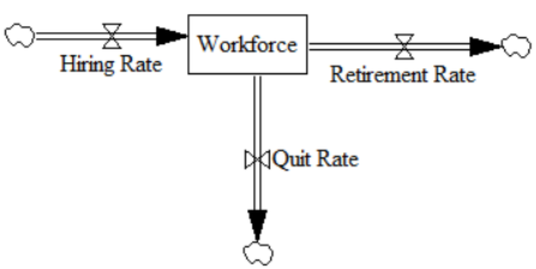
- examples of flows:
  - births, deaths (stock: population)
  - production, sales (stock: inventory of goods)
  - heat in, heat out (stock: temperature)
  - glucose intake, glucose consumption (stock: blood glucose level)
  - learning, forgetting (stock: knowledge level)
  - hiring rate, quit rate, retirement rate (stock: workforce)

Stocks and Flows

- stock: *population*
- flows: *births*, *deaths*
  - inflow: *births* (*fills in* the population stock)
  - outflow: *deaths* (*drains* the population stock)



- there can be more than one inflow and outflow of a stock!
  - e.g., workforce; hiring (inflow), quit rate, retirement rate (outflows)

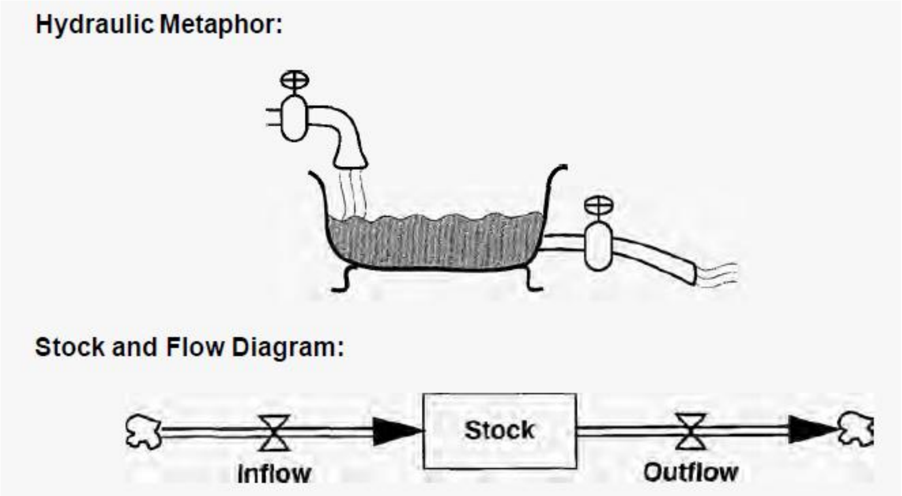


Stocks and Flows

- Summary;
  - Stocks are quantities of material or other accumulations. They are the states of the system.
  - The flows are the rates at which these system states change.

Stocks and Flows

- How to identify stocks and flows?
  - use "bathtub" metaphor
  - water in the bathtub → stock
  - "water flowing in through water tap" and "water flowing out through drainer" → flows



Stocks and Flows

- Distinguishing stocks and flows:
  - units of measure
  - snapshot (freeze) test

- units of measure
  - item, people, TL, … → stocks
  - item/hour, people/day, TL/month, … → flows
    - flows show the rate at which stocks change!

Stocks and Flows

- snapshot (freeze) test
  - freeze time and take a "snapshot" of the system
  - stocks persist but flows "disappear"
  - e.g., population
    - freeze time → you can "count" the number of people, but cannot identify births and deaths
  - e.g., bathtub
    - freeze time → you can "measure" the volume of the water, but cannot say anything about the (net) rate of change
  - the reason is that flows are defined as *units/time* while stocks are defined as *units*
  - not all of the variables that pass the snapshot test are stocks!
    - auxiliary variables (exogenous inputs, functions of stocks)

Stocks and Flows

- How to mathematically represent stocks and flows?
  - SD models are a set of differential equations

Stock and Flow Diagram:

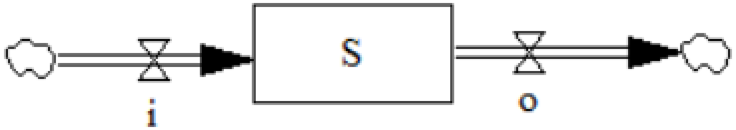
Integral Equation:

$$\text{Stock}(t) = \int_{t_0}^{t} [\text{Inflow}(s) - \text{Outflow}(s)]ds + \text{Stock}(t_0)$$

Differential Equation:

$$d(\text{Stock})/dt = \text{Net Change in Stock} = \text{Inflow}(t) - \text{Outflow}(t)$$

Stocks and Flows



- differential equation

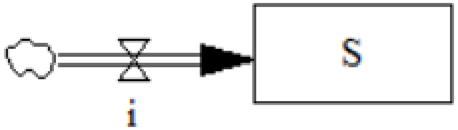
$$\frac{dS(t)}{dt} = i(t) - o(t) \quad \Rightarrow \quad \frac{dS}{dt} = i - o$$

- integral equation

$$S(t) = S(0) + \int_0^t \big(i(s) - o(s)\big)ds$$

Basic Stock – Flow Dynamics

- consider an SD model with one stock and one constant inflow;

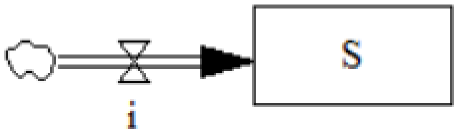


- differential equation;

- integral equation;

Basic Stock – Flow Dynamics

- consider an SD model with one stock and one constant inflow;



- differential equation;

$$\frac{dS}{dt} = i$$

- integral equation;

$$S(t) = S(0) + \int_0^t i(s)ds$$

Basic Stock – Flow Dynamics

- think about the differential equation (one more time)

$$\frac{dS}{dt} = i$$

  - order → highest order of any differential
    - first order
  - ordinary/partial → derivatives with respect to one or more variables
    - ordinary
  - linear/nonlinear → higher powers of the function (dependent variable) or its differentials
    - linear
  - homogenous/heterogeneous → homogeneous if it contains no non-differential terms
    - heterogeneous

https://users.aber.ac.uk/ruw/teach/260/classification.php

Basic Stock – Flow Dynamics

- $\frac{dS}{dt} = i$ is analytically solvable.
  - solution;

$$\frac{dS}{dt} = i$$

$$dS = i dt$$

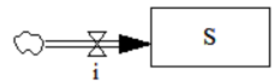
$$\int dS = \int i dt$$

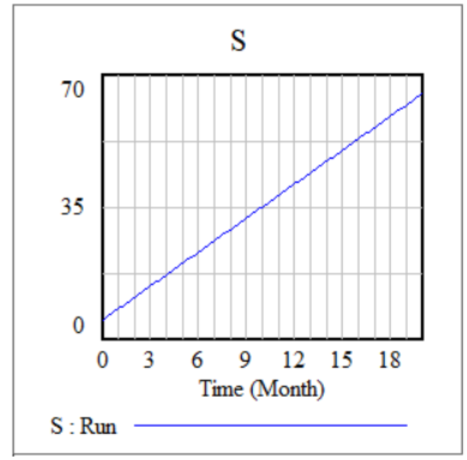
$$S = it + C$$

- S is linearly increasing with respect to *t*
- to find C, we need to know S(0)

Basic Stock – Flow Dynamics

- Assume S(0) = 5 and i = 3
- S(20) = ?





$$S(t) = it + C$$

$$S(0) = i * 0 + C$$

$$S(0) = C = 5$$

$$S(t) = 3t + 5$$

$$S(20) = 3 * 20 + 5 = 65$$

Table Time Down

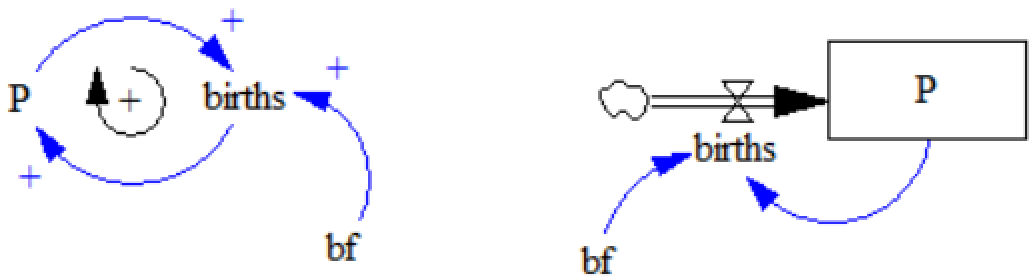
Time (Month	"S" Runs:	S
18	Run	59
18.125		59.375
18.25		59.75
18.375		60.125
18.5		60.5
18.625		60.875
18.75		61.25
18.875		61.625
19		62
19.125		62.375
19.25		62.75
19.375		63.125
19.5		63.5
19.625		63.875
19.75		64.25
19.875		64.625
20		65

Basic Stock – Flow Dynamics

- consider an SD model with a positive feedback loop
- assume that *births* are proportional to the population (*P*)

$$births = P * bf$$

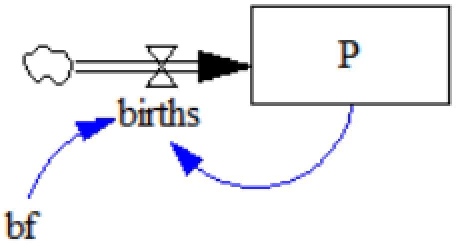
  - *bf*: birth fraction



- units:
  - *P*: people
  - *births*: people/time
  - *bf*: 1/time

Basic Stock – Flow Dynamics

- write down the differential equation



Basic Stock – Flow Dynamics

- Homework:
  - write down the differential and integral equations for the previous stock – flow diagram
  - solve the differential equation and try to guess the dynamics
  - wait for the reading assignment
- Reference

Barlas, Y. "System Dynamics: Systemic Feedback Modeling for Policy Analysis" in Knowledge for Sustainable Development - An Insight into the Encyclopedia of Life Support Systems, UNESCO-EOLSS Publishers, Paris, Oxford, UK. 2002, pp.1131-1175.

Sterman, J. Business Dynamics. Systems Thinking and Modeling for a Complex World. McGraw-Hill, U.S.A., 2000.