Static Equilibrium

The Conditions for Equilibrium

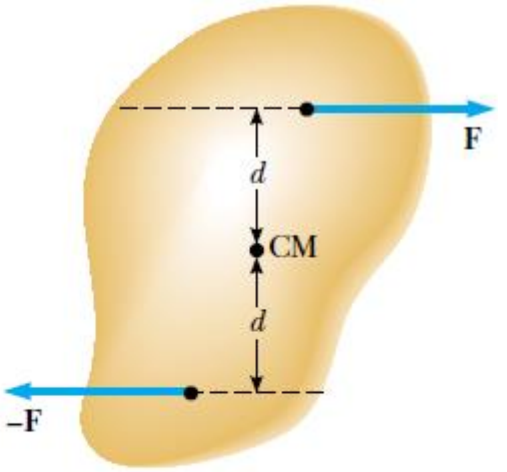
1. The resultant external force must equal zero:

$$\sum \mathbf{F} = 0$$

2. The resultant external torque about *any* axis must be zero:

$$\sum \boldsymbol{\tau} = 0$$

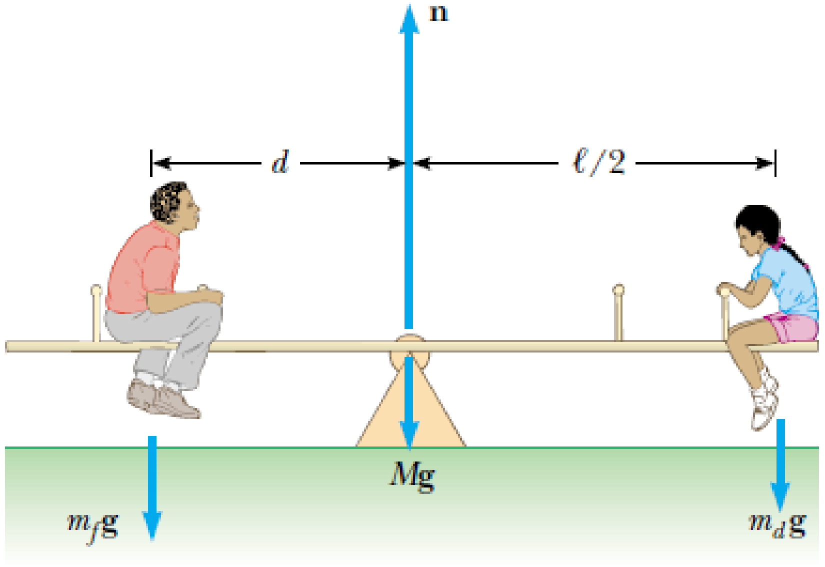
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau_z = 0$$



Ex/ A seesaw consisting of a uniform board of mass M and length ℓ supports a father and daughter with masses m_f and m_d, respectively, as shown in Figure. The support (called the *fulcrum*) is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance $\ell/2$ from the center.

(A) Determine the magnitude of the upward force **n** exerted by the support on the board.

(B) Determine where the father should sit to balance the system.



(A)

$$\Sigma F_y = 0,$$

$$n - m_f g - m_d g - Mg = 0$$

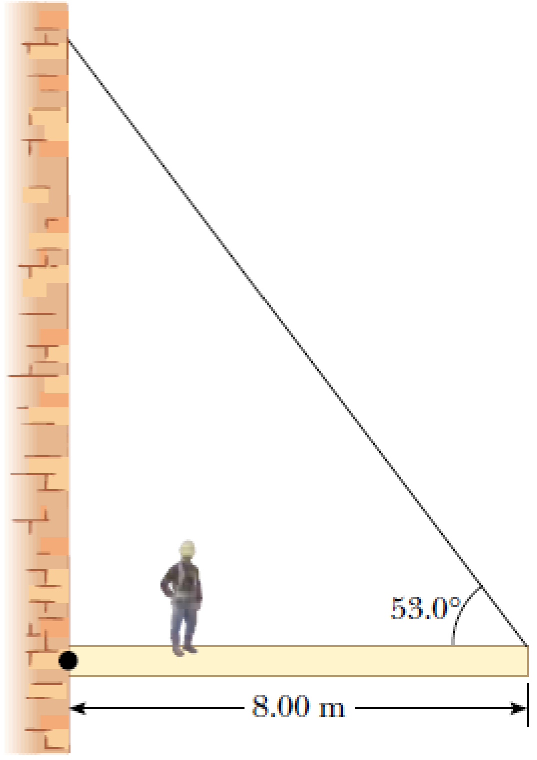
$$n = m_f g + m_d g + Mg$$

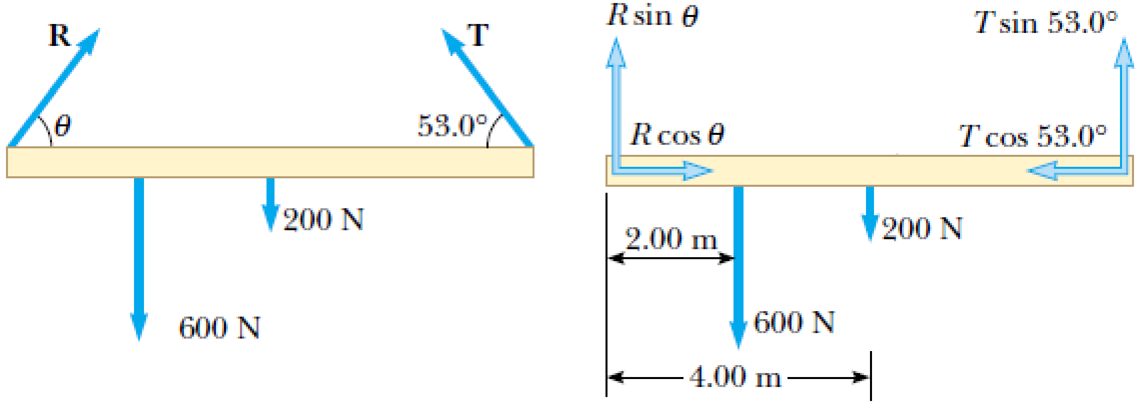
(B) $$\Sigma \tau = 0$$

$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell$$

Ex/ A uniform horizontal beam with a length of 8 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam. If a 600-N person stands 2 m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.





$$\sum F_x = R\cos\theta - T\cos 53.0^\circ = 0$$

$$\sum F_y = R\sin\theta + T\sin 53.0^\circ - 600\text{ N} - 200\text{ N} = 0$$

$$\sum \tau = (T\sin 53.0^\circ)(8.00\text{ m}) - (600\text{ N})(2.00\text{ m}) - (200\text{ N})(4.00\text{ m}) = 0$$

$$T = \boxed{313\text{ N}}$$

$$R\cos\theta = 188\text{ N}$$

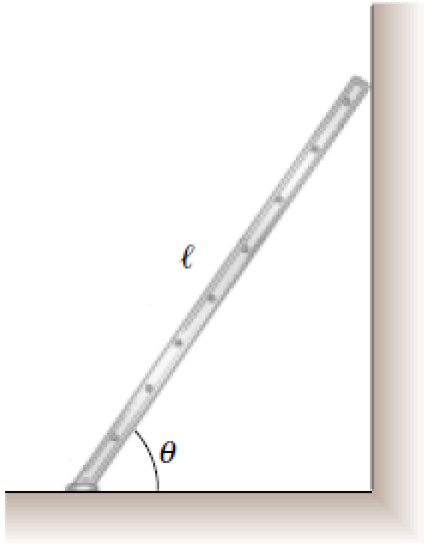
$$R\sin\theta = 550\text{ N}$$

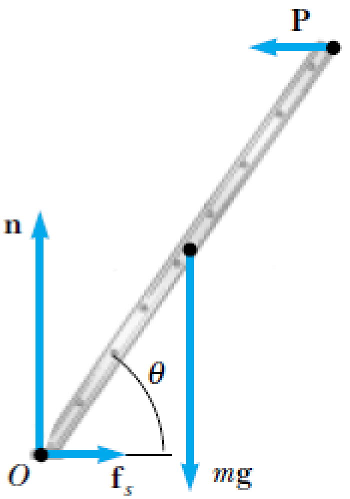
$$\tan\theta = \frac{550\text{ N}}{188\text{ N}} = 2.93$$

$$\theta = \boxed{71.1^\circ}$$

$$R = \frac{188\text{ N}}{\cos\theta} = \frac{188\text{ N}}{\cos 71.1^\circ} = \boxed{580\text{ N}}$$

Ex/ A uniform ladder of length ℓ rests against a smooth, vertical wall. If the mass of the ladder is *m* and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$, find the minimum angle θ_{min} at which the ladder does not slip.





$$\sum F_x = f_s - P = 0$$
$$\sum F_y = n - mg = 0$$

$$P = f_s. \qquad n = mg.$$

$$f_{s,\,\max} = \mu_s n.$$

$$P = f_s = \mu_s n = \mu_s mg.$$

$$\sum \tau_O = P\ell \sin\theta - mg\frac{\ell}{2}\cos\theta = 0$$

$$\tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s} = 1.25$$

$$\theta_{\min} = 51°$$