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Research Article

Fuzzy Γ-Hyperideals in Γ-Hypersemirings by Using Triangular Norms

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The concept of Γ-semihyperrings was introduced by Dehkordi and Davvaz as a generalization of semirings, semihyperrings, and Γ-semiring. In this paper, by using the notion of triangular norms, we define the concept of triangular fuzzy sub-Γ-semihyperrings as well as triangular fuzzy Γ-hyperideals of a Γ-semihyperring, and we study a few results in this respect.

1. Introduction

In [1], Nobusawa introduced Γ-rings as a generalization of ternary rings. Let M be an additive group whose elements are denoted by $a, b, c, \ldots$ and Γ another additive group whose elements are $\gamma, \beta, \alpha, \ldots$. Suppose that $a\gamma b$ is defined to be an element of M and that $\gamma a\beta$ is defined to be an element of Γ for every a, b, γ, and β. If the products satisfy the following three conditions: (1) $(a_1 + a_2)\gamma b = a_1\gamma b + a_2\gamma b$, $a(\gamma_1 + \gamma_2)b = a\gamma_1 b + a\gamma_2 b$, $a\gamma(b_1 + b_2) = a\gamma b_1 + a\gamma b_2$; (2) $(a\gamma b)\beta c = a\gamma(b\beta c) = a(\gamma b\beta)c$; (3) if $a\gamma b = 0$ for any a and b in M, then $\gamma = 0$; then M is called a Γ-ring in the sense of Nobusawa [1]. Barnes [2] weakened slightly the conditions in the definition of Γ-ring and gave a new definition of a Γ-ring. Let M and Γ be two additive abelian groups. Suppose that there is a mapping from $M \times \Gamma \times M \to M$ (sending (a, γ, b) into $a\gamma b$ such that (1) $(a_1 + a_2)\gamma b = a_1\gamma b + a_2\gamma b$, $a(\gamma_1 + \gamma_2)b = a\gamma_1 b + a\gamma_2 b$, $a\gamma(b_1 + b_2) = a\gamma b_1 + a\gamma b_2$; (2) $(a\gamma b)\beta c = a\gamma(b\beta c)$; then M is called a Γ-ring in the sense of Barnes [2]. Nowadays, Γ-rings mean the Γ-rings due to Barnes and other Γ-rings are known as Γ_N-rings, that is, gamma rings in the sense of Nobusawa. Barnes [2], Luh [3], and Kyuno [4] studied the structure of Γ-rings and obtained various generalization analogous to corresponding parts in ring theory. The notion of Γ-semirings was introduced by Rao [5] as a generalization of semirings as well as Γ-rings. Subsequently, by introducing the notion of operator semirings of a Γ-semiring, Dutta and Sardar [6] enriched the theory of Γ-semirings. Algebraic hyperstructures represent a natural extension of classical algebraic structures and they were introduced by the French mathematician Marty [7]. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic structure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, for example, see [8–10]. In [11, 12], Dehkordi and Davvaz studied the notion of a Γ-semihyperring as a generalization of semiring, semihyperring, and Γ-semiring.

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets have been introduced by Zadeh (1965) as an extension of the classical notion of sets [13]. Let X be a set. A fuzzy subset A of X is characterized by a membership function $\mu_A : X \to [0, 1]$ which associates with each point $x \in X$ its grade or degree of membership $\mu_\mu(x) \in [0, 1]$. Fuzzy sets generalize classical sets since the characteristic functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. After the introduction of fuzzy sets by Zadeh, reconsideration of the concept of classical mathematics began. In 1971, Rosenfeld [14] introduced fuzzy sets in the context of group theory and formulated

the concept of a fuzzy subgroup of a group. Das characterized fuzzy subgroups by their level of subgroups in [15], since then many notions of fuzzy group theory can be equivalently characterized with the help of notion of level subgroups. The concept of a fuzzy ideal of a ring was introduced by Liu [16]. In 1992, Jun and Lee [17] introduced the notion of fuzzy ideals in Γ-rings and studied a few properties. In [6], Dutta and Sardar studied the structures of fuzzy ideals of Γ-rings. Also, see [18]. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. In [19], Davvaz introduced the concept of fuzzy H_v-ideals of H_v-rings. Then, this concept was studied in depth in several papers, for example, see [20]. Also, see [21]. In [22, 23], Ersoy and Davvaz investigated some properties of fuzzy Γ-hyperideals of Γ-semihyperring. Now, in this paper, we define the concept of triangular fuzzy sub-Γ-semihyperrings and fuzzy Γ-hyperideals of a Γ-semihyperring by using triangular norms, and we study a few results in this respect.

2. Basic Concepts

Let H be a nonempty set and let $\wp^*(H)$ be the set of all nonempty subsets of H. A hyperoperation on H is a map $\circ : H \times H \to \wp^*(H)$ and the couple $(H, \circ)$ is called a hypergroupoid. If A and B are nonempty subsets of H, then we denote $A \circ B = \bigcup_{a\in A, b\in B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$. A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all x, y, z of H we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that $\bigcup_{u\in x\circ y} u \circ z = \bigcup_{v\in y\circ z} x \circ v$. A semihyperring is an algebraic structure $(R, +, \cdot)$ which satisfies the following properties: (1) $(R, +)$ is a commutative semihypergroup; that is, $(x + y) + z = x + (y + z)$ and $x + y = y + x$ for all $x, y, z \in R$; (2) $(R, \cdot)$ is a semihypergroup; (3) the hyperoperation $\cdot$ is distributive with respect to the hyperoperation $+$; that is, $x \cdot (y + z) = x \cdot y + x \cdot z, (x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$; (4) the element $0 \in R$ is an absorbing element; that is $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$. A semihyperring $(R, +, \cdot)$ is called commutative if and only if $a \cdot b = b \cdot a$ for all $a, b \in R$. Vougiouklis in [24] studied the notion of semihyperrings in a general form; that is, both the sum and product are hyperoperations; also see [25]. A semihyperring $(R, +, \cdot)$ with identity $1_R \in R$ means that $1_R \cdot x = x \cdot 1_R = x$ for all $x \in R$. The concept of Γ-semihyperring is introduced and studied by Dehkordi and Davvaz [11]. Let R be a commutative semihypergroup and Γ be a commutative group. Then, R is called a Γ-semihyperring if there exists a map $R \times \Gamma \times R \to \wp^*(R)$ (image to be denoted by $a\alpha b$ for $a, b \in R$ and $\alpha \in \Gamma$) and $\wp^*(R)$ is the set of all nonempty subsets of R satisfying the following conditions:

(1) $a\alpha(b + c) = a\alpha b + a\alpha c$,

(2) $(a + b)\alpha c = a\alpha c + b\alpha c$,

(3) $a(\alpha + \beta)b = a\alpha b + a\beta b$,

(4) $a\alpha(b\beta c) = (a\alpha b)\beta c$,

for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$. One can find many examples of Γ-semihyperrings in [11, 12]. In the above definition if R is a semigroup, then R is called a multiplicative Γ-semihyperring. A Γ-semihyperring R is called commutative if $x\alpha y = y\alpha x$ for every $x, y \in R$ and $\alpha \in \Gamma$. We say that Γ-semihyperring R with zero if there exists $0 \in R$ such that $a \in a + 0$, $0 \in 0\alpha a, 0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$. Let A and B be two nonempty subsets of Γ-semihyperring R and $x \in R$. We define $A + B = \{x \mid x \in a + b;\ a \in A,\ b \in B\}$ and $A\Gamma B = \{x \mid x \in a\alpha b; a \in A,\ b \in B,\ \alpha \in \Gamma\}$. A nonempty subset R_1 of Γ-semihyperring R is called a sub-Γ-semihyperring if it is closed with respect to the multiplication and addition. In other words, a nonempty subset R_1 of Γ-semihyperring R is a sub-Γ-semihypergroup if $R_1 + R_1 \subseteq R_1$ and $R_1\Gamma R_1 \subseteq R_1$.

A right (left) Γ-hyperideal I of a Γ-semihyperring R is an additive sub-semihypergroup $(R, +)$ such that $I\Gamma R \subseteq I$ $(R\Gamma I \subseteq I)$. If I is both right and left Γ-hyperideal of R, then we say that I is a two-sided Γ-hyperideal or simply a Γ-hyperideal of R. In [22, 23], Ersoy and Davvaz studied fuzzy Γ-hyperideals of Γ-semihyperrings. We recall the notion of a fuzzy Γ-hyperideal of a Γ-semihyperring. Let R be a Γ-semihyperring and μ be a fuzzy subset of R. Then (1) μ is called a fuzzy left Γ-hyperideal of R if $\min\{\mu(x), \mu(y)\} \leq \inf_{z\in x+y}\{\mu(z)\}$ and $\mu(y) \leq \inf_{z\in x\gamma y}\{\mu(z)\}$ for all $x, y \in R$ and $\gamma \in \Gamma$; (2) μ is called a fuzzy right Γ-hyperideal of R if $\min\{\mu(x), \mu(y)\} \leq \inf_{z\in x+y}\{\mu(z)\}$ and $\mu(x) \leq \inf_{z\in x\gamma y}\{\mu(z)\}$ for all $x, y \in R$ and $\gamma \in \Gamma$; (3) μ is called a fuzzy Γ-hyperideal of R if it is both a fuzzy left Γ-hyperideal and a fuzzy right Γ-hyperideal of R.

The concept of a triangular norm was introduced by Menger [26] in order to generalize the triangular inequality of a metric. The current notion of a t-norm and its dual operation is due to Schweizer and Sklar [27]. By a t-norm T we mean a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions: (T1) $T(x, y) = T(y, x)$; (T2) $T(x, T(y, z)) = T(T(x, y), z)$; (T3) $T(x, y) \leq T(x, z)$ if $y \leq z$; (T4) $T(x, 1) = x$, for all $x, y, z \in [0, 1]$. For every t-norm T, we set $\Delta_T = \{x \in [0, 1], T(x, x) = x\}$. A t-norm on $[0, 1]$ is called a continuous t-norm if T is a continuous function from $[0, 1] \times [0, 1]$ to $[0, 1]$ with respect to the usual topology. Note that the function "Min" is a continuous t-norm. A triangular conorm (t-norm for short) is a binary operation S on the unit interval $[0, 1]$, that is, a function $S : [0, 1] \times [0, 1] \to [0, 1]$, which for all $x, y, z \in [0, 1]$ satisfies (T1)–(T3) and (S4) $S(x, 0) = x$. From an axiomatic point of view, t-norms and t-conorms differ only with respect to their respective boundary conditions. In fact, the concepts of t-norms and t-conorms are dual in some sense. Anthony and Sherwood [28] redefined a fuzzy subgroup of a group by using the notion of t-norm.

3. T-Fuzzy Sub-Γ-Semihyperrings and T-Fuzzy Γ-Hyperideals

In this section, we define the notion of T-fuzzy sub-Γ-semihyperrings and T-fuzzy Γ-hyperideals of a Γ-semihyperring and we study some of their properties. Let T be a t-norm and μ be a fuzzy subset of a Γ-semihyperring R. Then, we say μ has imaginable property if $\operatorname{Im}\mu \subseteq \Delta_T$.

Definition 1. Let R be a Γ-semihyperring, T be a t-norm, and μ be a fuzzy subset of R. Then, μ is called a T-fuzzy sub-Γ-semihyperring of R if

(1) $T(\mu(x), \mu(y)) \leq \inf_{z \in x+y}\{\mu(z)\}$,

(2) $T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y}\{\mu(z)\}$,

for all $x, y \in R$ and for all $\gamma \in \Gamma$.

A T-fuzzy sub-Γ-semihyperring of R is said to be imaginable if it satisfies the imaginable property. Clearly, if R is a Γ-semiring, then μ is a T-fuzzy sub-Γ-semiring of R when

$(1')$ $T(\mu(x), \mu(y)) \leq \mu(x + y)$,

$(2')$ $T(\mu(x), \mu(y)) \leq \mu(x\gamma y)$,

for all $x, y \in R$ and for all $\gamma \in \Gamma$.

Example 2. Suppose that $R = \mathbb{N}$, the set of natural numbers, and Γ is a nonempty subset of R. For any $x, y \in R$ and $\gamma \in \Gamma$, we define $x + y = \{x, y\}$ and $x\gamma y = \{x, \gamma, y\}$. Then, R is a Γ-semihyperring. We define the fuzzy subset μ of R by

$$\mu(x) = \begin{cases} \frac{3}{4} & \text{if } x \in \Gamma \\ \frac{5}{9} & \text{otherwise} \end{cases} \tag{1}$$

and we consider the t-norm $T(r, s) = rs/(2 - (r + s - rs))$, where $r, s \in [0, 1]$. Then, for any $x, y \in R$ and $\gamma \in \Gamma$, we have

$$\begin{aligned} \inf_{z \in x+y}\{\mu(z)\} &= \min\{\mu(x), \mu(y)\} \\ &= \begin{cases} \frac{3}{4} & \text{if } x, y \in \Gamma \\ \frac{5}{9} & \text{otherwise,} \end{cases} \\ \inf_{z \in x\gamma y}\{\mu(z)\} &= \min\{\mu(x), \mu(\gamma), \mu(y)\} = \frac{5}{9}. \end{aligned} \tag{2}$$

On the other hand, we have the following cases:

(1) $x, y \in \Gamma$,

(2) $x \notin \Gamma$ and $y \in \Gamma$ (or, $x \in \Gamma$ and $y \notin \Gamma$),

(3) $x, y \notin \Gamma$.

Regarding the above cases, we have $T(3/4, 3/4) = 9/17$, $T(3/4, 5/9) = 15/40$, and $T(5/9, 5/9) = 25/81$. Thus, in every case, we obtain

$$\begin{aligned} T(\mu(x), \mu(y)) &\leq \inf_{z \in x+y}\{\mu(z)\}, \\ T(\mu(x), \mu(y)) &\leq \inf_{z \in x\gamma y}\{\mu(z)\}. \end{aligned} \tag{3}$$

Therefore, μ is a T-fuzzy sub-Γ-semihyperring of R.

Lemma 3. *Let R be a Γ-semihyperring, T be a t-norm, and μ be a T-fuzzy sub-Γ-semihyperring of S. Then*

$$\begin{aligned} T_n(\mu(x_1), \ldots, \mu(x_n)) &\leq \inf_{z \in x_1 + \cdots + x_n}\{\mu(z)\}, \\ T_n(\mu(x_1), \ldots, \mu(x_n)) &\leq \inf_{z \in x_1\gamma \cdots \gamma x_n}\{\mu(z)\}, \end{aligned} \tag{4}$$

for all $x_1, \ldots, x_n \in R$ and $\gamma \in \Gamma$, where

$$T_n(t_1, \ldots, t_n) = \begin{cases} t_1 & \text{if } n = 1 \\ T(t_1, t_2) & \text{if } n = 2 \\ T(t_i, T_{n-1}(t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)) & \text{if } n > 2. \end{cases} \tag{5}$$

Proof. The proof is straightforward by mathematical induction. □

Lemma 4. *Let R be a Γ-semihyperring, T be a t-norm, and μ be a T-fuzzy sub-Γ-semihyperring of S. Let A and B be nonempty subsets of R. Then*

$$\begin{aligned} T\left(\inf_{a \in A}\{\mu(a)\}, \inf_{b \in B}\{\mu(b)\}\right) &\leq \inf_{z \in A+B}\{\mu(z)\}, \\ T\left(\inf_{a \in A}\{\mu(a)\}, \inf_{b \in B}\{\mu(b)\}\right) &\leq \inf_{z \in A\gamma B}\{\mu(z)\}, \end{aligned} \tag{6}$$

for all $\gamma \in \Gamma$.

Proof. The proof is straightforward. □

Theorem 5. *Let R be a Γ-semihyperring, T be a t-norm, and μ be a fuzzy subset of S with imaginable property and b the maximum of $\operatorname{Im}\mu$. Then, the following two statements are equivalent:*

(1) *μ is a T-fuzzy sub-Γ-semihyperring of S,*

(2) *$\mu^{-1}[a, b]$ is a sub-Γ-semihyperring of S whenever $a \in \Delta_T$ and $0 < a \leq b$.*

Proof. (1) $\Rightarrow$ (2): Suppose that $a \in \Delta_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a, b]$, then $\inf_{z \in x+y}\{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$, which implies that $x + y \subseteq \mu^{-1}[a, b]$. Similarly, assume that $a \in \Delta_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a, b]$ and $\gamma \in \Gamma$, then $\inf_{z \in x\gamma y}\{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$. Then, we have $x\gamma y \subseteq \mu^{-1}[a, b]$, and so $\mu^{-1}[a, b]$ is a sub-Γ-semihyperring of R.

(2) $\Rightarrow$ (1): Suppose that $x, y \in S$ and $\gamma \in \Gamma$. Since $\operatorname{Im}\mu \subseteq \Delta_T$, both $\mu(x)$ and $\mu(y)$ are in Δ_T. Now, we have

$$\begin{aligned} &T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y))) \\ &\quad = T(T(\mu(x), T(\mu(y), \mu(x))), \mu(y)) \\ &\quad = T(T(\mu(x), T(\mu(x), \mu(y))), \mu(y)) \\ &\quad = T(T(\mu(x), \mu(x)), T(\mu(y), \mu(y))) \\ &\quad = T(\mu(x), \mu(y)), \end{aligned} \tag{7}$$

and so $T(\mu(x), \mu(y)) \in \Delta_T$. Assume that $a = T(\mu(x), \mu(y))$. If $a = 0$, then

$$\begin{aligned} T(\mu(x), \mu(y)) = 0 &\leq \inf_{z \in x+y}\{\mu(z)\}, \\ T(\mu(x), \mu(y)) = 0 &\leq \inf_{z \in x\gamma y}\{\mu(z)\}. \end{aligned} \tag{8}$$

Now, let $0 < a = T(\mu(x), \mu(y)) \leq \mu(x) \wedge \mu(y) \leq \mu(x) \leq b$. Hence $x, y \in \mu^{-1}[a, b]$, which implies that $x + y \subseteq \mu^{-1}[a, b]$ and $x\gamma y \subseteq \mu^{-1}[a, b]$. Therefore $T(\mu(x), \mu(y)) \leq \inf_{z \in x+y}\{\mu(z)\}$ and $T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y}\{\mu(z)\}$. □

Definition 6. Let R be a Γ-semihyperring, T be a t-norm, and μ be a fuzzy subset of R. Then

(1) μ is called a T-fuzzy left Γ-hyperideal of R if

$$T(\mu(x), \mu(y)) \leq \inf_{z \in x+y}\{\mu(z)\}, \quad \forall x, y \in R,$$
$$\mu(y) \leq \inf_{z \in x\gamma y}\{\mu(z)\}, \quad \forall x, y \in R, \ \forall \gamma \in \Gamma. \tag{9}$$

(2) μ is called a T-fuzzy right Γ-hyperideal of R if

$$T(\mu(x), \mu(y)) \leq \inf_{z \in x+y}\{\mu(z)\}, \quad \forall x, y \in R,$$
$$\mu(x) \leq \inf_{z \in x\gamma y}\{\mu(z)\}, \quad \forall x, y \in R, \ \forall \gamma \in \Gamma. \tag{10}$$

(3) μ is called a T-fuzzy Γ-hyperideal of R if it is both a T-fuzzy left Γ-hyperideal and a T-fuzzy right Γ-hyperideal of R.

Theorem 7. *Let R be a Γ-semihyperring, T be a t-norm, and μ be a fuzzy subset of S with imaginable property and b the maximum of $\operatorname{Im}\mu$. Then, the following two statements are equivalent:*

(1) *μ is a T-fuzzy Γ-hyperideal of R,*

(2) *$\mu^{-1}[a, b]$ is a Γ-hyperideal of R whenever $a \in \Delta_T$ and $0 < a \leq b$.*

Proof. The proof is similar to the proof of Theorem 5. □

Let μ be a fuzzy subset of R and $t \in [0, 1]$. The set $U(\mu; t) = \{x \in R \mid \mu(x) \geq t\}$ is called a level subset of μ. So, we obtain the following corollary.

Corollary 8. *Let R be a Γ-semihyperring and μ be a fuzzy subset of R. Then*

(1) *μ is a Min-fuzzy sub-Γ-semihyperring of R if and only if every nonempty level subset is a sub-Γ-semihyperring of R;*

(2) *μ is a Min-fuzzy Γ-hyperideal of R if and only if every nonempty level subset is a Γ-hyperideal of R.*

Corollary 9. *Let A be a subset of R. Then*

(1) *the characteristic function χ_A is a T-fuzzy sub-Γ-semihyperring of R if and only if A is a sub-Γ-semihyperring of R;*

(2) *the characteristic function χ_A is a T-fuzzy Γ-hyperideal of R if and only if A is a Γ-hyperidealof R.*

Theorem 10. *Let R be a Γ-semihyperring and K be a sub-Γ-semihyperring of R. Let T be the t-norm defined by $T(a, b) = \max\{0, a + b - 1\}$ and μ be a fuzzy subset of R defined by*

$$\mu(x) = \begin{cases} r & \text{if } x \in K \\ s & \text{otherwise,} \end{cases} \tag{11}$$

for all $a, b \in [0, 1]$ and $x \in R$, where $r, s \in [0, 1]$ such that $s < r$. Then, μ is a T-fuzzy sub-Γ-semihyperring of R. In particular, if $r = 1$ and $s = 0$, then μ is imaginable.

Proof. The proof is similar to the proof of Theorem 2.6 in [29]. □

Definition 11. Let R_1 and R_2 be Γ_1 and Γ_2-semihyperrings, respectively. If there exists a map $\varphi : R_1 \to R_2$ and a bijection $f : \Gamma_1 \to \Gamma_2$ such that

$$\varphi(x + y) = \{\varphi(z) \mid z \in x + y\} = \varphi(x) + \varphi(y),$$
$$\varphi(x\gamma y) = \{\varphi(z) \mid z \in x\gamma y\} = \varphi(x) f(\gamma) \varphi(y), \tag{12}$$

for all $x, y \in R_1$ and $\gamma \in \Gamma$, then we say (φ, f) is a homomorphism from R_1 to R_2. Also, if φ is a bijection then (φ, f) is called an isomorphism and R_1 and R_2 are isomorphic.

Proposition 12. *Let R_1 and R_2 be Γ_1 and Γ_2-semihyperrings, respectively. Let (φ, f) be a homomorphism from R_1 to R_2. If λ is a T-fuzzy sub-Γ-semihyperring of R_2, then $\varphi^{-1}(\lambda)$ is a T-fuzzy sub-Γ-semihyperring of R_1 too.*

Proof. Suppose that $x, y \in R_1$ and $\gamma \in \Gamma$. Then, we have

$$\begin{aligned}
\inf_{z \in x+y}\left\{\varphi^{-1}(\lambda)(z)\right\} &= \inf_{z \in x+y}\{\lambda(\varphi(z))\} \\
&\geq \inf_{\varphi(z) \in \varphi(x+y)}\{\lambda(\varphi(z))\} \\
&\geq \inf_{\varphi(z) \in \varphi(x)+\varphi(y)}\{\lambda(\varphi(z))\} \\
&\geq T(\lambda(\varphi(x)), \lambda(\varphi(y))) \\
&= T\left(\varphi^{-1}(\lambda)(x), \varphi^{-1}(\lambda)(y)\right); \\
\inf_{z \in x\gamma y}\left\{\varphi^{-1}(\lambda)(z)\right\} &= \inf_{z \in x\gamma y}\{\lambda(\varphi(z))\} \\
&\geq \inf_{\varphi(z) \in \varphi(x\gamma y)}\{\lambda(\varphi(z))\} \\
&\geq \inf_{\varphi(z) \in \varphi(x) f(\gamma) \varphi(y)}\{\lambda(\varphi(z))\} \\
&\geq T(\lambda(\varphi(x)), \lambda(\varphi(y))) \\
&= T\left(\varphi^{-1}(\lambda)(x), \varphi^{-1}(\lambda)(y)\right).
\end{aligned} \tag{13}$$

Therefore, $\varphi^{-1}(\lambda)$ is a T-fuzzy sub-Γ-semihyperring of R_1. □

Proposition 13. *Let R_1 and R_2 be Γ_1 and Γ_2-semihyperrings, respectively. Let (φ, f) be a homomorphism from R_1 to R_2. If λ is a T-fuzzy Γ-hyperideal of R_2, then $\varphi^{-1}(\lambda)$ is a T-fuzzy Γ-hyperideal of R_1 too.*

Proof. The proof is similar to the proof of Proposition 12. □

Let $\{a_i\}_{i\in I}$ and $\{b_j\}_{j\in J}$ be two sets of real numbers in $[0,1]$. Then, we say T is infinitely distributive if

$$T\left(\sup_{i\in I}\{a_i\},\sup_{j\in J}\{b_j\}\right)=\sup_{\substack{i\in I\\ j\in J}}\{T(a_i,b_j)\}. \tag{14}$$

If T is continuous, then T is infinitely distributive [30].

Lemma 14. *Let T be a continuous t-norm and $\{\mu_i\}_{i\in I}$ be a family of T-fuzzy sub-Γ-semihyperring of R. Then, $\bigcap_{i\in I}\mu_i$ is a T-fuzzy sub-Γ-semihyperring of R.*

Proof. For any $x, y\in R$ and $\gamma\in\Gamma$, we have

$$\begin{aligned}
\inf_{z\in x+y}\left\{\left(\bigcap_{i\in I}\mu_i\right)(z)\right\}&=\inf_{z\in x+y}\left\{\inf_{i\in I}\{\mu_i(z)\}\right\}\\
&=\inf_{i\in I}\left\{\inf_{z\in x+y}\{\mu_i(z)\}\right\}\\
&\geq\inf_{i\in I}\{T(\mu_i(x),\mu_i(y))\}\\
&=T\left(\inf_{i\in I}\{\mu_i(x)\},\inf_{i\in I}\{\mu_i(y)\}\right)\\
&=T\left(\left(\bigcap_{i\in I}\mu_i\right)(x),\left(\bigcap_{i\in I}\mu_i\right)(y)\right);\\
\inf_{z\in xyy}\left\{\left(\bigcap_{i\in I}\mu_i\right)(z)\right\}&=\inf_{z\in xyy}\left\{\inf_{i\in I}\{\mu_i(z)\}\right\}\\
&=\inf_{i\in I}\left\{\inf_{z\in xyy}\{\mu_i(z)\}\right\}\\
&\geq\inf_{i\in I}\{T(\mu_i(x),\mu_i(y))\}\\
&=T\left(\inf_{i\in I}\{\mu_i(x)\},\inf_{i\in I}\{\mu_i(y)\}\right)\\
&=T\left(\left(\bigcap_{i\in I}\mu_i\right)(x),\left(\bigcap_{i\in I}\mu_i\right)(y)\right).
\end{aligned}\tag{15}$$

□

Lemma 15. *Let R_1 and R_2 be Γ_1 and Γ_2-semihyperrings, respectively, and (φ, f) be an onto homomorphism from R_1 to R_2. Then, for every $t\in[0,1]$, we have $\varphi(\mu)_t=\bigcap_{t>\epsilon>0}\varphi(\mu_{t-\epsilon})$.*

Proof. The proof is similar to the proof of Lemma 3.5 in [31]. □

Proposition 16. *Let R_1 and R_2 be Γ_1 and Γ_2-semihyperrings, respectively, and let μ be a fuzzy subset of R_1. Let (φ, f) be an onto homomorphism from R_1 to R_2. If μ is a Min-fuzzy sub-Γ-semihyperring of R_1, then $\varphi(\mu)$ is a Min-fuzzy sub-Γ-semihyperring of R_2 too.*

Proof. Suppose that μ is a Max-fuzzy sub-Γ-semihyperring of R_1. By Corollary 8, $\varphi(\mu)$ is a Max-fuzzy sub-Γ-semihyperring of R_2 if every nonempty level subset $U(\varphi((\mu);t))$ is a sub-Γ-semihyperring of R_2. Thus, assume that $U(\varphi((\mu);t))$ is any nonempty level subset. If $t=0$, then $U(\varphi((\mu);t))=R_2$, and if $0<t\leq 1$, then by Lemma 15, we have $U(\varphi((\mu);t))=\bigcap_{t>\epsilon>0}\varphi(U(\mu;t-\epsilon))$. By Corollary 8, $U(\mu;t-\epsilon)$ for each $t>\epsilon>0$ is a sub-Γ-semihyperring of R_1. Hence, $\varphi(U(\mu;t-\epsilon))$ is a sub-Γ-semihyperring of R_2. By Lemma 14, $\varphi(U(\mu;t))$ being an intersection of a family of sub-Γ-semihyperrings is also a sub-Γ-semihyperring of R_2 and the proof is completed. □

Definition 17. Let R_1 and R_2 be two Γ-semihyperrings and let μ and λ be fuzzy subsets of R_1 and R_2, respectively. The product of μ and λ is defined to be the fuzzy subset $\mu\times\lambda$ of $R_1\times R_2$ with $(\mu\times\lambda)(x,y)=T(\mu(x),\lambda(x))$, for all $(x,y)\in R_1\times R_2$.

Proposition 18. *Let R_1 and R_2 be two Γ-semihyperrings and let μ and λ be fuzzy subsets of R_1 and R_2, respectively. Then*

(1) *if μ and λ are T-fuzzy sub-Γ-semihyperrings of R_1 and R_2, respectively, then $\mu\times\lambda$ is a T-fuzzy sub-Γ-semihyperring of $R_1\times R_2$;*

(2) *if μ and λ are T-fuzzy Γ-hyperideals of R_1 and R_2, respectively, then $\mu\times\lambda$ is a T-fuzzy Γ-hyperideal of $R_1\times R_2$.*

Proof. It is straightforward. □

In [12], Dehkordi and Davvaz studied Noetherian and Artinian Γ-semihyperrings in crisp case. A collection $\mathscr{A}$ of subsets of a Γ-semihyperring R satisfies the ascending chain condition (or Acc) if there does not exist a properly ascending infinite chain $A_1\subset A_2\subset\cdots$ of subsets from $\mathscr{A}$. Recall that a subset $B\in\mathscr{A}$ is a maximal element of $\mathscr{A}$ if there does not exist a subset in $\mathscr{A}$ that properly contains B. Similar to [18], in the following, we obtain some results related to fuzzy sets and Noetherian Γ-semihyperrings.

Proposition 19 (see [12]). *Let R be a Γ-semihyperring. Then, the following conditions are equivalent:*

(1) *R satisfying the Acc condition on right (left) Γ-hyperideals,*

(2) *every nonempty family of right (left) Γ-hyperideals has a maximal element,*

(3) *every right (left) Γ-hyperideals is finitely generated.*

Definition 20 (see [12]). A Γ-semihyperring R is right (left) Noetherian if the equivalent conditions of the above proposition are satisfied. In the same way, we can define an Artinian Γ-semihyperring. Let I be a Γ-hyperideal of a Γ-semihyperring R and I be a Noetherian Γ-semihyperring. Then, I is called a Noetherian Γ-hyperideal of R.

Example 21 (see [12]). Let $A_n=[n,n_{n+1}]$ for every $n\in\mathbb{Z}$, $R=\bigcup_{n\in\mathbb{Z}}A_n$, and $\Gamma=\mathbb{Z}$. Then, R is a Noetherian Γ-semihyperring with respect to the following hyperoperations:

$$x\oplus y=A_{n+m},\qquad x\gamma y=A_{n\gamma m}, \tag{16}$$

where $x\in A_n$ and $y\in A_m$.

Theorem 22. *Let $\{A_k \mid k \in \mathbb{N}\}$ be a family of Γ-hyperideals of a Γ-semihyperring R, where $A_1 \supset A_2 \supset A_3 \cdots$. Let μ be a fuzzy subset of R defined by*

$$\mu(x) = \begin{cases} \dfrac{k}{k+1} & \text{if } x \in A_k \setminus A_{k+1},\ k = 0, 1, 2, \ldots \\ 1 & \text{if } x \in \bigcap_{k=0}^{\infty} A_k, \end{cases} \tag{17}$$

for all $x \in R$, where A_0 stands for R. Let T be a t-norm with $\operatorname{Im}\mu \subseteq \Delta_T$. Then, μ is a T-fuzzy Γ-hyperideal of R.

Proof. Let $x, y \in R$. Suppose that $x \in A_k \setminus A_{k+1}$ and $y \in A_r \setminus A_{r+1}$ for $k = 0, 1, 2, \ldots$ and $r = 0, 1, 2, \ldots$. Without loss of generality we may assume that $k \le r$. Then, obviously $y \in A_k$. Since A_k is a Γ-hyperideal of R, it follows that $x + y \subseteq A_k$ and $x\alpha y \subseteq A_k$ which imply that $\inf_{z \in x+y}\mu(z) \ge k/(k+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) \ge k/(k+1) = \mu(y)$ for all $\alpha \in \Gamma$. If $x \in \bigcap_{k=0}^{\infty} A_k$ and $y \in \bigcap_{k=0}^{\infty} A_k$, then $x + y \subseteq \bigcap_{k=0}^{\infty} A_k$. Hence, $\inf_{z \in x+y}\mu(z) = 1 = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) = 1 \ge \mu(y)$. If $x \notin \bigcap_{k=0}^{\infty} A_k$ and $y \in \bigcap_{k=0}^{\infty} A_k$, then there exists $n \in N$ such that $x \in A_n \setminus A_{n+1}$. It follows that $x + y \subseteq A_n$ which implies that $\inf_{z \in x+y}\mu(z) \ge n/(n+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) \ge n/(n+1) = \mu(y)$ for all $\alpha \in \Gamma$. Finally, assume that $x \in \bigcap_{k=0}^{\infty} A_k$ and $y \notin \bigcap_{k=0}^{\infty} A_k$, then $y \in A_n \setminus A_{n+1}$ for some $n \in \mathbb{N}$. Therefore, $x + y \subseteq A_n$, and thus $\inf_{z \in x+y}\mu(z) \ge n/(n+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) \ge n/(n+1) = \mu(y)$, for all $\alpha \in \Gamma$. Hence, μ is a T-fuzzy Γ-hyperideal of R. □

Theorem 23. *Let R be a Γ-semiring satisfying descending chain condition, let μ be a fuzzy subset of R, and let T be a t-norm with $\operatorname{Im}\mu \subseteq \Delta_T$. Let μ be a T-fuzzy Γ-hyperideal of R. If a sequence of elements of $\operatorname{Im}\mu$ is strictly increasing, then μ has finite number of values.*

Proof. Let $\{t_k\}$ be a strictly increasing sequence of elements of $\operatorname{Im}\mu$. Then $0 \le t_1 \le t_2 \le \cdots \le 1$. Then, $U(\mu; t_r)$ is an ideal of M for all $r = 2, 3, \ldots$ Let $x \in U(\mu; t_r)$. Then $\mu(x) \ge t_r \ge t_{r-1}$, and so $x \in U(\mu; t_{r-1})$. Hence $U(\mu; t_r) \subseteq U(\mu; t_{r-1})$. Since $t_{r-1} \in \operatorname{Im}\mu$, there exists $x_{r-1} \in M$ such that $\mu(x_{r-1}) = t_{r-1}$. It follows that $x_{r-1} \in U(\mu; t_{r-1})$ but $x_{r-1} \notin U(\mu; t_r)$. Thus, $U(\mu; t_r) \subset U(\mu; t_{r-1})$ and so we obtain a strictly descending sequence $U(\mu; t_1) \supset U(\mu; t_2) \supset U(\mu; t_3) \supset \cdots$ of Γ-hyperideals of R which is not terminating. This contradicts the assumption that M satisfies the descending chain condition. Consequently, μ has finite number of values. □

Theorem 24. *Let R be a Γ-semiring, μ be a fuzzy subset of R, and T be a t-norm with $\operatorname{Im}\mu \subseteq \Delta_T$. Then, the following conditions are equivalent:*

(1) *R is a Noetherian Γ-semihyperring,*

(2) *the set of values of any Γ-hyperideal of R is a well-ordered subset of $[0, 1]$.*

Proof. (1) $\Rightarrow$ (2): Let μ be a T-fuzzy Γ-hyperideal of R. Suppose that the set of values of μ is not a well-ordered subset of $[0, 1]$. Then, there exists a strictly decreasing sequence $\{t_k\}$ such that $\mu(x_k) = t_k$. It follows that $U(\mu; t_1) \subset U(\mu; t_2) \subset \cdots$ is a strictly ascending chain of Γ-hyperideals of M, where $U(\mu; t_r) = \{x \in M \mid \mu(x) \ge t_r\}$, for every $r = 1, 2, \ldots$. This contradicts the assumption that R is a Noetherian Γ-semihyperring.

(2) $\Rightarrow$ (1): Suppose that the condition (2) is satisfied and R is not a Noetherian Γ-hyperring. There exists a strictly ascending chain

$$A_1 \subset A_2 \subset A_3 \subset \cdots \tag{18}$$

of Γ-hyperideals of R. Note that $A = \bigcap_{k=0}^{\infty} A_k$ is a Γ-hyperideal of R. Define a fuzzy subset in R by

$$\mu(x) = \begin{cases} \alpha & \text{if } x \notin A, \\ \dfrac{1}{r} & \text{where } r = \min\{k \in \mathbb{N} \mid x \in A_k\}. \end{cases} \tag{19}$$

We claim that μ is a T-fuzzy Γ-hyperideal of R. Let $x, y \in R$. If $x \in A_k \setminus A_{k-1}$ and $y \in A_k \setminus A_{k-1}$, then $x - y \subset A_k$ and $x\alpha y \subset A_k$. It follows that $\inf_{z \in x+y}\mu(z) = 1/k = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) \ge 1/k \ge \mu(y)$, for all $\beta \in \Gamma$. Suppose that $x \in A_k$ and $y \in A_k \setminus A_r$ for all $r < k$. Since A_k is a Γ-hyperideal of R, it follows that $x + y \subset A_k$. Hence, $\inf_{z \in x+y}\mu(z) \ge 1/k \ge 1/(k+1) \ge \mu(y)$ and so $\inf_{z \in x+y}\mu(z) \ge \min\{\mu(x), \mu(y)\} \ge T(\mu(x), \mu(y))$. Similarly, for the case $x \in A_k \setminus A_r$ and $y \in A_k$, we have $\inf_{z \in x+y}\mu(z) \ge \min\{\mu(x), \mu(y)\} \ge T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y}\mu(z) \ge \mu(y)$, for all $\beta \in \Gamma$. Thus, μ is a T-fuzzy Γ-hyperideal of R. Since the chain is not terminating, μ has a strictly descending sequence of values. This contradicts the assumption that the value set of any ideal is well-ordered. Therefore, R is a Noetherian Γ-semihyperring. □

For a family $\{\mu_\alpha \mid \alpha \in \Lambda\}$ of fuzzy subsets in R, we define the join $\vee_{\alpha \in \Lambda}\mu_\alpha$ and the meet $\vee_{\alpha \in \Lambda}\mu_\alpha$ as follows:

$$\begin{aligned} \left(\bigvee_{\alpha \in \Lambda} \mu_\alpha\right)(x) &= \sup\{\mu_\alpha(x) \mid \alpha \in \Lambda\}, \\ \left(\bigwedge_{\alpha \in \Lambda} \mu_\alpha\right)(x) &= \inf\{\mu_\alpha(x) \mid \alpha \in \Lambda\}, \end{aligned} \tag{20}$$

for all $x \in R$, where Λ is any index set.

Theorem 25. *The family of T-fuzzy Γ-hyperideals in R is a completely distributive lattice with respect to meet "$\wedge$" and join "$\vee$".*

Proof. Since $[0, 1]$ is a completely distributive lattice with respect to the usual ordering in $[0, 1]$, it is sufficient to show that $\vee_{\alpha \in \Lambda}\mu_\alpha$ and $\wedge_{\alpha \in \Lambda}\mu_\alpha$ are T-fuzzy Γ-hyperideals of R for family $\{\mu_\alpha \mid \alpha \in \Lambda\}$ of T-fuzzy Γ-hyperideals of R.

For any $x, y \in R$, we have

$$\begin{aligned}\inf_{z\in x+y}\left(\bigvee_{\alpha\in\Lambda}\mu_\alpha\right)(z) &\geq \sup\left\{T\left(\mu_\alpha(x),\mu_\alpha(y)\right) \mid \alpha\in\Lambda\right\}\\ &\geq T\left(\sup\left\{\mu_\alpha(x)\mid\alpha\in\Lambda\right\},\right.\\ &\qquad \left.\sup\left\{\mu_\alpha(y)\mid\alpha\in\Lambda\right\}\right)\\ &= T\left(\left(\bigvee_{\alpha\in\Lambda}\mu_\alpha\right)(x),\left(\bigvee_{\alpha\in\Lambda}\mu_\alpha\right)(y)\right),\\ \inf_{z\in x+y}\left(\bigwedge_{\alpha\in\Lambda}\mu_\alpha\right)(z) &\geq \inf\left\{T\left(\mu_\alpha(x),\mu_\alpha(y)\right) \mid \alpha\in\Lambda\right\}\\ &= T\left(\left(\bigwedge_{\alpha\in\Lambda}\mu_\alpha\right)(x),\left(\bigwedge_{\alpha\in\Lambda}\mu_\alpha\right)(y)\right).\end{aligned} \tag{21}$$

Now, let $x, y \in M$ and $\beta \in \Gamma$. Then

$$\begin{aligned}\inf_{z\in x\beta y}\left(\bigvee_{\alpha\in\Lambda}\mu_\alpha\right)(z) &\geq \sup\left\{\mu_\alpha(y)\mid\alpha\in\Lambda\right\}\\ &= \left(\bigvee_{\alpha\in\Lambda}\mu_\alpha\right)(y),\end{aligned} \tag{22}$$

$$\begin{aligned}\inf_{z\in x\beta y}\left(\bigwedge_{\alpha\in\Lambda}\mu_\alpha\right)(z) &\geq \inf\left\{\mu_\alpha(y)\mid\alpha\in\Lambda\right\}\\ &= \left(\bigwedge_{\alpha\in\Lambda}\mu_\alpha\right)(y).\end{aligned} \tag{23}$$

Hence, $\vee_{\alpha\in\Lambda}\mu_\alpha$ and $\wedge_{\alpha\in\Lambda}\mu_\alpha$ are T-fuzzy Γ-hyperideals of R. This completes the proof. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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