Yıldız Technical University
Civil Engineering Department
Construction Management Division

Engineering Economy

Time value of money

Time value of money

- Money makes money which is known as earning power of money.
- The change in the amount of money over a given time period is called the time value of money; it is the most important concept in engineering economy.
- Interest is the manifestation of the time value of money.
- Computationally, interest is the difference between an ending amount of money and the beginning amount.
- Interest is paid when a person or organization borrowed money. Interest is earned when a person or organization lent money.

  Interest = amount owed now- original amount
- Interest, interest period and interest rate

Simple and compound interest

- Simple interest: is calculated using the principal only, ignoring any interest accrued in preceding interest periods.
- Compound interest: the interest accrued for each interest period is calculated on the principal plus the total amount of interest accumulated in all previous periods. Thus, compound interest means interest on top of interest.

Period	Initial amount	Interest	Final amount
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$80	$1,160
3	$1,160	$80	$1,240

Period	Initial amount	Interest	Final amount
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$86.40	$1,166.40
3	$1,166.40	$93.31	$1,259.71

Simple Interest

F = Future Value

P = Present Value

i = interest ratio

n = Interest period

$$\mathbf{F = P + [(n.i.P) / 100]}$$

Compound Interest

F = Future Value

P = Present Value

i = interest ratio

n = Interest period

$$F = P * (1+i/100)^n$$

Example 1

- Mr. A borrows 100 000 Turkish Lira at %40 per year for 5 year period,
  - If the interest is calculated by using simple interest, how much will he pay at the end of 5 year interest period?
  - If the interest is compounded, how much will he pay at the end of 5 year interest period?

Example 1

- $F= P+ P.i.n$

$F= 100000+100000*0,4*5$

$F=300000$ TL

- $F= P(1+i)^n$

$F = 100000\ (1+0,4)^5$

$F= 100000*5,37824$

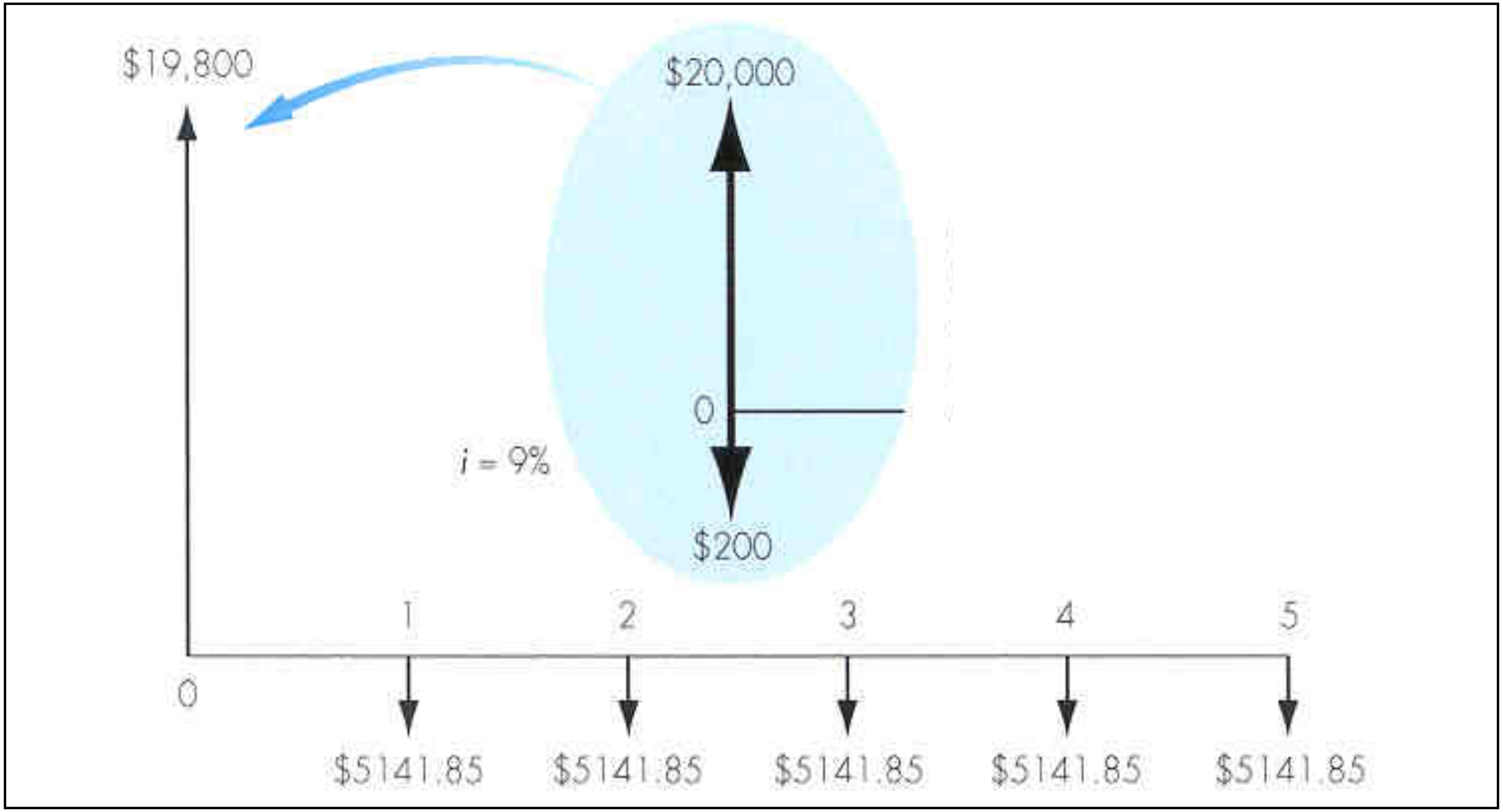
$F= 537824$ TL

Cash Flow Diagrams

- The estimated inflows (revenues) and outflows (costs) of money are called cash flows.
- The cash flow is fundamental to every economy study.
- Without cash flow estimates over a stated time period, no engineering economy study can be conducted.
- They can be considered as free body diagrams.
- Every person or company has cash receipts-revenue and income (inflows); and cash disbursements-expenses, and costs (outflows).

Cash Flow Diagram

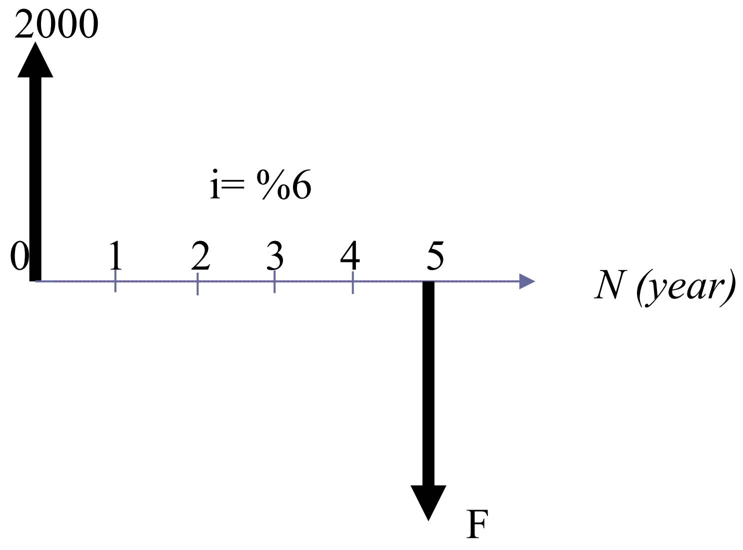
An example of cash flow diagram is shown. The inflows and outflows of the company are illustrated as arrows. The direction of the arrows indicate the direction of the money. The time period is shown on the horizontal line. Finally, the interest rate valid for this cash flow diagram is stated.



Example 2

- 2000 TL is borrowed today for 5 year period at 6% interest rate compounded yearly. Show these transactions on the cash flow diagram.

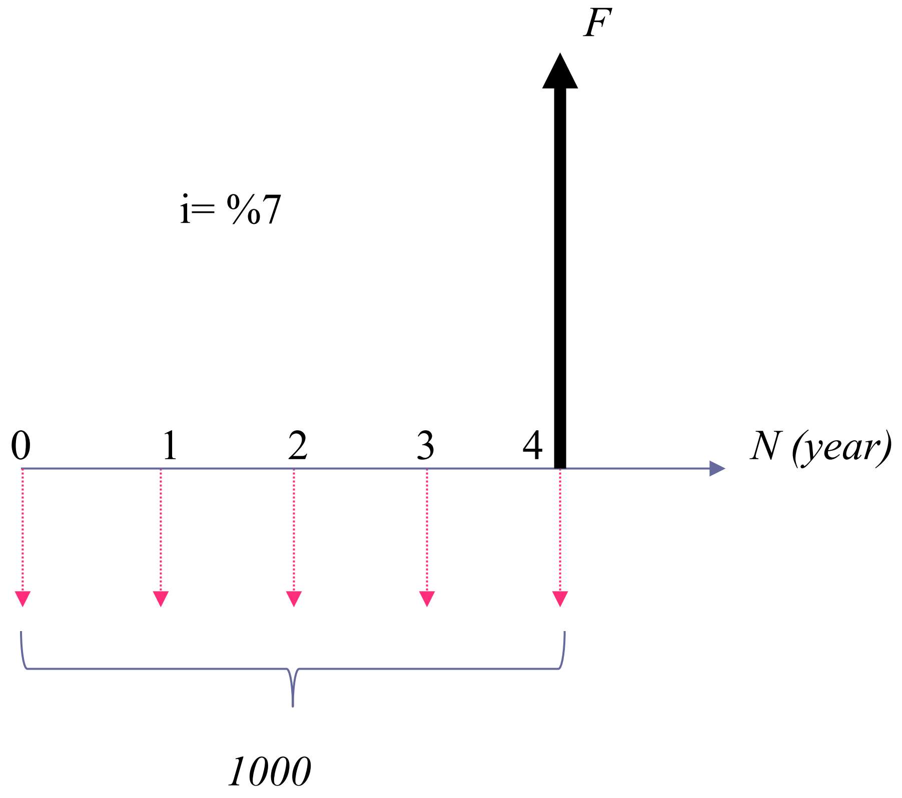
Example 2



Example 3

- 1000 TL is saved for 5 times starting from today at %7 interest rate per year. The saved total amount is withdrawn with the last saving. Draw the cash flow diagram of these transactions.

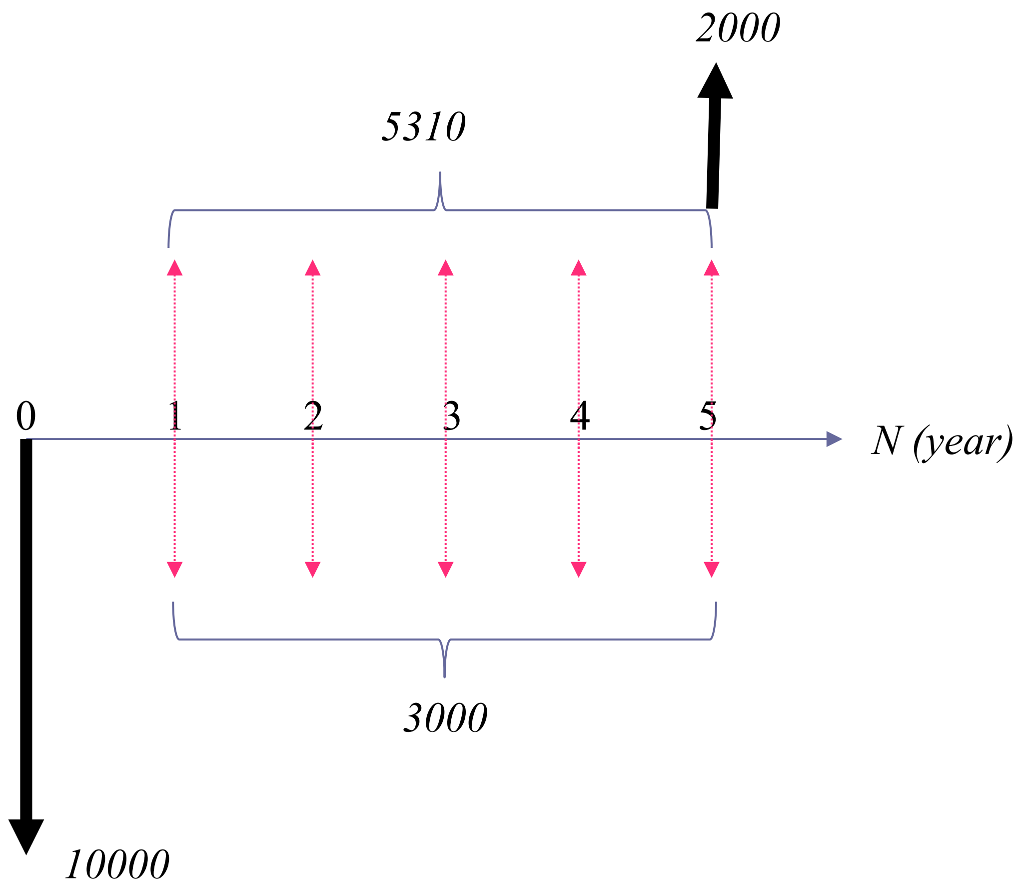
Example 3



Example 4

- A company invests 10,000 TL today. The income of this investment is predicted as 5,310 TL per year for 5 years, and the salvage value of this investment is 2,000 TL at the end of 5^{th} year. The operation and maintenance cost of this investment is calculated as 3,000 TL per year. According to these data, please draw the cash flow diagram of this investment.

Example 4



Economic Equivalence

- Economic equivalence, means that different sums of money at different times would be equal in economic value.
- Economic equivalence exists between cash flows that have the same economic effect..
- For example, if the interest rate is 6% per year, $100 today (present time) is equivalent to $106 one year from today.
- Each payment has different economic equivalences for different times.
- In other words, by selecting different times, the economic value of the payment can change. If the present time is selected, then the present worth is calculated, on the other hand the if future time is selected, future worth is calculated.

Economic Equivalence

- In comparing of different cash flows in terms of economic equivalence, the economic equivalences of each inflows and outflows should be calculated for a specific time.
- In control of economic equivalence, all transactions in the cash flow could be required to be converted into one transaction at the targeted time.
- Equivalence depends on selected interest rate.
- Two cash flows which seem different due to the different transactions can be economically equivalent due to the interest rate.

Example 5

- A company owes 8,000 TL at 10% compound interest rate per year for 4 year period. According to the following payback plans, which payback plan is best alternative.
  - 2,000 TL and the interest are paid annually.
  - The interest is paid annually, and principal repaid at the end of the period.
  - All payment is made at the end of the period.

2000 TL and the interest are paid annually

End of year	Total owed at the beginning of the year	Interest owed for the year	Total owed at the end of the year	Payment from principal	Total payment at the end of the year
1	8000	800	8800	2000	2800
2	6000	600	6600	2000	2600
3	4000	400	4400	2000	2400
4	2000	200	2200	2000	2200
Total		2000			10000

The interest is paid annually, and principal repaid at the end of the period

End of year	Total owed at the beginning of the year	Interest owed for the year	Total owed at the end of the year	Payment from principal	Total payment at the end of the year
1	8000	800	8800	0	800
2	8000	800	8800	0	800
3	8000	800	8800	0	800
4	8000	800	8800	8000	8800
Total		3200			11200

All payment is made at the end of the period

End of year	Total owed at the beginning of the year	Interest owed for the year	Total owed at the end of the year	Payment from principal	Total payment at the end of the year
1	8000	800	8800	0	0
2	8800	880	9680	0	0
3	9680	968	10648	0	0
4	10648	1064,8	11712,8	8000	11712,8
Total		3712,8			11712,8

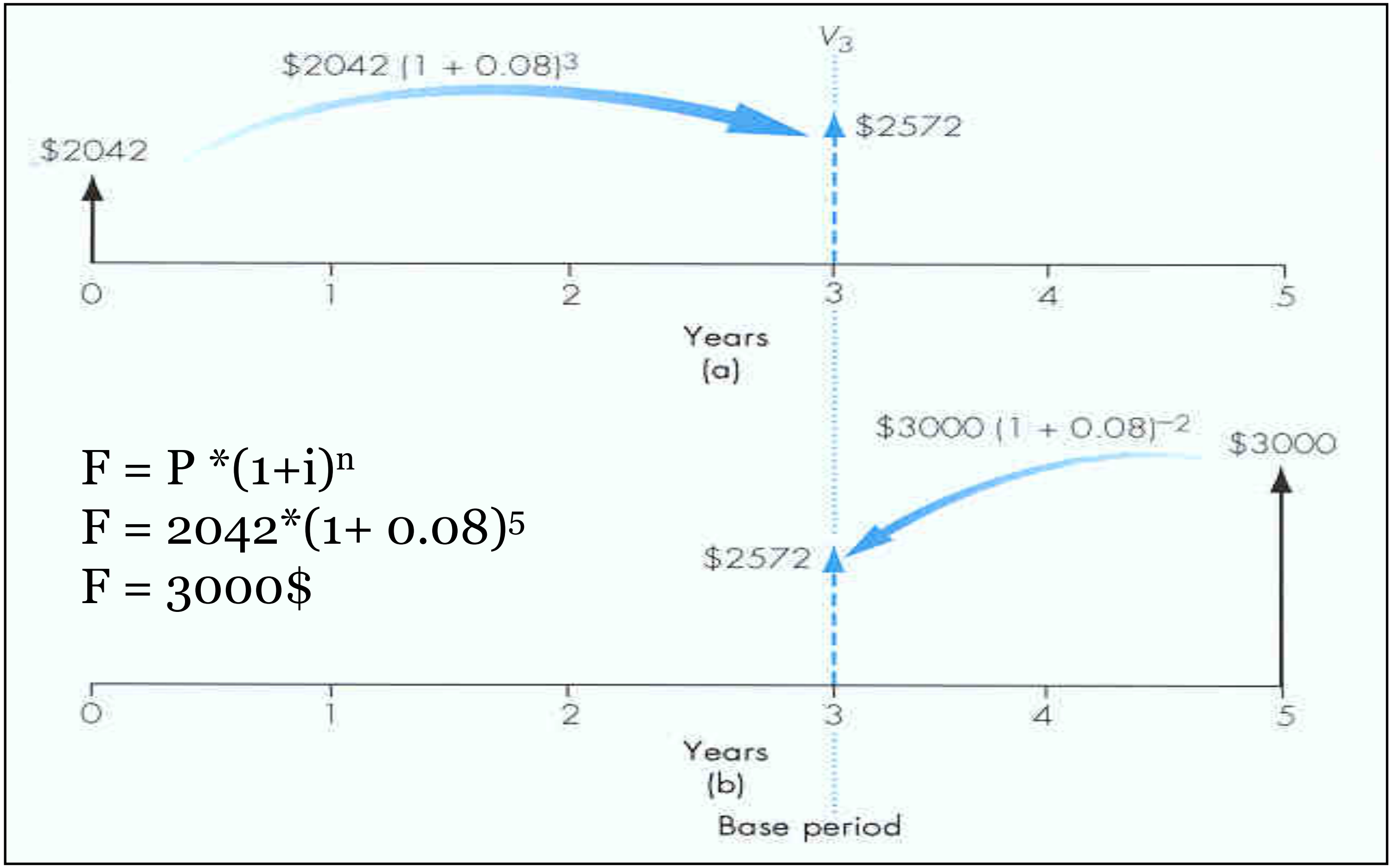
Economic Equivalence

- As you can see all these three alternatives have different cash flows and the total amount of payments made at these alternatives are different.
- Actually, the present worth of all these alternatives is equivalent to 8000 TL, in other words they have same attractiveness. This shows that it is impossible to reach a correct conclusion by just considering the cash flows at this stage. In order to reach a conclusion, these cash flows are required to be rearranged in order to compare with each other and the equivalent values of all cash flows at a specific time should be calculated.

Example 6

- If you deposit $2,042 today in a savings account that pays 8% interest annually, how much would you have at the end of 5 years?

“Equivalent cash flows are equivalent at any common point in time”

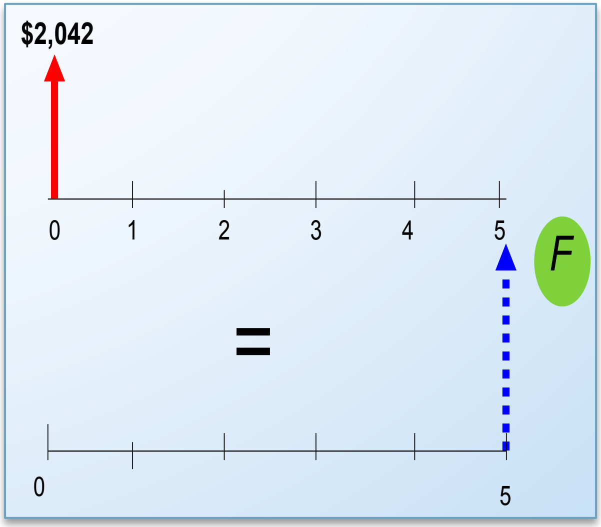


$$F = P *(1+i)^n$$
$$F = 2042*(1+ 0.08)^5$$
$$F = 3000\$$$

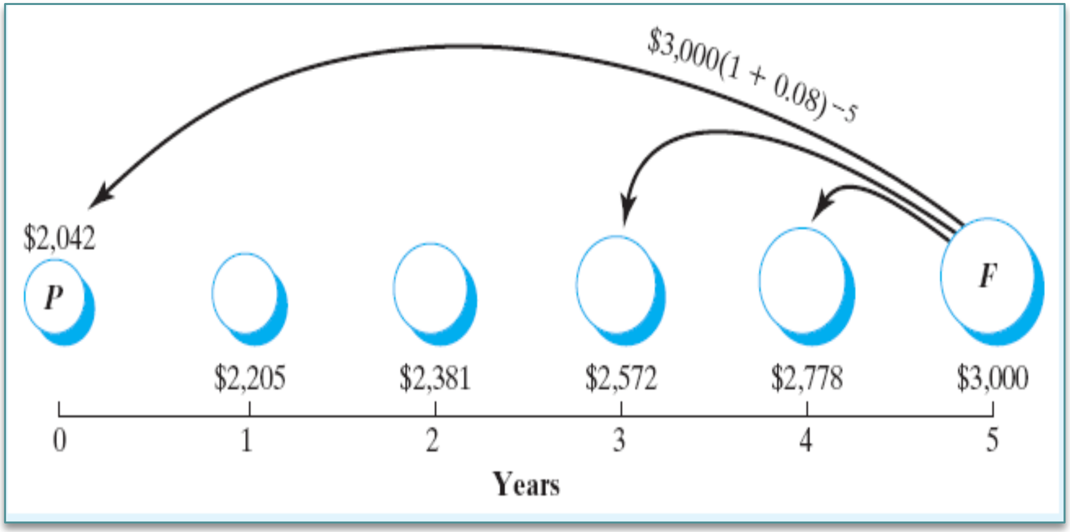
P value which satisfies the equivalence of these two cash flows is calculated as shown. Since these two cash flows are equivalent, they are equivalent at any common point in time. This is also valid for third year.

- If you deposit $2,042 today in a savings account that pays an 8% interest annually, how much would you have at the end of 5 years?
- At an 8% interest, what is the equivalent worth of $2,042 now in 5 years?

$$F = \$2{,}042(1+0.08)^5 = \$3{,}000$$

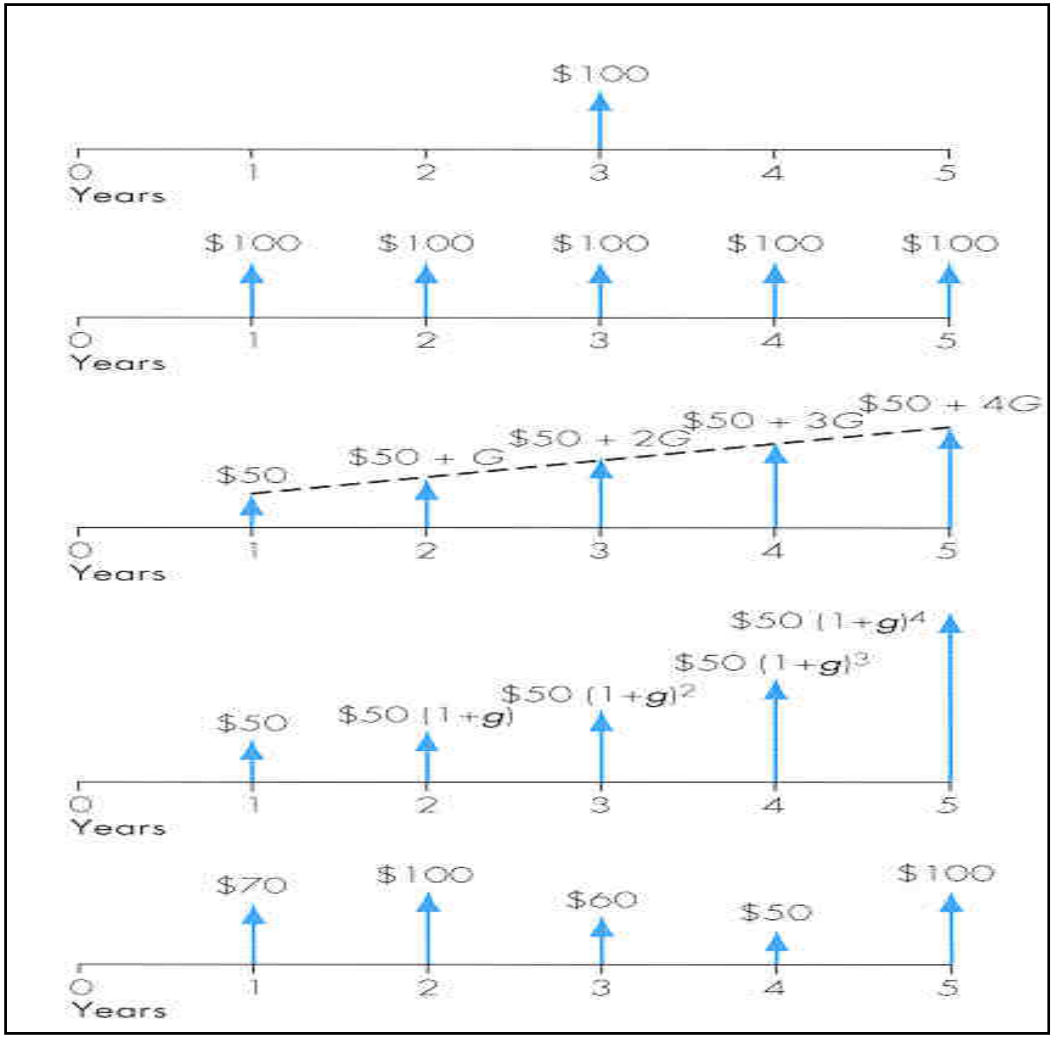


- Various dollar amounts that will be economically equivalent to $3,000 in five years, at an interest rate of 8%.



Types of Cash Flows

- Single payment
- Uniform payment series
- Lineer gradient series
- Geometrical gradient series
- Irregular payment series



Single Payment

- Single Payment, compound interest, <u>future value</u>
- Given:

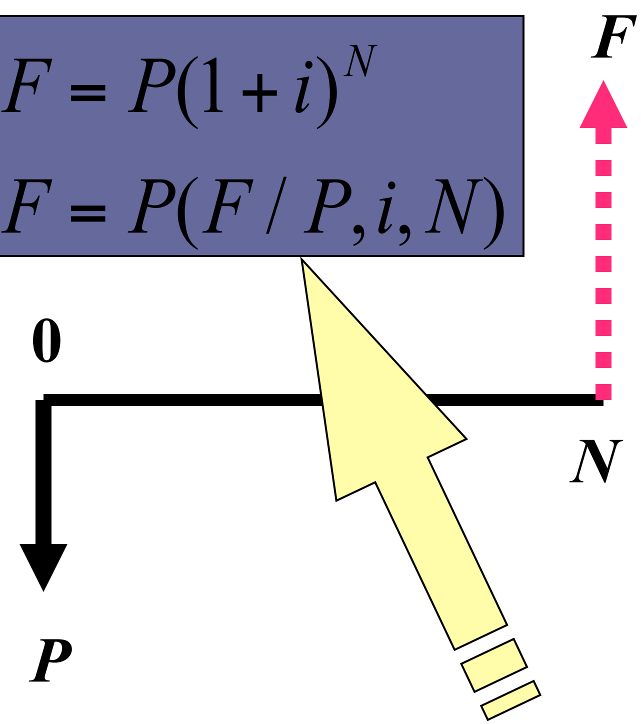
$$i = 10\%$$
$$N = 8 \text{ years}$$
$$P = \$2{,}000$$

- Demand:

$$F = \$2{,}000(1+0.10)^{8}$$
$$= \$2{,}000(F/P, 10\%, 8)$$
$$= \$4{,}287.18$$

- *Compound factor*

$$F = P(1+i)^{N}$$
$$F = P(F/P, i, N)$$



The data obtained from interest tables can be used instead of formulas!!!

$$F = P \times (1+i)^n$$

- **P** : Present value; value or amount of money at a time designated as the present or time 0. (TL, ...)
- **F** : future value; value or amount of money at some future time. (TL, ...)
- **n** : number of interest periods; years, months, days
- **i** : interest rate or rate of return per time period; percent per year, percent per month; percent per day

Interest table for 1% interest rate

1% TABLE 4 Discrete Cash Flow: Compound Interest Factors 1%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	F/P Compound Amount	P/F Present Worth	A/F Sinking Fund	F/A Compound Amount	A/P Capital Recovery	P/A Present Worth	P/G Gradient Present Worth	A/G Gradient Uniform Series
1	1.0100	0.9901	1.00000	1.0000	1.01000	0.9901		
2	1.0201	0.9803	0.49751	2.0100	0.50751	1.9704	0.9803	0.4975
3	1.0303	0.9706	0.33002	3.0301	0.34002	2.9410	2.9215	0.9934
4	1.0406	0.9610	0.24628	4.0604	0.25628	3.9020	5.8044	1.4876
5	1.0510	0.9515	0.19604	5.1010	0.20604	4.8534	9.6103	1.9801
6	1.0615	0.9420	0.16255	6.1520	0.17255	5.7955	14.3205	2.4710
7	1.0721	0.9327	0.13863	7.2135	0.14863	6.7282	19.9168	2.9602
8	1.0829	0.9235	0.12069	8.2857	0.13069	7.6517	26.3812	3.4478
9	1.0937	0.9143	0.10674	9.3685	0.11674	8.5660	33.6959	3.9337
10	1.1046	0.9053	0.09558	10.4622	0.10558	9.4713	41.8435	4.4179
11	1.1157	0.8963	0.08645	11.5668	0.09645	10.3676	50.8067	4.9005
12	1.1268	0.8874	0.07885	12.6825	0.08885	11.2551	60.5687	5.3815
13	1.1381	0.8787	0.07241	13.8093	0.08241	12.1337	71.1126	5.8607
14	1.1495	0.8700	0.06690	14.9474	0.07690	13.0037	82.4221	6.3384
15	1.1610	0.8613	0.06212	16.0969	0.07212	13.8651	94.4810	6.8143
16	1.1726	0.8528	0.05794	17.2579	0.06794	14.7179	107.2734	7.2886
17	1.1843	0.8444	0.05426	18.4304	0.06426	15.5623	120.7834	7.7613
18	1.1961	0.8360	0.05098	19.6147	0.06098	16.3983	134.9957	8.2323
19	1.2081	0.8277	0.04805	20.8109	0.05805	17.2260	149.8950	8.7017
20	1.2202	0.8195	0.04542	22.0190	0.05542	18.0456	165.4664	9.1694

Single payment

- Single Payment, compound interest, future value
- Given:

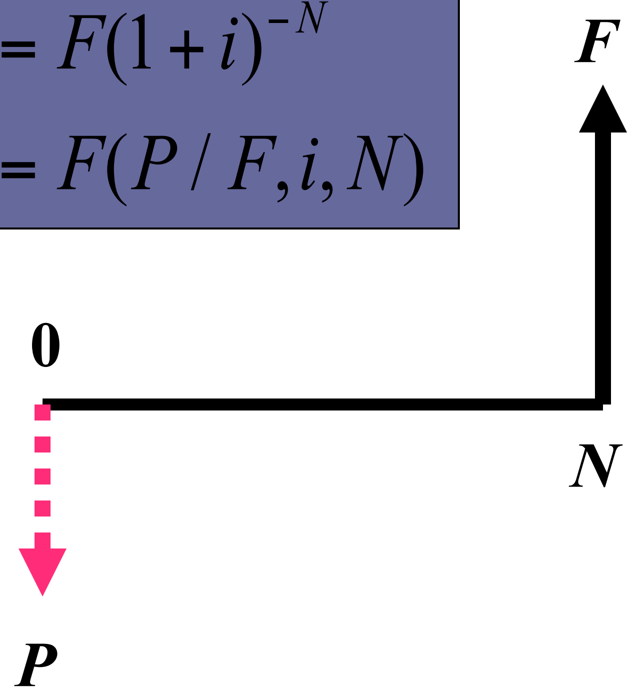
$$i = 12\%$$
$$N = 5 \text{ years}$$
$$F = \$1{,}000$$

- Demand:

$$P = \$1{,}000(1+0.12)^{-5}$$
$$= \$1{,}000(P/F, 12\%, 5)$$
$$= \$567.40$$

- *Discount factor*

$$P = F(1+i)^{-N}$$
$$P = F(P/F, i, N)$$



Single Payment

- Example: What is the value of 3680 $ at the end of 8 year at 12% compound interest rate per year?

  - P = 3680 $
  - i = %12
  - n = 8 year

$$F = P \times (1+i)^n$$

$$= 3680 \times (1 + 0.12)^8$$

$$= 3680 \times 2.476$$

$$= \mathbf{9112\ \$}$$

Single Payment

- **Example: The stock bought at $10 is sold at $20 after 5 years. Then what is the interest rate of this operation?**

- Solution:

$$\boxed{\begin{array}{l} F=P(1+i)^N \\ 20 = 10(1+i)^5 \\ i=\%14.87 \end{array}}$$

Single Payment

- **Example:** XYZ company buys 100 stocks at \$60/stock price. The plan of this company is selling these stocks when its value increases to \$120 \$/stock. If it is predicted that the value of stocks will increase at %20 per year, how many years should the company wait for selling the stocks?

$F=P(1+i)^N = P(F/P, i,N)$

$12{,}000 = 6{,}000\ (1+0.20)^N$

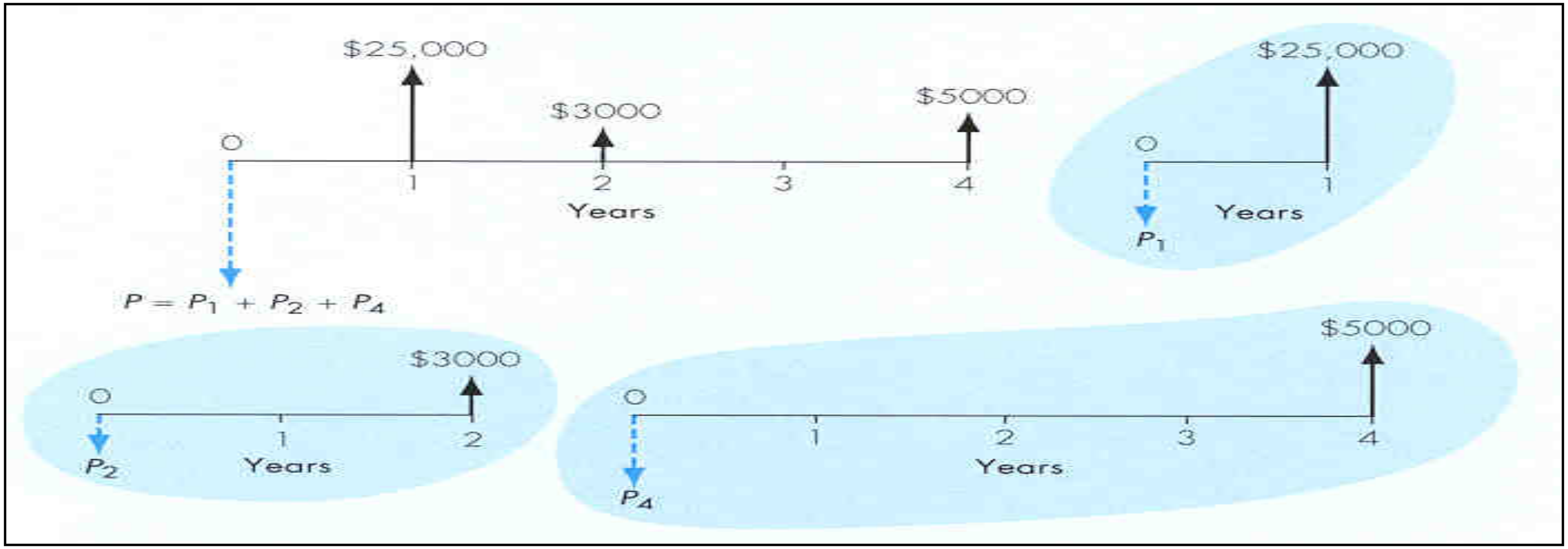
$\log 2 = N \cdot \log 1.2$

$N=3.80$ or approximately 4 year

Irregular payment series

- **Example:** The payments for 4 year stated below, how much money should be deposited to the bank (interest ratio is %10 per year)?
  - Year 1: Computer and software for customer services $25,000
  - Year 2: Upgrade the existing system $3000
  - Year 3: No payment
  - Year 4: Upgrade software $5,000

Düzensiz ödeme serisi



$P = F / (1+i)^n$

$P_1 = 25000*(P/F, 10\%,1) \rightarrow P_1 = 25000*0,9091 = 22727,5$

$P_2 = 3000*(P/F, 10\%,2) \rightarrow P_2 = 3000*0,8264 = 2479,2$

$P_3 = 5000*(P/F, 10\%,1) \rightarrow P_3 = 5000*\ 0,7513 = 3756,5$

$P = \$28623,2$

These can be calculated by using the formula or interest table of 10% interest rate