Problem Set for Midterm Exam

1. The utility function of a consumer that consumes only the good X is given as

$$\mathrm{U(X)} = 10\mathrm{X} - \mathrm{X}^2$$

Graph utility and marginal utility functions. Find X when utility is maximum. Is the law of diminishing marginal utility valid?

2. The utility functions of various consumers that consume only X and Y are given below.

i. $U(X,Y) = X\sqrt{\mathrm{Y}}$
ii. $U(X,Y) = XY^2$
iii. $U(X,Y) = XY$
iv. $U(X,Y) = lnX + lnY$
v. $U(X,Y) = 3X + Y$
vi. $U(X,Y) = 5X^{0.4}Y^{0.6}$
vii. $U(\mathrm{X},\mathrm{Y}) = minX, Y$

Answer the questions below for each function:

a. Is the law of diminishing marginal utility valid for both goods?
b. Is the law of diminishing marginal rate of substitution valid?
c. Graph two indifference curves briefly. Explain why the two indifference curves do not intersect.

3. Cansel loves cheese sandwiches. A cheese sandwich is made of 2 loafs of bread and 1 slice of cheese. The price of bread is P_B and the price of cheese is P_C. Cansel's income is $I = 18$ TL for preparing cheese sandwiches.

a. Write down a utility function of Cansel for consuming cheese sandwich.
b. Plot indifference curves for Cansel's utility function at $U = 4$ and $U = 6$.
c. Write down her budget constraint.
d. Compute the demand curve for cheese.
e. Are cheese and bread complements or substitutes (Hint: You should look at the cross-price elasticity)
f. Assume the price of cheese falls to $P_C = 2$ from $P_C = 4$. Calculate the substitution effect given that $P_B = 1$. Interpret your result.

4. A consumer has the utility function $U(x, y) = xy$. Income of the consumer is 72 TL, while price of good *y* is 1 TL per unit. Suppose that the price of *x* is initially 9 TL. Then the price falls to 4 TL. Find the numerical values of the income effect and substitution effect. Graph your findings and write down the economic meaning of each result.

5. Suzie purchases two goods, food and clothing. She has the utility function $U(x, y) = xy$ where *x* denotes the amount of food consumed and *y* the amount of clothing.

a. Show that the equation for her demand curve for clothing is $y = I/2P_y$.
b. Is clothing a normal good? Draw her demand curve for clothing when the level of income is $I = 200$. Label this demand curve D_1. Draw the demand curve when $I = 300$ and label this demand curve D_2.
c. What can be said about the cross-price elasticity of demand of food with respect to the price of clothing?

6. A utility function is given as $U(x, y) = xy$. Then,

- **a.** Derive Marshallian demand function.
- **b.** Derive indirect utility function.
- **c.** Derive Hicksian demand function.
- **d.** Derive expenditure function.

7. There are two types of consumers in a market for sheet metal. Let P represent the market price. The total quantity demanded by Type I consumers is $Q_1 = 100 - 2P$. The total quantity demanded by Type II consumers is $Q_2 = 40 - P$. Draw the total market demand on a clearly labeled graph.

8. Consider the linear demand curve $Q = 360 - 6P$.

- **a.** What is the price elasticity of demand at *P=40*?
- **b.** In what direction and at what rate should the price be changed to maximize total revenue?

9. Ginger's utility function is $U(\mathrm{x}, \mathrm{y}) = \mathrm{x}^2 y$. She has income *I* = 240 and faces prices $P_x = 8$ and $P_{\mathrm{y}} = 2$.

- **a.** Determine Ginger's optimal basket given these prices and her income.
- **b.** If the price of *y* increases to 8 and Ginger's income is unchanged, what must the price of *x* fall to for her to be exactly as well off as before the change in P_y?

10. If the Cobb-Douglas production function is $q = L^{0.75}K^{0.25}$, and $\overline{K} = 16$, what is the elasticity of output with respect to labor?

11. By studying, Will can produce a higher grade, G_W, on an upcoming economics exam. His production function depends on the number of hours he studies marginal analysis problems, *A*, and the number of hours he studies supply and demand problems, *R*. Specifically, $G_W = 2.5\mathrm{A}^{0.36}\mathrm{R}^{0.64}$. His roommate David's grade production function is $G_{\mathrm{D}} = 2.5\mathrm{A}^{0.25}\mathrm{R}^{0.75}$.

- **a.** What is Will's marginal productivity from studying supply and demand problems? What is David's?
- **b.** What is Will's marginal rate of technical substitution between studying the two types of problems? What is David's?
- **c.** Is it possible that Will and David have different marginal productivity functions but the same marginal rate of technical substitution functions? Explain.

12. Under what conditions do the following production functions exhibit decreasing, constant, or increasing returns to scale?

- **a.** $q = L + K$, a linear production function,
- **b.** $q = AL^{\alpha}K^{\beta}$, a general Cobb-Douglas production function
- **c.** $q = L + L^{\alpha}K^{\beta} + K$,
- **d.** $q = (\alpha L^{\rho} + [1 - \alpha]K^{\rho})^{d/\rho}$, a CES production function.