REPUBLIC OF TURKEY
YILDIZ TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
MASTER OF ARTS PROGRAMME IN ECONOMICS

MASTER THESIS

3 SHADES OF GRAY; POLLUTION ABATEMENT IN COURNOT, BERTRAND AND COURNOT-BERTRAND DUOPOLIES

ALİ ÖZENÇ GÜNDOĞDU
ÖĞRENCİ NUMARASI
21729029

THESIS SUPERVISOR
DOÇ. DR. TOLGA AKSOY

2024

REPUBLIC OF TURKEY
YILDIZ TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
MASTER OF ARTS PROGRAMME IN ECONOMICS

MASTER THESIS

3 SHADES OF GRAY; POLLUTION ABATEMENT IN COURNOT, BERTRAND AND COURNOT-BERTRAND DUOPOLIES

ALİ ÖZENÇ GÜNDOĞDU
21729029
ORCID NO: 0009-0006-8442-8398

THESIS SUPERVISOR
DOÇ. DR. TOLGA AKSOY

JULY 2024

Ali Özenç Gündoğdu tarafından hazırlanan “3 Shades of Gray: Pollution Abatement In Cournot, Bertrand and Cournot-Bertrand Duopolies” başlıklı çalışma, **05/07/2024** tarihinde yapılan savunma sınavı sonucunda **oybirliği** ile başarılı bulunmuş ve jürimiz tarafından İktisat Ana Bilim Dalı İngilizce İktisat Programında **YÜKSEK LİSANS** tezi olarak kabul edilmiştir.

Danışman	**İmza**
Doç. Dr. Tolga AKSOY	……

Jüri Üyeleri	**İmza**
Dr. Öğr. Üyesi, Hasan Ağan Karaduman	……
Doç. Dr. Şeref Bozoklu	……

ABSTRACT

3 SHADES OF GRAY; POLLUTION ABATEMENT IN COURNOT, BERTRAND AND COURNOT-BERTRAND DUOPOLIES

Environmental pollution worldwide has increased with the acceleration of industrial production processes. It has become a necessity to implement solutions that will minimize the damage to nature from environmental pollution caused by industrial production processes. In this context, advanced technologies for the treatment of industrial wastes should be included in industrial production processes. In this study, the profitability of firms, social welfare and social welfare-enhancing tax are examined with which production function variables are related to the inclusion of emission-reducing technologies in industrial production processes. In this context, Cournot, Bertrand and Cournot-Bertrand duopoly were selected as the market model within the three-stage non-cooperative game model, and the structure of the game was formed under the presence of a regulator that applies tax reductions to firms that incorporate emission-reducing technologies into their production processes. Firms make investment decisions for emission-reducing policies in the first stage of the game, in the second stage the regulator imposes the tax that maximizes social welfare and In the third stage, firms choose the amount of production they will make within the market structure they are in. It is seen that profitability increases if firms invest in emission-reducing technology, emission-reducing policies affect social welfare positively in all cases, and since the competitive aspect of the Bertrand duopoly is more competitive than Cournot and Cournot-Bertrand duopolies, it is most positively affected in terms of social welfare. Moreover, in the second stage of the game, the amount of tax that maximizes social welfare increases as environmental pollution increases and technological efficiency decreases. In actual practice, this model can be considered as a prediction of the output of firms in industries with relevant market structures when they are included in environmental policies.

Keywords: Emission reduction policies, Cournot-Bertrand duopoly, Social Welfare

ÖZET

GRİNİN 3 TONU; COURNOT, BERTRAND VE COURNOT-BERTRAND DÜOPOLLERİNDE KİRLİLİĞİN AZALTILMASI

Dünya genelinde çevre kirliliği, endüstriyel üretim süreçlerinin hızlanmasıyla ile birlikte artmıştır. Doğanın endüstriyel üretim süreçlerinin yarattığı çevre kirliliğinden gördüğü zararı en aza indirgeyecek çözümlerin uygulanması gereklilik haline dönüşmüştür.Bu baglamda endüstriyel atıkların ıslahı için gelişmiş teknolojilerin endüstriyel üretim süreçlerine dahil edilmesi gerekmektedir. Bu çalışmada, emisyon azaltıcı teknolojilerin endüstriyel üretim süreçlerine dahil edilmesiyle firmaların karlılık durumları , toplumsal refah durumu ve sosyal refahı arttırıcı verginin hangi üretim fonksiyonu değişkenleri ile ilişkili olduğu incelenmiştir. Bu kapsamda üç aşamalı işbirlikçi olmayan oyun modeli içerisinde Cournot, Bertrand ve Cournot-Bertrand duopolü piyasa modeli olarak seçilmiş, emisyon azaltıcı teknolojileri üretim süreçlerine dahil eden firmalara vergi indirimi uygulayan bir regülatorun varlığı altında oyunun yapısı oluşturulmuştur. Firmalar oyunun ilk aşamasında emisyon azaltıcı politikalar için yatırım kararlarını verir, ikinci aşamada regulator toplumsal refahı maksimize edecek vergiyi uygular ve üçüncü aşamada firmalar içinde bulundukları piyasa yapısı içerisinde yapacakları üretim miktarlarını seçerler.Firmaların emisyon azaltıcı teknolojiye yatırım yapmaları halinde karlılıkların arttığı, emisyon azaltıcı politikaların, toplumsal refahı tüm durumlarda olumlu etkilediği, Bertrand duopolünün rekabetçi yönü Cournot ve Cournot-bertrand duopollerinden daha fazla olduğu için toplumsal refah yönünden , en olumlu etkilendiği görülmüştür. Ayrıca Oyunun ikinci aşamasında toplumsal refahı maksimize edecek vergi miktarı, çevre kirliliği arttıkça ve teknolojik verimlilik azaldıkça artmaktadır.Güncel yaşamda bu model, ilgili piyasa yapılarının görüldüğü endüstriler içerisindeki firmaların çevre politikalarına dahil edildiklerinde yarattığı çıktıların bir öngörüsü olarak düşünülebilir.

Anahtar Kelimeler: Emisyon azaltıcı politikalar, Cournot-Bertrand duopolü, Sosyal Refah

ÖN SÖZ

Tez sürecimin başından itibaren gösterdiği ilgi, yaklaşımları ve katkılarıyla ile değerli tez danışmanım Sayın Doç. Dr. Tolga Aksoy'a, eğitim öğretim sürecim boyunca desteklerini esirgemeyen sevgili aileme, ve çalışmalarıyla zihin dünyamıza ve hayatımıza katkı sağlayan tüm bilim insanlarına teşekkürü borç bilirim.

Ali Özenç Gündoğdu
Temmuz,2024; İstanbul.

TABLE OF CONTENTS

ABSTRACT iii
ÖZET iv
ÖN SÖZ v
LIST OF TABLES vii
LIST OF FIGURES viii
LIST OF ABBREVATIONS ix
1. INTRODUCTION 1
2. CONCEPTUAL FRAMEWORK 3
2.1. Abatement 3
2.2. Abatement Process 4
2.3. Models' Terminology 6
3. RELATED LITERATURE 8
4. MODEL 13
4.1. Description 13
4.2. Cournot Duopoly 14
4.2.1. Firms Do Not Abate 14
4.2.2. Firms Abate 16
4.2.3. Asymmetric case: Abate / Not Abate 18
4.3. Bertrand Duopoly 20
4.3.1. Overview 20
4.3.2. Firms Do Not Abate 21
4.3.3. Firms Abate 23
4.3.4. Asymmetric case: Abate / Not Abate 26
4.4. Cournot-Bertrand Duopoly 28
4.4.1. Overview 28
4.4.2. Firms Do Not Abate 28
4.4.3. Firms Abate 30
4.4.4. Asymmetric case: Not Abate / Abate 32
4.4.5. Asymmetric case: Abate / Not Abate 34
5. DISCUSSIONS 37
6. CONCLUSION 41
REFERENCES 42
APPENDICES 44

LIST OF TABLES

Table 1: Results of Abatement Games Across the Duopolies In terms of Firms' Profit 37
Table 2 : Social Welfare Results of Abatement Game Across the Duopolies 38
Table 3: Social Welfare Results Comparison Between the Duopolies 39

LIST OF FIGURES

Figure 1: The Relationship Between Producers and Consumers In The Scope of Environmental Economics. 3

Figure 2 : The Relationship Between t* and ρ In the Cournot Not Abatement Case 16

Figure 3 : The Relationship Between t*, z and ρ In the Cournot Abatement Case... 18

Figure 4 : The Relationship Between t*, z and ρ In the Cournot Asymmetric Case. 20

Figure 5 : The Relationship Between t* and ρ In the Bertrand Not Abatement Case 23

Figure 6 : The Relationship Between t*, z and ρ In the Bertrand Abatement Case.. 25

Figure 7 : The Relationship Between t*, z and ρ In the Bertrand Abatement / Not Abatement Case 27

Figure 8 : The Relationship Between t* and ρ In the Cournot-Bertrand Not Abatement Case 30

Figure 9 : The Relationship Between t*, z and ρ In the Cournot-Bertrand Abatement Case 32

Figure 10 : The Relationship Between t*, z and ρ In the Cournot-Bertrand Not Abatement / Abatement Case 34

Figure 11 : The Relationship Between t*, z and ρ In the Cournot-Bertrand Abatement / Not Abatement Case 35

LIST OF ABBREVATIONS

CO_2 : Carbon dioxide
SO_2 : Sulphur dioxide
SW : Social Welfare
PS : Producer Surplus
CS : Consumer Surplus
GR : Government Revenue
ED : Environmental Damage

1. INTRODUCTION

Nature is fixed in a structure that is limited in terms of its resources. In its own balance, it has been in motion since the world has existed. It can renew and sustain itself without any external intervention. However, Human beings, one of the creatures living in nature, may not always do useful things for nature. For example, they may carry out their production processes carelessly in a way that harms nature, or they may damage the carbon cycle by destroying trees that are part of nature. As a result of these actions, the health of nature is jeopardized. Natural resources are crucial for the continuity of life on Earth. The water on Earth, the air we breathe, the soil we walk on are all part of natural resources. Therefore, without realizing it, we live our daily lives as much as nature's resources offer us. While natural resources are so important, the destruction of natural resources is just as important.

Since the industrial revolution, mankind has accelerated its production processes and increased the damage to nature with the methods it uses. Industrial production processes have brought with them consumption, and these two structures have progressed to the present day by strengthening their existence in a way that feeds each other. Unfortunately, this progress has not been in favor of natural resources.

Preventing this environmental damage arising from production and consumption processes will undoubtedly be possible with the implementation of various correct policies. These policies should be formulated in such a way as to include production and consumption processes, and with a view to social welfare.

In this context, the aim of this thesis is to examine regulations that will minimize the destruction of natural resources created in industrial production processes and to determine the benefits of the resulting outputs to nature and society.

The thesis continues as follows: Chapter 2 will examine the concepts subject to the model in a conceptual framework, Chapter 3 will consist of a review of the literature on the content of the model, chapter 4 will form the functioning and structure of the

model, and Chapter 5 will consist of a discussion of the results of the model. Finally, Chapter 6 concludes the thesis.

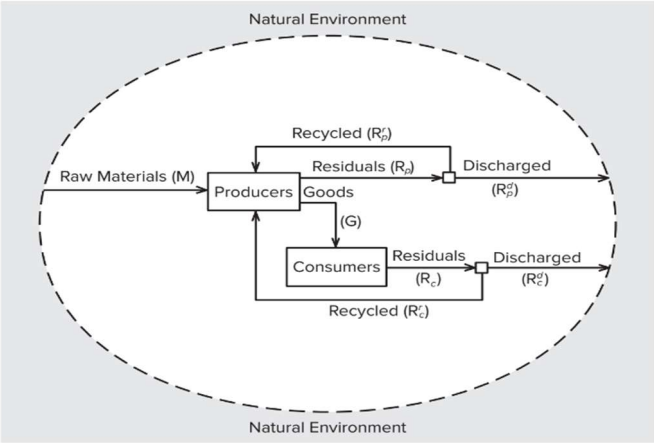
2. CONCEPTUAL FRAMEWORK

Since the industrial revolution, the industrial products have increased and diversified. This situation was in favor of the wealth of nations. Income increased with production and new job fields emerged. Many sectors within the industry have participated in production process. The proliferation of industrial products has given a new form to production-consumption relations. In this context, the relationship between production firms and consumers can be considered within the scope of environmental economics.

2.1. Abatement

Production processes use raw materials as a source of energy. When raw materials are involved in this process, environmentally harmful wastes are generated, for example, when coal is burned, carbon dioxide emission gas is produced. Below is a figure that illustrates this relationship.

Figure 1: The Relationship Between Producers and Consumers In The Scope of Environmental Economics.



Source: Field & Field, 2017, pg24.

According to model Field and Field (2017):

> ‘The economy has been divided into the two groups: producers and consumers. The producer’s category includes all private firms take inputs and convert them to outputs; The consumer category includes all of the private households to whom the vast collections of final goods and services are distributed.’ (pp.24)

Following the definition, Field and Field (2017) explains:

$$R_p^d + R_c^d = M = G + R_p - R_p^r - R_c^r$$

which means that the quantity of raw materials (M) is equal to output of goods and services (G) plus production residuals (R_p) minus amounts that are recycled from producers (R_p^r) and consumers (R_c^r) and the quantity of raw materials (M) is equal to discharged residual from production plus discharged residual from production (R_p^d+R_c^d).

According to this model, to reduce the environmental damage caused by the raw materials used in production processes, it is necessary to replace these raw materials with renewable energy sources or introduce new technologies into the production process. Since transforming raw materials into renewable energy sources will take time, implementing new technologies presents a viable solution to minimize environmental damage from production in the short run.

It is possible to improve environmental quality through applying abatement technologies into the industrial production processes, Operational Group, (2024) gives an example to the air pollution abatement like building a distillation towers to separate components in waste emissions, carbon bed abatement to absorb the pollutants and cleans the air.

After applying abatement technologies into the industrial production processes, the discharged residual from production and consumption will reduce.

2.2. Abatement Process

Abatement is one of the necessary steps for production and consumption processes to become sustainable in an environmentally friendly way. In terms of production, companies invest in abatement technologies to reduce the emissions of their products. Thus, the products are less harmful to the environment.

In real-world market conditions, firms are primarily motivated by profit maximization. Investing in abatement technologies, which incur additional costs, may seem contrary to this objective. Therefore, within the framework of the political instruments of environmental economics, it is essential to implement tools that incentivize firms to invest in abatement technologies by making such investments more profitable or advantageous in the market. In order to support abatement investments of firms, it would be positive for policy makers to support companies by providing tax deductions or abatement subsidy. In return for investing in abatement technologies, policy makers can give firms a tax deduction equal to the amount of environmentally beneficial products they produce. Or they can deduct the products they buy for abatement from the taxes they charge them. Thus, companies can invest in abatement technologies without losing their profit maximization motivation.

If we evaluate applying abatement technologies into the industry from an analytical framework, we need a model that symbolizes firms, where we can observe the effects of abatement decisions on their profitability and compare firms in this respect, as well as observe how environmental economics policy instruments affect social welfare together with firm outcomes. In this context, duopoly models have the features we are looking for. Duopoly models symbolize situations where two firms control the market, where firms compete with each other over strategic variables, where they produce with respect to their strategic variables, and where profitability of these firms depends on each other's choices. According to Kellog& Reguant (2021):

> For instance, wholesale electricity markets take the form of high-frequency multi-auctions, vehicle markets can be characterized by differentiated Bertrand competition, extractors of exhaustible resources face dynamic problems, and energy distribution firms are regulated natural monopolies. These market structures and forms of regulation are all areas where industrial organization economists hold expertise. (pp.5)

Also from an environmental economics perspective, duopoly models offer a realistic examination of how environmental policy instruments work.

Hoel's analysis (1991) of the concept of whether other countries following Norway's cutting green house gases will adjust themselves politically or take advantage of this

situation with the structure of the Stackelberg leader-follower game can also be given as an example of duopoly - environmental economics cooperation.

Considering all these examples, it is appropriate to analyze abatement decisions under the tax deduction in the industry within duopoly markets.

2.3. Models' Terminology

Related to the concept and process of abatement, which we have discussed in previous sections. Abatement technologies require a certain cost for their implementation. This is called abatement cost. According to Field and Field (2017): 'Abatement costs are the costs of reducing the quantity of residuals being emitted into the environment, or of lowering ambient concentrations.' (pp. 87)

Based on this definition, the abatement cost function we will use in the model is shown as: $\frac{z}{2}.k_i{}^2$,where z is a coefficient that represents the total cost of abatement. İt may be thought that decline in z is more efficient abatement process with lower cost of abatement. Respectively, increase in z is less efficient abatement process with higher cost of abatement.

To continue with, environmental damage, which is another concept we include in the model is proportional to the emissions emitted by companies. The relationship between firm emissions and environmental damage in the model is formulated as:

$$\mathrm{ED^{NA}} = \rho\,(q_1 + q_2)^2 \quad \text{and} \quad \mathrm{ED^{A}} = \frac{\rho}{2}\,[(q_1 - k_1) + (q_2 - k_2)]^2$$

In this formula ρ stands for perceived environmental damage. ρ symbolizes the awareness developed about environmental damage. As environmental damage increases, awareness of environmental damage also increases. When ρ is constant, firms are more beneficial to the environment when they invest in abatement technologies.

Another concept to be used in the model is tax deduction. Accordingly, when firms remediate their emissions by choosing abatement technology, the pollution they remediate is reflected as a tax deduction. Thus, firms will be taxed on the amounts they abate and the cost caused by the abatement process will be deducted from the tax.

All these concepts mentioned in this section are the concepts that will be used in the model and are examined in order to eliminate possible question marks.

The next section deals with the review of the environmental literature in the construction of the model that is the subject of our study.

3. RELATED LITERATURE

The following studies have been carried out on the policies under which the abatement process should be implemented, the possible consequences of these policies and the political ways of reducing carbon emissions on a country basis:

Yeldan and Voyvoda (2015) have analyzed which policies Turkey needs to reduce its carbon emissions in the period until 2030, named these policies as climate policy package and classified the related policies. They underlined that carbon tax and sectoral emission reductions are political ways that can offer solutions.

Yeldan and Acar (2016) conducted an analytical forecasting modelling on the macroeconomic and environmental impacts of existing carbon incentives in Turkey for the years 2015-2030 and showed that by eliminating these incentives, Turkey can integrate a production process that is less environmentally damaging with a GDP loss of 5%.

Yeldan and Acar (2018) analyzed the sectoral distribution of carbon dioxide emissions and showed that high-tech production, which creates added value within sectors, causes more carbon dioxide emissions than medium and low-tech production. At the same time, they underlined that it is the business cycle that causes carbon dioxide emissions.

Kat et al (2024) created an economic model that simulates the electricity market based on the conversion of coal with renewable energy in line with net zero emission targets, spreads the net zero carbon emissions policy to the sector and discusses the macroeconomic results and the results they found according to the scenario drawn by the model revealed that while the GDP and electricity demand increased, the production centers using coal as fuel decreased.

Theoretically, studies have been carried out on how the policies to be implemented will yield results in which market and which environmental economics instrument should be used. These studies extensively utilizes duopoly models to explain the relationship between firm behavior and environmental regulation outcomes. Many of

these studies specifically employ Cournot duopoly models to analyze these interactions:

Matsumoto and Szidarovszky (2020) investigated the case where the price function is hyperbolic in the n-firm Cournot oligopoly model. They have found that, if average abatement technology in the industry increases, it will also increases the output in Cournot model.

Xu Yuvan and Sang-Ho (2021) added the Cournot-Bertrand market to their model to examine the relationship between emission tax and environmental social responsibility investments. They compared the pre-tax investments of companies to reduce environmental damage with their post-tax investments and showed that making post-tax investments always results in a higher tax burden. At the same time, it was emphasized that the Cournot type would be more preferred in this model.

Dragone, Lambertini, and Palestini (2021) included pollution-emitting emissions in the differential Cournot game. In their study, they concluded that in equilibrium, market makers should impose an emission tax or control market entry and exit in order to implement environmentally friendly technology. In addition, they found that if environmental damage is caused by emissions, the relationship between total research and development effort and environmental damage is first increasing and then decreasing. They also concluded that an industry structure that supports green transformation technologies minimizes individual and societal emissions.

On the other hand, some studies focus on Cournot-Bertrand and mix duopoly models to analyze environmental regulation and comparing its outcomes. For instance,

Barcena and Garzon (2010) show that a double equilibrium situation arises in an industry where firms first select quantities and then compete on price. In conclusion, social welfare is higher when the public firm is the leader.

Fanti and Meccheri (2011) examined Cournot and Bertrand equilibrium situations within differentiated products on the background of labor economics. They revealed that profitability is higher in Cournot type companies.

Kangsik (2012) investigate within the framework of the Cournot Bertrand market model, they developed the contract game in line with the preferences of private and public companies under the differentiated mixed duopoly model. According to their

results, the market structure of the Bertrand company is greater than the situation of the Cournot company in terms of social welfare.

Tremblay and Tremblay (2019) have extensively examined the studies conducted under the Cournot-Bertrand model, and also compared the social welfare in a way to include the previous models separately. As a result, they stated that Cournot firm prefers environmentally friendly technology more than its rival Bertrand firm, and that it makes more profit by producing more.

Kopel, Putz and Eva (2021) designed a 3-stage game in which the owners of the companies share the managers' contract details. Accordingly, they showed how the contract sharing decisions taken by the companies were shaped within the Cournot-Bertrand market structure. If regulator make it mandatory to share contracts, the company in the Bertrand market will make more profit and social welfare will be higher in this market.

Asproudis and Filippiadis (2021) analyzed abatement preferences under the Cournot-Bertrand model and showed that the Bertrand firm is less willing to choose environmentally friendly technologies than the Cournot firm and that environmentally friendly technology is more preferred under quantity competition than price competition. They also stated that abatement technology increases total output and reduces emissions.

In addition to these, some studies have added Hotelling duopoly and Stackelberg model to their models. Examples of these studies are:

Wei, Yi and Fu (2019) designed the Stackelberg differential game between the market maker and the market within the scope of green innovation. Considering the impact of investments on production capacities and the impact of R&D on environmental pollution amelioration, they concluded that the relationship between green innovation and competition first decreased and then increased, and that learning by doing had a positive impact on companies' research and development processes and capacity investments.

Bimonte et al (2021) examined the investment incentives of companies under non-simultaneous sequential game. They found that if competing companies act together in research and development processes, they can have a positive impact on investing in green innovation.

Pan, Cao and Liu (2023) examined firms' investments in green innovation choices under the Hotelling duopoly model. According to the results of their study, the market maker should take decisions in parallel with privacy regulation when regulating the market. Projects that create renewable energy should be financed, weak privacy regulation should be applied on these projects, and competition should be encouraged through uniform pricing on renewable energy products.

Finally, some studies use both Cournot & Bertrand models to compare results of environmental regulation:

Singh and Vives (1984) examined the abatement game under horizontal product differentiation in markets with quantity and price competition. According to the results of their study, in this context, price competition is more efficient than quantity competition.

Buccella, Fanti and Gori (2021) examined the investments in abatement technology separately in the Cournot and Bertrand market structures. They designed a non-cooperative duopoly game. In that game, companies that choose abatement are subject to tax exemption. According to the results of their studies, it was stated that social welfare was higher when investments in abatement technologies were made than when this investment was not made.

Chen, Lee and Sang-Ho (2022) examined R&D competition under quantity and R&D incentive policies. They emphasized that in the Cournot duopoly, companies invest more in R&D and benefit more from output incentives. While they revealed that the output subsidy policy gave the same result in terms of social welfare in the two duopolies, they showed that the Bertrand duopoly caused more social welfare in R&D incentives.

Considering the related literature, the difference of the study to be included in the thesis from the current literature will be to examine the industry in a more dimensional and inclusive way by adding the Cournot-Bertrand market structure to the model in which the abatement preferences is examined. With the addition of the Cournot-Bertrand model, the outputs of abatement preferences will be compared with each other in terms of firm profitability and social welfare, bringing us one step closer to the reality of the current industrial market than previous studies in this context.

The Cournot-Bertrand duopoly can be observed in real life in various sectors. Examples of such sectors are natural gas, telecommunications, electricity and renewable energy. In these sectors, while firms compete with each other in the same industry, they may exhibit Cournot-type and Bertrand-type firm characteristics. For example, two firms from the natural gas sector, Gazprom and Luft-oil, may compete on price, while Gazprom maintains its market share by choosing the quantity for natural gas production.

Another example is the airline industry. One of the two airline companies may increase investments in new infrastructure to increase interaction, while the other may seek to increase customer demand through price competition.

At the same time, the cement sector can also be recognized as a study conducted in this context. Miller et al (2010) studied the effect of the location of cement factories on competition, while the domestic firm competes with the foreign firm by choosing the production capacity, the other firm in the industry may choose to keep the demand in the market through price competition. In addition, location differences are effective on the amount of production to be made and may affect production processes by considering transport costs.

Given the relevant literature and the practical application of the Cournot-Bertrand model, the next part of the paper will discuss the results of applying the abatement process to industrial production within the Cournot-Bertrand, Cournot and Bertrand duopolies.

4. MODEL

4.1. Description

Building on the model Buccella, Fanti and Gori (2021) study's model, we added Cournot-Bertrand duopoly model to their industry structure where two firms, i =1,2 produce a differentiated product with d∈(0,1) represents product differentiation. İn this situation, two firms choose their strategic variable, For example, firm 1 chooses quantity and firm 2 choose price then they start to compete in their strategic variables in the same industry. In this model, firm i produce q_i units of output. In this model firm i produce q_i units of output. While they are producing, the production process causes e_i units of emissions : which can be symbolized by $e_i = q_i$ when firms don't involve abatement process, $e_i = q_i - k_i$ where firms involve abatement process. The cost of abatement technology is modeled as half of the coefficient z multiplied by the abatement quantity. The z coefficient here represents the technological part of the abatement cost. If the z coefficient increases, the cost of abatement will increase, so the z coefficient represents the efficiency of the technology used in the abatement process. On the other hand, if we talk about the tax structure in the model; Since firms that use abatement technology are taxed by deducting the amount they abate from the amount they produce, rather than the amount they produce, the tax liability according to the abatement preference is a piecewise function in the model. Another element in the model, perceived environmental damage, is designed considering the perception of environmental pollution for each firm and is multiplied by the square of the amount of emissions produced to form the environmental pollution function.

In the model, firms are integrated into a three-phase competition process. In the first phase, firms make abatement choices, in the second phase, the regulator imposes taxes, and in the last phase, firms determine their production capacity by making strategic variable choices under the conditions of the market structure they are in. Solution of the game contains Backward induction method.

4.2. Cournot Duopoly

4.2.1. Firms Do Not Abate

If both firms choose not to involve abatement process in the Cournot duopoly, their profit functions at the last phase can be expressed as follows:

$$\pi_i^{NA} = P_i Q_i - tq_i$$

For firm 1:

$$\pi_1^{NA} = P_1 Q_1 - tq_1$$

$$\pi_1^{NA} = (1 - q_1 - q_2 - t)q_1$$

For firm 2:

$$\pi_2^{NA} = P_2 Q_2 - tq_2$$

$$\pi_2^{NA} = (1 - q_1 - q_2 - t)q_2$$

To find firm's best response functions, when they are competing each other in the last phase, we need to take first derivatives of profit functions according to their strategic variables which are q_1 and q_2 :

$$\frac{d_{\pi_1 NA}}{d_{q_1}} = 1 - 2q_1 - q_2 - t$$

$$q_1^{BR} = \frac{1-q_2-t}{2} ;$$

$$\frac{d_{\pi_2 NA}}{d_{q_2}} = 1 - 2q_2 - q_1 - t$$

$$q_2^{BR} = \frac{1 - q_1 - t}{2}$$

To find Cournot equilibrium (q_1^*, q_2^*) we need to solve best response functions simultenously: After the solution, we find [1]:

$$q_1^* = \frac{1-t}{3},\ q_2^* = \frac{1-t}{3}$$

[1] See Appendix 2 for the details.

At the second phase, government levies a tax t^{*CNA} that maximizes social welfare; to reach out the optimal tax rate, we need to derive social welfare functions. Following Buccella, Fanti and Gori (2021) model, social welfare function can be described as:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{NA}}+\mathrm{CS}^{\mathrm{NA}}+\mathrm{GR}^{\mathrm{NA}}-\mathrm{ED}^{\mathrm{NA}}$$

which is the sum of producer surplus, consumer surplus, government revenue minus environmental damage. It can be shown as;

$$\mathrm{PS}^{\mathrm{NA}}=\pi_1{}^{\mathrm{NA}}+\pi_2{}^{\mathrm{NA}}$$

$$\mathrm{CS}^{\mathrm{NA}}=\frac{(q_1+q_2)^2}{2}$$

$$\mathrm{GR}^{\mathrm{NA}}=\mathrm{t}.(q_1+q_2)$$

$$\mathrm{ED}^{\mathrm{NA}}=\rho\,(q_1+q_2)^2$$

So; according to these formulas; values will be as follows:

$$\mathrm{PS}^{\mathrm{NA}}=\frac{2}{9}(1-\mathrm{t})^2$$

$$\mathrm{CS}^{\mathrm{NA}}=\frac{2}{9}(1-\mathrm{t})^2$$

$$\mathrm{GR}^{\mathrm{NA}}=\mathrm{t}\left(\frac{2}{3}\right)(1-\mathrm{t})$$

$$\mathrm{ED}^{\mathrm{NA}}=\rho\left(\frac{2-2\mathrm{t}}{3}\right)^2$$

After calculation, we get:

$$\mathrm{SW}=\frac{2}{9}(1-t)^2+\frac{2}{9}(1-t)^2+\mathrm{t}\left(\frac{2}{3}\right)(1-t)-\rho\left(\frac{2-2t}{3}\right)^2$$

$$\mathrm{SW}=-\frac{2}{9}(-1+t)(2+t-2\rho+2t\rho)$$

After finding the social welfare function, we need to take first derivative of social welfare function according to t:

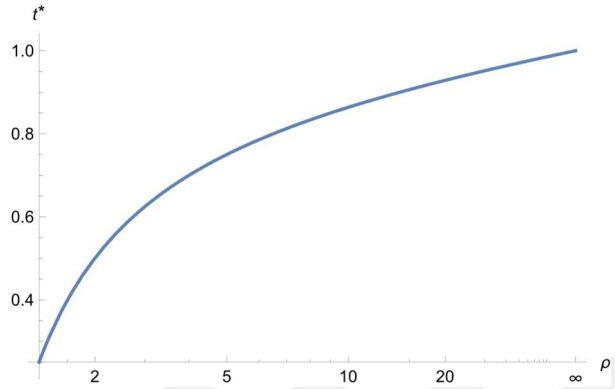
$$\frac{dSW}{dt}=-\frac{2}{9}(-1+t)(1+2\rho)-\frac{2}{9}(2+t-2\rho+2t\rho)$$

And solving $\frac{dSW}{dt}=0$ with respect to t gives us; optimal tax rate that government levies on at the second stage;

$$t^{*NA}=\frac{-1+4\rho}{2(1+2\rho)}$$

We can see the relationship between $\boldsymbol{t}^{*}$and **ρ** as follows:

Figure 2 : The Relationship Between t^{*} and ρ In the Cournot Not Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. There is positive relationship between taxation and environmental damage in the situation of firms choose not abatement in the pure Cournot duopoly.

When we substitute t^{*NA} optimal tax into the profit functions; (${\pi_1}^{*NA}{\pi_2}^{*NA}$) for first stage.

$$({\pi_1}^{*NA}{\pi_2}^{*NA}) = (\frac{1}{4(1+2\rho)^2}, \frac{1}{4(1+2\rho)^2}$$

We find the same level profit for both firms;

$${\pi_1}^{*NA} = {\pi_2}^{*NA}$$

4.2.2. Firms Abate

If two firms in pure Cournot duopoly adopt themselves the abatement technology, with $k_i \in (0, q_i)$, According to firm's preferences, firm's profit functions take the following form:

$$\pi_i^{A} = P_i Q_i - t(q_i - k_i) - \frac{z{k_i}^2}{2}$$

For firm 1:

$$\pi_1^{A} = (1 - q_1 - q_2)q_1 - t(q_1 - k_1) - \frac{z{k_1}^2}{2}$$

For firm 2:

$$\pi_2{}^A = (1 - q_1 - q_2)q_2 - t(q_2 - k_2) - \frac{zk_2{}^2}{2}$$

To find firm's best response functions and optimal emission levels, when they competing each other in the last phase, we need to take first derivatives of profit functions according to their strategic variables which are q_1 and q_2 and k_1 and k_2:

For firm 1:

$$\frac{d_{\pi_1{}^A}}{d_{q_1}} = 1 - 2q_1 - q_2 - t = 0$$

$$q_1{}^{BR} = \frac{1 - q_2 - t}{2}$$

$$\frac{d_{\pi_1{}^A}}{d_{k_1}} = t - zk_1 = 0$$

$$k_1{}^{opt} = \frac{t}{z}$$

For firm 2:

$$\frac{d_{\pi_2{}^A}}{d_{q_2}} = 1 - 2q_2 - q_1 - t = 0$$

$$q_2{}^{BR} = \frac{1 - q_1 - t}{2}$$

$$\frac{d_{\pi_2{}^A}}{d_{k_2}} = t - zk_2 = 0$$

$$k_2{}^{opt} = \frac{t}{z}$$

To find Cournot equilibrium $(q_1{}^*, q_2{}^*)$ we need to solve best response functions simultaneously: After the solution, we find[2] :

$$q_1{}^* = \frac{1 - t}{3},\ q_2{}^* = \frac{1 - t}{3}$$

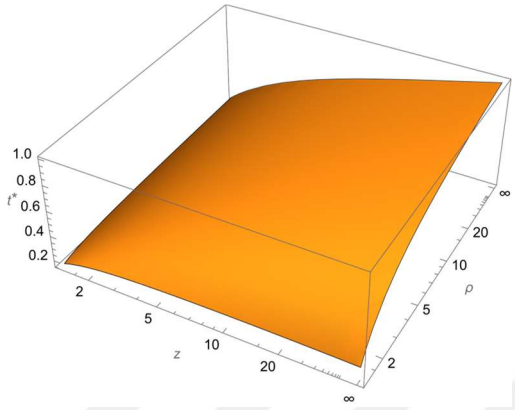
At the second phase, government levies a tax t^{*CNA} that maximizes social welfare, to reach out the optimal tax rate[3]:

[2] See Appendix 2 for the details.
[3] See Appendix 3 for the details.

$$t^{*A} = \frac{z(-z+6\rho+2z\rho)}{9z+2z^2+18\rho+12z\rho+2\ ^2\rho}$$

We can see the relationship between t^*, z and ρ as follows:

Figure 3 : The Relationship Between t^*, z and ρ In the Cournot Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of firms choose abatement in the pure Cournot duopoly.

When we substitute t^{*A} optimal tax into the profit functions; (${\pi_1}^{*A}{\pi_2}^{*A}$) for first stage; We find the same level profit for both firms[4] ;

$${\pi_1}^{*CA} = {\pi_2}^{*CA}$$

4.2.3. Asymmetric case: Abate / Not Abate

If one firm in pure Cournot duopoly choose not abatement and the other firm adopt themselves the abatement technology, with $k_i \in (0, q_i)$, In the last phase, firm 2 chooses both the output and the abatement level whereas firm 1 chooses only output. According to firm's preferences, firm's profit functions take the following form:

$$\pi_1 = P_1 Q_1 - t q_1$$

[4] See Appendix 3 for the details.

$$\pi_2 = P_2Q_2 - \frac{Z}{2}k_2 - t(q_2 - k_2)$$

For firm 1:

$$\pi_1{}^{NA} = P_1Q_1 - tq_1$$

$$\pi_1{}^{NA} = (1 - q_1 - q_2 - t)q_1$$

Best response function of firm 1 is shaped like:

$$\frac{d_{\pi_1{}^{NA}}}{d_{q_1}} = 1 - 2q_1 - q_2 - \mathrm{t} = 0$$

$$q_1{}^{BR} = \frac{1 - q_2 - t}{2}$$

Profit function of firm 2 is shaped like:

$$\pi_2{}^{A} = P_2Q_2 - \frac{Z}{2}k_2 - t(q_2 - k_2)$$

$$\pi_2{}^{A} = (1 - q_1 - q_2)q_2 - t(q_2 - k_2) - \frac{zk_2{}^2}{2}$$

Also, best response function and optimum level of firm 2 is :

$$\frac{d_{\pi_2{}^{A}}}{d_{q_2}} = 1 - 2q_2 - q_1 - \mathrm{t} = 0$$

$$q_2{}^{BR} = \frac{1 - q_1 - t}{2}$$

$$\frac{d_{\pi_2{}^{A}}}{d_{k_2}} = \mathrm{t\text{-}z}k_2 = 0$$

$$k_2{}^{opt} = \frac{t}{z}$$

To find Cournot equilibrium $(q_1{}^*, q_2{}^*)$ we need to solve best response functions simultenously: After the solution, we find[5] :

$$q_1{}^* = \frac{1 - t}{3},\ q_2{}^* = \frac{1 - t}{3}$$

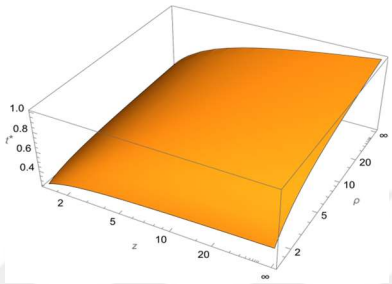
Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second stage.

[5] See Appendix 2 for the details.

$$t^{*A/NA} = \frac{z(2z+3\rho+z\rho)}{9z+8z^2+9\rho+6z\rho+z^2\rho} \quad 6$$

We can see the relationship between t^*, z and ρ as follows:

Figure 4 : The Relationship Between t^*, z and ρ In the Cournot Asymmetric Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of one firm choose abate the other one choose not abate in the pure Cournot duopoly.

When we substitute t^{*A} optimal tax into the profit functions; (${\pi_1}^{*NA}{\pi_2}^{*A}$) for first stage; We find the same level profit for both firms.

$${\pi_2}^{*A} > {\pi_1}^{*NA}{}_7$$

4.3. Bertrand Duopoly

4.3.1. Overview

Now we continue with Bertrand duopoly. In this section, there will be different strategic variable for firms than the Cournot duopoly. In this case firms selects price

[6] See Appendix 4 for the details.
[7] See Appendix 4 for the details

rather than quantity to maximize their profits. Also, we add product differentiation as a parameter to our calculation.

With differentiated products, the inverse demand will be:

$$p_1 = 1 - q_1\text{-d}q_2$$

Bertrand setting implies:

$$q_1 = \frac{1-p_1-d(1-p_2)}{(1-d)^2}$$

So, profit functions of firm1 according to Bernard settings will be;

$$\pi_1{}^{BA} = p_1\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right] - t\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2} - k_1\right] - \frac{zk_1{}^2}{2}$$

and;

$$\pi_1{}^{BNA} = p_1\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right] - t\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right]$$

For firm 2:

$$\pi_2{}^{BA} = p_2\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right] - t\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2} - k_2\right] - \frac{zk_2{}^2}{2}$$

And

$$\pi_2{}^{BNA} = p_1\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right] - t\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right]$$

where subscript of BNA represents 'Not Abate' and BA represents 'Abate' in Bertrand settings.

4.3.2. Firms Do Not Abate

If two firms choose not abatement in the Bertrand duopoly, their profit functions will be at the last phase:

$$\pi_i{}^{BNA} = P_iQ_i - tq_i$$

$$\pi_1{}^{BNA} = P_1Q_1 - tq_1$$

For Firm 1:

$$\pi_1{}^{BNA} = p_1\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right] - t\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right]$$

Best response function is shaped like:

$$\frac{d_{\pi_1{}^{BNA}}}{d_{p_1}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_1}{(1-d)^2} + \frac{dp_2}{(1-d)^2} = 0$$

$$P_1{}^{BR} = \frac{1-d-t+dp_2}{2}$$

To continue with firm 2, first we need to derive firm 2's profit function:

$$\pi_2{}^{BNA} = P_2 Q_2 - tq_2$$

$$\pi_2{}^{BNA} = p_2\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right] - t\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right]$$

Best response function is shaped like:

$$\frac{d_{\pi_2{}^{BNA}}}{d_{p_2}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_2}{(1-d)^2} + \frac{dp_1}{(1-d)^2} = 0$$

$$P_2{}^{BR} = \frac{1-d-t+dp_1}{2}$$

To find Bertrand equilibrium $(p_1{}^*, p_2{}^*)$ we need to solve best response functions simultenously: After calculation we find:

$$P_1{}^* = \frac{1-d+t}{2-d},\ P_2{}^* = \frac{1-d+t}{2-d}$$

Since $q_1 = \frac{1-p_1-d(1-p_2)}{(1-d)^2}$ and $q_2 = \frac{1-p_2-d(1-p_1)}{(1-d)^2}$ and we have equilibrium prices, we plug this prices into the quantity functions and then we reach the quantities in that price level.

$$Q_1{}^* = \frac{1-t}{2-3d+d^2},\ Q_2{}^* = \frac{1-t}{2-3d+d^2}$$

At the second phase, government levies a tax t^{*BNA} that maximizes social welfare; to reach out the optimal tax rate, we need to derive social welfare functions.

Social welfare can be found as we did it in the previous calculations in the Cournot duopoly model, after finding the social welfare function, we need to take first derivative of this function according t to get optimal tax rate ;

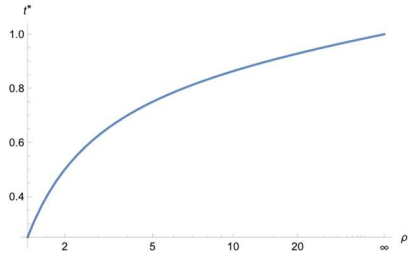
After calculation we find:

$$t^{*BNA} = \frac{1-4d+d^2+4d\rho-4\rho}{-7+2d+4\rho}$$ [8]

[8] See Appendix 6 for the details.

We can see the relationship between $t^*$and **ρ** as follows:

Figure 5 : The Relationship Between t^* and ρ In the Bertrand Not Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. There is positive relationship between taxation and environmental damage in the situation of firms choose not abatement in the pure bertrand duopoly.

When we substitute t^{*BNA} optimal tax into the profit functions; (${\pi_1}^{*BNA}{\pi_2}^{*BNA}$) for first stage; We find the same level profit for the both firms ;

$$ {\pi_1}^{*BNA} = {\pi_2}^{*BNA}{}^{9} $$

4.3.3. Firms Abate

If two firms in Bertrand duopoly adopt themselves the abatement technology, with $k_i \in (0, q_i)$. According to firm's preferences, firm's profit functions take the following form:

$$ {\pi_i}^{BA} = P_i Q_i \ - t(q_i - k_i) - \frac{z{k_i}^2}{2} $$

For firm 1

$$ {\pi_1}^{BA} = p_1 \left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right] - t\left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2} - k_1\right] - \frac{z{k_1}^2}{2} $$

For firm 2:

$$ {\pi_2}^{BA} = p_2 \left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right] - t\left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2} - k_2\right] - \frac{z{k_2}^2}{2} $$

Best response function of firm 1 is shaped like:

[9] See Appendix 6 for the details.

$$\frac{d_{\pi_1 BA}}{d_{p_1}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_1}{(1-d)^2} + \frac{dp_2}{(1-d)^2} = 0$$

$$P_1^{BR} = \frac{1-d-t+dp_2}{2}$$

And optimum abatement level:

$$\frac{d_{\pi_1 BA}}{d_{k_1}} = \text{t-z}k_1 = 0$$

$$k_1^{BAopt} = \frac{t}{z}$$

Best response and optimum level of firm 2 is shaped like:

$$\frac{d_{\pi_2 BA}}{d_{p_1}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_2}{(1-d)^2} + \frac{dp_1}{(1-d)^2} = 0$$

$$p_2^{BR} = \frac{1-d-t+dp_1}{2}$$

And optimum abatement level:

$$\frac{d_{\pi_2 BA}}{d_{k_2}} = \text{t-z}k_2 = 0$$

$$k_2^{Bopt} = \frac{t}{z}$$

To find Bertrand equilibrium (p_1^*, p_2^*) we need to solve best response functions simultenously: After calculation we find[10]

$$P_1^* = \frac{1-d+t}{2-d},\ P_2^* = \frac{1-d+t}{2-d}$$

Since $q_1 = \frac{1-p_1-d(1-p_2)}{(1-d)^2}$ and $q_2 = \frac{1-p_2-d(1-p_1)}{(1-d)^2}$ and we have equilibrium prices, we plug this prices into the quantity functions and then we reach the quantities in that price level.

$$Q_1^* = \frac{1-t}{2-3d+d^2},\ Q_2^* = \frac{1-t}{2-3d+d^2}$$

At the second phase, government levies a tax t^{*CNA} that maximizes social welfare, to reach out the optimal tax rate, differently, now two firms choose abatement and their optimal emission level, so we need to take optimal emission level, k_1^{opt} and k_2^{opt} into calculation. Also, we need to adjust PS, CS, GR, ED formulas to abatement situation.

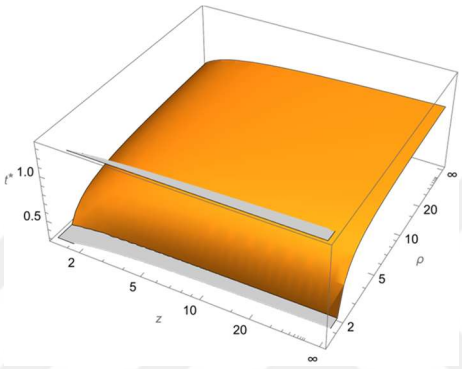
[10] See Appendix 5 for the details.

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second phase. After the calculation we find[11]:

$$t^{*BA} = \frac{2z\left(-4z+11dz-10d^2z+2d^3z-8\rho+20d\rho-1\quad{}^2\rho+4d^3\rho+4z\rho-8dz\rho+4d^2z\rho\right)}{\left(-16z+32dz-20d^2z+4d^3z+16z^2-24dz^2+3d^2z^2+d^3z^2-32\rho+64d\rho-40\ {}^2\rho+8d^3\rho+32z\rho-48dz\rho+\quad{}^2z\rho-8z^2\rho+8dz^2\rho\right)}$$

For d = 0.5, we can see the relationship between t^*,z and ρ as follows:

Figure 6 : The Relationship Between t^*, z and ρ In the Bertrand Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of abatement in the pure Bertrand duopoly.

When we substitute t^{*NA} optimal tax into the profit functions; (${\pi_1}^{*BA}{\pi_2}^{*BA}$) for first stage; We find the same level profit for the both firms[12] ;

$${\pi_1}^{*BA} = {\pi_2}^{*BA}$$

[11] See Appendix 7 for the details.
[12] See Appendix 7 for the details.

4.3.4. Asymmetric case: Abate / Not Abate

If one firm in pure Bertrand duopoly choose not abatement and the other firm adopt themselves the abatement technology, with $k_i \in (0, q_i)$ According to firm's preferences, firm's profit functions take the following form:

$$\pi_1 = P_1 Q_1 - t q_1$$

$$\pi_2 = P_2 Q_2 - \frac{Z}{2} k_2 - t(q_2 - k_2)$$

For firm 1:

$$\pi_1{}^{BNA} = p_1 \left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right] - t \left[\frac{(1-p_1-d(1-p_2))}{(1-d)^2}\right]$$

For firm 2:

$$\pi_2{}^{BA} = p_2 \left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2}\right] - t \left[\frac{(1-p_2-d(1-p_1))}{(1-d)^2} - k_2\right] - \frac{z{k_2}^2}{2}$$

Best response function of firm 1 is shaped like:

$$\frac{d_{\pi_1 B^{NA}}}{d_{p_1}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_1}{(1-d)^2} + \frac{dp_2}{(1-d)^2} = 0$$

$$P_1{}^{BR} = \frac{1-d-t+dp_2}{2}$$

Bes response function of firm 2 is shaped like:

$$\frac{d_{\pi_2 BA}}{d_{p_1}} = \frac{1}{(1-d)^2} - \frac{d}{(1-d)^2} + \frac{t}{(1-d)^2} - \frac{2p_2}{(1-d)^2} + \frac{dp_1}{(1-d)^2} = 0$$

$$p_2{}^{BR} = \frac{1-d-t+dp_1}{2}$$

And optimum abatement level:

$$\frac{d_{\pi_2 BA}}{d_{k_2}} = \text{t-z}k_2 = 0$$

$$k_2{}^{Bopt} = \frac{t}{z}$$

To find Bertrand equilibrium $(p_1{}^*, p_2{}^*)$ we need to solve best response functions simultenously: After calculation we find[13]

[13] See Appendix 5 for the details.

$$P_1^* = \frac{1-d+t}{2-d},\ P_2^* = \frac{1-d+t}{2-d}$$

Since $q_1 = \frac{1-p_1-d(1-p_2)}{(1-d)^2}$ and $q_2 = \frac{1-p_2-d(1-p_1)}{(1-d)^2}$ and we have equilibrium prices, we plug this prices into the quantity functions and then we reach the quantities in that price level.

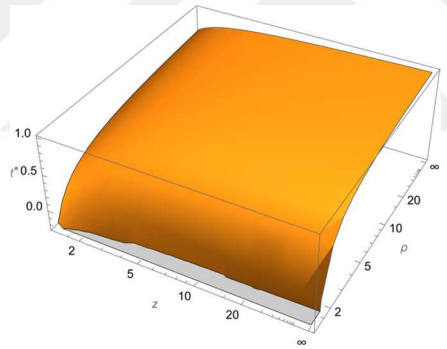
$$Q_1^* = \frac{1-t}{2-3d+d^2},\ Q_2^* = \frac{1-t}{2-3d+d^2}$$

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second phase[14]:

$$t^{*BA/NA} = \frac{4z(z-4dz+{}^2z+2\rho-3d\rho+{}^2\rho-2z\rho+2dz\rho)}{4z-4dz+d^2z-28z^2+8dz^2+8\rho-8d\rho+2d^2\rho-16z\rho+8dz\rho+{}^2\rho}$$

For d=0.5: we can see the relationship between t^*,z and ρ as follows:

Figure 7 : The Relationship Between t^*, z and ρ In the Bertrand Abatement / Not Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of abatement in the asymmetric Bertrand duopoly.

[14] See Appendix 6 for the details.

When we substitute $t^{*BA/NA}$ optimal tax into the profit functions; (${\pi_1}^{*BA/NA}{\pi_2}^{*BA/NA}$) for first stage; We find[15] :

$${\pi_2}^{*BA/NA} > {\pi_1}^{*BA/NA}$$

4.4. Cournot-Bertrand Duopoly

4.4.1. Overview

Now we continue pollution abatement with Cournot-Betrand duopoly. In this case, two firms choose their strategic variable, for instance, firm 1 chooses quantity and firm 2 choose price then they start to compete in their strategic variables.

According to Tremblay and Tremblay (2019), we consider a Cournot-Bertrand oligopoly market where two firms, i =1,2 produce a differentiated product. We assume that firm 1 's strategic variable is quantity facing an inverse demand with $a = 1$, so the inverse demand function of firm 1 is shaped like:

$$p_1 = (1-d)a - (1-d^2)q_1 + \mathrm{d}p_2$$

While firm 2's strategic variable is price facing a demand:

$$q_2 = 1 - p_2 - \mathrm{d}q_1$$

Where $a = 1$ represents the size of the market and d∈(0,1) represents product differentiation.

4.4.2. Firms Do Not Abate

If two firms choose not abatement in the Cournot-Bertrand duopoly, their profit functions will be at the last phase:

$${\pi_i}^{CBNA} = P_i Q_i - tq_i$$

For firm 1;

$${\pi_1}^{CBNA} = P_1 Q_1 - tq_1$$

$${\pi_1}^{CBNA} = [(1-d) - (1-d^2)q_1 + \mathrm{d}p_2]\, q_1 - tq_1$$

Best response function is[16] :

[15] See Appendix 8 for the details.
[16] See Appendix 9 for the details.

$$q_1{}^{BR} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

For firm 2:

$$\pi_2{}^{CBNA} = P_2 Q_2 \ - tq_2$$

$$\pi_2{}^{CBNA} = P_2(1 - p_2 - \mathrm{d}q_1) - t(a - p_2 - \mathrm{d}q_1)$$

Best response function is shaped like:

$$p_2{}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

To find Cournot Bernard equilibrium $(q_1{}^*, p_2{}^*)$ we need to solve best response functions simultenously[17] : So the Cournot- Bertrand equilibrium is: $\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} \ , \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\right)$

Substituting the above points into functions for p_1 and q_2[18]:

$$p_1{}^* = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2}$$

And

$$q_2{}^* = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second stage[19]:

$$t^{*CBNA} = \frac{(-2d^5-4(-4+\mathrm{dt})\rho^2+d(-14+3\mathrm{dt}-8\rho^2)+2d^4(-1+\rho^2)+d^3(11-\mathrm{dt}+4\rho^2)+d^2(5+2(-6+\mathrm{dt})\rho^2))}{(2(-2+d^2)(-2d+d^3+2(1+\rho^2)-d^2(2+\rho^2)))}$$

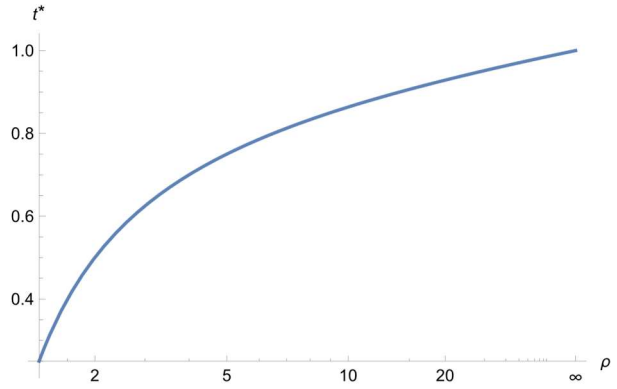
For d=0.5. we can see the relationship between $t^*$and ρ as follows:

[17] See Appendix 9 for the details.

[18] See Appendix 9 for the details.

[19] See Appendix 10 for the details.

Figure 8 : The Relationship Between t^* and ρ In the Cournot-Bertrand Not Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. There is positive relationship between taxation and environmental damage in the situation of firms choose not abatement in the cournot-bertrand duopoly.

When we substitute t^{*NA} optimal tax into the profit functions; (${\pi_1}^{*CBNA}{\pi_2}^{*CBNA}$) for first stage; We find that counot earns more profit than the Bertrand firm in the not abatement case[20].

$$ {\pi_1}^{*CBNA} > {\pi_2}^{*CBNA} $$

4.4.3. Firms Abate

If two firms in Cournot-Bertrand duopoly adopt themselves the abatement technology, with $k_i \in (0, q_i)$. According to firm's preferences, firm's profit functions take the following form:

$$ {\pi_i}^{CBA} = P_i Q_i \ - t(q_i - k_i) - \frac{z{k_i}^2}{2} $$

For firm 1:

$$ {\pi_1}^{CBA} = P_1 Q_1 \ - t(q_1 - k_1) - \frac{z{k_1}^2}{2} $$

$$ {\pi_1}^{CBA} = [(1-d) - (1-d^2)q_1 + \mathrm{d}p_2]q_1 - t(q_1 - k_1) - \frac{z({k_1}^2)}{2} $$

[20] See Appendix 11 for the details.

Best response function and optimum level of firm 1 is shaped like[21]:

$$q_1{}^{BR} = \frac{1-d-t+\mathrm{d}p_2}{2(1-d^2)}$$

$$k_1{}^{CBopt} = \frac{t}{z}$$

To continue with firm 2, first we need to derive firm 2's profit function:

For firm 2:

$$\pi_2{}^{CBA} = P_2Q_2 - t(q_2 - k_2) - \frac{z(k_2{}^2)}{2}$$

$$\pi_2{}^{CBA} = P_2(1 - p_2 - \mathrm{d}q_1) - t(1 - p_2 - \mathrm{d}q_1) + \mathrm{t}k_2 - \frac{z(k_2{}^2)}{2}$$

Best response function and optimum level of firm 2 is shaped like:

$$p_2{}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

$$k_2{}^{CBopt} = \frac{t}{z}$$

To find Cournot Bernard equilibrium $(q_1{}^*, p_2{}^*)$ we need to solve best response functions simultenously: So, the Cournot- Bertrand equilibrium is: $\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2}, \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\right)$

Substituting the above points into functions for p_1 and q_2[22]:

$$p_1{}^* = -\frac{(-1+d^2)(-2+d+d\ )}{-4+3d^2}$$

And

$$q_2{}^* = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second stage[23]:

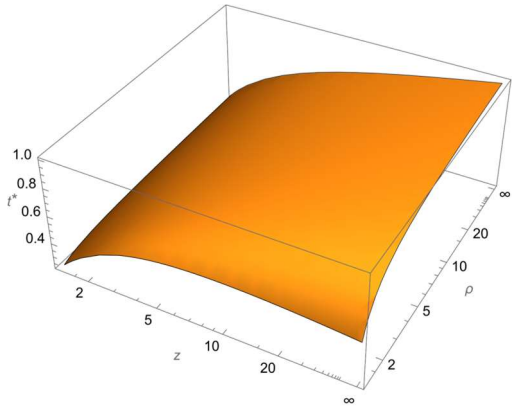
$$t^{*CBA} =$$

$$\frac{-((z(2d^5z+2(-4+\mathrm{dt})(-4+z)\rho+d^4(-z(-2+\rho)+6\rho)+d^3(z(-11+\mathrm{dt}-2\rho)+12\rho)+d(-16\rho+\ (14-3\mathrm{dt}+4\rho))-d^2(32\rho-6\mathrm{dt}\rho+z(5+(-6+\mathrm{dt})\rho))))}{(-4(4-3d^2)^2\rho+(-2+d^2)z^2(-4d+2d^3+2(2+\rho)-d^2(4+\rho))+2(-4+3d^2)z(4-4\rho+d^2(-3+2\rho)}$$

For d=0,5. we can see the relationship between t^*,z and ρ as follows:

[21] See Appendix 9 for the details.
[22] See Appendix 9 for the details.
[23] See Appendix 13 for the details.

Figure 9 : The Relationship Between t^*, z and ρ In the Cournot-Bertrand Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of abatement in Cournot Bertrand duopoly.

When we substitute t^{*NA} optimal tax into the profit functions; (${\pi_1}^{*CBA}{\pi_2}^{*CBA}$) for first stage; We find that Cournot firm earns higher profit than the Bertrand firm[24].

$$ {\pi_1}^{*CBA} > {\pi_2}^{*CBA} $$

4.4.4. Asymmetric case: Not Abate / Abate

In the Cournot-Bertrand dupoly, if firm 1 decides aganist abatement and firm 2 decides to follow the abatement process, with $k_i \in (0, q_i)$, Following the firm's preferences, the profit functions of the firm are as follows:

$$ {\pi_i}^{CBNA} = P_i Q_i \ - t q_i $$

For firm 1;

$$ {\pi_1}^{CBNA} = P_1 Q_1 \ - t q_1 $$

$$ {\pi_1}^{CBNA} = [(1-d) - (1-d^2) q_1 + \mathrm{d} p_2] \, q_1 \ - t q_1 $$

[24] See Appendix 14 for the details.

Best response function is shaped like[25] :

$$q_1{}^{BR} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

To continue with firm 2, first we need to derive firm 2's profit function:

For firm 2;

$$\pi_2{}^{CBA} = P_2Q_2 - t(q_2 - k_2) - \frac{z(k_2{}^2)}{2}$$

$$\pi_2{}^{CBA} = P_2(1 - p_2 - \mathrm{d}q_1) - t(1 - p_2 - \mathrm{d}q_1) + \mathrm{t}k_2 - \frac{z(k_2{}^2)}{2}$$

Best response function and optimum level of firm 2 is shaped like :

$$p_2{}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

$$k_2{}^{CBopt} = \frac{t}{z}$$

To find Cournot-Bernard equilibrium $(q_1{}^*, p_2{}^*)$ we need to solve best response functions simultenously:

So the Cournot- Bertrand equilibrium is: $\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2}\ , \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\right)$ [26]

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second phase[27]:

$t^{*CBNA/A}$

$$= \frac{-((z(2d^5z-8(-2+z)\rho+d^4(-z(-2+\rho)+3\rho)+d^3(z(-11+\mathrm{dt}-3\rho)+6\rho)+d^2(-16\rho+\ (-5+4\rho))+d(-8\rho+z(14-3\mathrm{dt}+8\rho))))}{(-(4-3d^2)^2\rho+z^2(2d^5-4(2+\rho)+4d(2+\rho)+3d^2(4+\rho)-2d^3(4+\rho)-d^4(4+\rho))+(-4+3d^2)z(4-4\rho+2d\rho+\ {}^2(-3+2\rho))))}$$

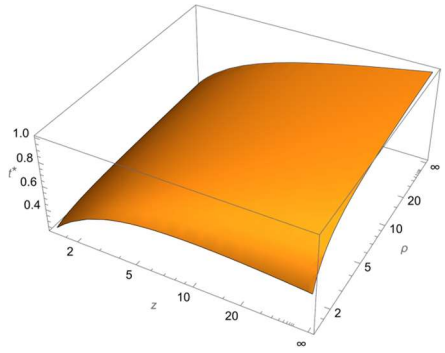
For d=0.5, we can see the relationship between t^*,z and ρ as follows:

[25] See Appendix 9 for the details.

[26] See Appendix 9 for the details.

[27] See Appendix 15 for the details.

Figure 10 : The Relationship Between t^*, z and ρ In the Cournot-Bertrand Not Abatement / Abatement Case



Source: This figure is generated using Mathematica

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second stage. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of abatement in Cournot Bertrand duopoly.

When we substitute $t^{*CBNA/A}$ optimal tax into the profit functions; $({\pi_1}^{*CBNA/A}{\pi_2}^{*CBNA/A})$ for first stage; We find that bertrand firm earns higher profit than the cournot firm in the Not abatement / abatement situation[28].

4.4.5. Asymmetric case: Abate / Not Abate

If firm 1 in Cournot-Bertrand duopoly choose abatement and adopt themselves the abatement technology, with $k_i \in (0, q_i)$, According to firm's preferences, firm's profit functions take the following form:

For firm 1;

$$ {\pi_i}^{CBA} = P_i Q_i \ - t(q_i - k_i) - \frac{z{k_i}^2}{2} $$

$$ {\pi_1}^{CBA} = [(1-d) - (1-d^2)q_1 + \mathrm{d}p_2]q_1 - t(q_1 - k_1) - \frac{z({k_1}^2)}{2} $$

Best response function and optimum level of firm 1 shaped like :

[28] See Appendix 16 for the details.

$$q_1{}^{BR} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

$$k_1{}^{CBopt} = \frac{t}{z}$$

For firm 2;

$$\pi_2{}^{CBNA} = P_2Q_2 - tq_2$$

$$\pi_2{}^{CBNA} = P_2(1 - p_2 - dq_1) - t(a - p_2 - dq_1)$$

Best response function is[29] :

$$p_2{}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

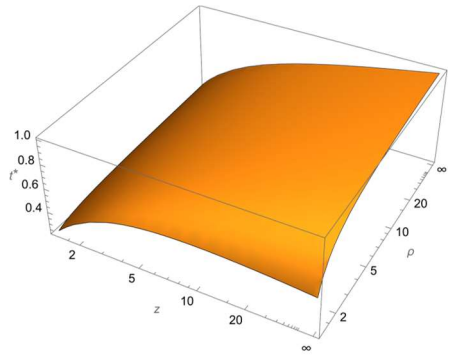
To find Cournot-Bertrand equilibrium $(q_1{}^*, p_2{}^*)$ we need to solve best response functions simultaneously: So the Cournot- Bertrand equilibrium is: $\left(\frac{2-d-dt}{4-3d^2}, \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\right)$ [30]

Taking these values into the SW formula and taking the SW with respect to t gives us the optimal tax rate that government levies on at the second phase[31]:

$$\frac{-\left(z(14dz-5d^2z-11d^3z+2d^4z+2d^5z-3d\,dtz+{}^3dtz+16\rho-8d\rho-16d^2\rho+6d^3\rho+3d^4\rho-8z\rho+8dz\rho+4d^2z\rho-3d^3z\rho-d^4z\rho)\right)}{(-16z+24d^2z-9d^4z-8z^2+8dz^2+12d^2z^2-8d^3z^2-4d^4z^2+2d^5z^2-16\rho+24d^2\rho-9d^4\rho+16z\rho-8dz\rho-20\ {}^2z\rho+6d^3z\rho+6d^4z\rho-4z^2\rho+4dz^2\rho+3d^2z^2\rho-2d^3z^2\rho-d^4}$$

For d=0.5, we can see the relationship between t^*,z and ρ as follows:

Figure 11 : The Relationship Between t^*, z and ρ In the Cournot-Bertrand Abatement / Not Abatement Case



Source: This figure is generated using Mathematica.

[29] See Appendix 9 for the details.
[30] See Appendix 12 for the details.
[31] See Appendix 17 for the details.

We see in the graph that when perceived environmental damage (ρ) increases, the tax rate in the second stage also will increase. Also, when abating becomes expensive for firms, upper values for z, the tax rate also will increase in the second phase. There is positive relationship between taxation and environmental damage. Also there is positive relationship between taxation and the cost of abatement in the situation of abatement in Cournot-Bertrand duopoly.

When we substitute $t^{*CBA/NA}$ optimal tax into the profit functions; (${\pi_1}^{*CBA/NA}{\pi_2}^{*CBA/NA}$) for first stage; Cournot firm that chooses abatement makes more profit than the Bertrand firm chooses not abatement.[32]

$$ {\pi_1}^{*CBA/NA} > {\pi_2}^{*CBA/NA} $$

[32] See Appendix 18 for the details.

5. DISCUSSIONS

The results realized within duopolies are summarized in the table below.

Table 1: Results of Abatement Games Across the Duopolies In terms of Firms' Profit

$\boldsymbol{\pi}^*$	Cournot	Bertrand	Cournot- Bertrand
Abate / Abate	${\pi_1}^{*\mathrm{CA}} = {\pi_2}^{*\mathrm{CA}}$	${\pi_1}^{*\mathrm{BA}} = {\pi_2}^{*\mathrm{CBA}}$	${\pi_1}^{*\mathrm{CBA}} > {\pi_2}^{*\mathrm{CBA}}$
Abate / Not Abate	${\pi_1}^{*\mathrm{CA}} > {\pi_2}^{*\mathrm{CNA}}$	${\pi_1}^{*\mathrm{BA}} > {\pi_2}^{*\mathrm{BNA}}$	${\pi_1}^{*\mathrm{CBA}} > {\pi_2}^{*\mathrm{CBNA}}$
Not Abate / Abate	${\pi_2}^{*\mathrm{CA}} > {\pi_1}^{*\mathrm{CNA}}$	${\pi_2}^{*\mathrm{BA}} > {\pi_1}^{*\mathrm{BNA}}$	${\pi_1}^{*\mathrm{CBA}} < {\pi_2}^{*\mathrm{CBNA}}$
Not Abate / Not Abate	${\pi_1}^{*\mathrm{CA}} = {\pi_2}^{*\mathrm{CA}}$	${\pi_1}^{*\mathrm{BA}} = {\pi_2}^{*\mathrm{CBA}}$	${\pi_1}^{*\mathrm{CBA}} > {\pi_2}^{*\mathrm{CBA}}$

In this table, firms are compared according to their abatement preferences. In the Cournot duopoly, firms with the same choice, i.e. when both firms chose to apply abatement technology or when both firms chose not to apply abatement technology, produced similar results in terms of profitability, while differences in choice differentiated the firms in terms of profitability. Similarly, the Bertrand duopoly provides a sample in which firm profitability results according to the symmetry of preferences. However, in the Cournot-Bertrand duopoly, when firms act by choosing different variables within the same industry, the asymmetric situation, that is, when one firm chooses to apply abatement technology while the other does not, differentiates the profitability levels of firms. In this respect, it differs from pure Cournot and pure Bertrand duopoly because of its market demand structure. In the Cournot-Bertrand duopoly, the firm that engages in the Cournot competition is more

profitable than the firm that engages in the Bertrand competition. In the Cournot-Bertrand model, since the pricing of the Cournot company is lower and the production amount is higher than the Bertrand company, Cournot-type companies are more profitable in the Cournot-Bertrand model. This result is consistent with Asproudis and Filippiadis (2020) and Tremblay and Tremblay (2019).

Considering the results realized in all duopoly structures, since companies receive tax deductions in return for remediating the emissions they create, investing in abatement technology is more profitable than when they do not invest in abatement technology.

In all three duopoly models, the tax imposed by the regulator to maximize social welfare in the second process of the abatement game is directly related to the variables z and p in the model[33]. In this case, as the cost of abatement increases, i.e. as the technology becomes inefficient, the regulator needs to impose more taxes. Also, increasing environmental pollution increases the amount of taxes that the regulator imposes to maximize social welfare.

Another conclusion to be drawn from this model is the comparison of social welfare within the three duopolies according to whether abatement technologies are implemented or not. The table below contains the relevant results.

Table 2 : Social Welfare Results of Abatement Game Across the Duopolies

SW^*	Rankings
Cournot	$SW^{*A} > SW^{*A/NA} > SW^{*NA}$
Bertrand	$SW^{*A} > SW^{*A/NA} > SW^{*NA}$
Cournot- Bertrand	$SW^{*A} > SW^{*A/NA} > SW^{*NA}$

In terms of social welfare, in all three duopoly models, social welfare is highest in the case where abatement technologies are applied, and second highest in the not

[33] See Appendix 21 for the details.

symmetric case. The application of abatement technologies reduces the burden on companies by providing emission improvement, which positively affects company profitability. At the same time since reducing emissions will reduce environmental pollution, social welfare will be greatest when two companies apply abatement technologies together.

Based on this result, we know that social welfare is higher in the case of abatement than the other situations. So, we can compare the social welfare situations of duopolies with the abatement choices they made.

Table 3: Social Welfare Results Comparison Between the Duopolies

SW^*	Rankings
Abate / Abate	$SW^{*BA} > SW^{*CBA} > SW^{*CA}$

In the abatement symmetric case, social welfare is highest in the case of the Bertrand duopoly, second highest in the case of the Cournot-Bertrand duopoly, and lowest in the Cournot duopoly.

The Bertrand duopoly involves more competition than the Cournot duopoly due to the differentiation of products produced by the companies. On the other hand, in the Cournot-Bertrand duopoly, since one company acts as Cournot and the other company acts as Bertrand, the competition in the market will be more than the Cournot duopoly but it will be less than in Bertrand.

Therefore, social welfare is ranked from the duopoly with the most competition to the duopoly with the least competition.

The findings of the thesis are valid in real life within the scope of environmental economics.

In the real industry structure, companies can act in the Cournot type, Bertrand typer, or Cournot-Bertrand type. For example, firms within the oil industry may consider maintaining their market share by choosing the amount that maximizes their profits. On the other hand, companies providing airline services can compete by keeping their prices lower.

At the same time, one company within the telecommunications industry may compete in a Cournot-type manner to develop its own infrastructure while another company may act in a Bertrand-type manner by keeping the price low to increase subscriptions.

Based on the competitive structures of companies operating in the industry, the policymaker can consider how the pollution emitted by companies in such industries can be corrected, with the industry dynamics in which the companies are located. Since social welfare increases with abatement, the policymaker can search for answers to how companies in these industries can be included in the abatement process? or what the potential policy tools should be**.** Based on the characteristics of the duopolies she examined, decides on the appropriate policy for the appropriate industry.

6. CONCLUSION

The model considered in this study involves a comparison of outcomes across duopolies with and without the implementation of abatement technologies under the policy instrument of tax deductions for emission-reducing investments. In doing so, Cournot, Bertrand and Cournot-Bertrand duopolies are used as the market structure. Firm profits are higher when abatement technologies are applied than when they are not, in addition, under the same market structure, the profitability of firms with symmetric preferences remained the same in Cournot and Bertrand duopolies, whereas firm profitability due to the market demand structure of Cournot-Bertrand differed in this duopoly. At the same time, the tax applied in the phase of the game was directly proportional to the technological efficiency coefficient z in the abatement cost function and to the perceived environmental damage p in the environmental damage function. While social welfare is maximized when abatement technologies are applied among the three duopolies. In addition, social welfare in the Bertrand duopoly is found to be higher than in other duopolies. All these results can provide clues about the industrial consequences in terms of outputs for a possible environmentally friendly regulation.
On the other hand, this study has some limitations. Duopoly market models may be insufficient to represent the real industry. For future research, the model can be extended to the case where firms engage in monopolistic competition and horizontally differentiate their products. Also, the environmental damage function can be modified by updating the initial perceived environmental damage coefficient with the cumulative intra-industry environmental damage coefficient to include the abatement of emissions emitted by firms.

With these additions, the model can more realistically represent industry dynamics and the surrounding economic framework than the current model.

REFERENCES

Acar Sevil & Yeldan A.Erinç. (2016). Environmental impacts of coal subsidies in Turkey: A general equilibrium analysis. *Energy Policy*, (90), 1-15.https://doi.org/10.1016/j.enpol.2015.12.003

Acar, Sevil & A. Erinç Yeldan. (2018). Investigating Patterns Of Carbon Convergence In An Uneven Economy: The Case Of Turkey. *Structural Change and Economic Dynamics,* (46), 96-106. https://doi.org/10.1016/j.strueco.2018.04.006

Asproudis Elias & Filippiadis Eleftherios. (2021). Environmental Technological Choice in a Cournot-Bertrand Model, *Journal of Industry, Competition & Trade,* (21): 43–58, https://doi.org/10.1007/s10842-020-00339-z.

Barcena Carlos Juan & Garzon Begona Marina. (2010). Endogenous Timing in a Mixed Duopoly with Capacity Choice, *The Manchester School,* (78), 93-109, https://doi.org/10.1111/j.1467-9957.2009.02137.x

Bimonte Giovanna, Romano Grazia Maria. & Russolillo Maria. (2021). Green Innovation and Competition: R&D Incentives in a Circular Economy. *Games,* (12), 68 https://doi.org/10.3390/g12030068

Buccela Domenica, Fanti Luciano & Gori Luca. (2021). To Abate, or not to abate? A strategic approach on green production in Cournot and Bertrand duopolies, *Energy Econ*. (96), 105164. https://doi.org/10.1016/j.eneco.2021.105164

Chen Jiaqi & Sang-Ho Lee. (2022). Cournot–Bertrand comparisons under R&D competition: Output versus R&D subsidies, *International Journal of Economic Theory,* (19), 77-100, https://doi.org/10.1111/ijet.12336

Chen Yuvan, Xu Lili & Sang-Ho Lee. (2021). Emission tax and strategic environmental corporate social responsibility in a Cournot–Bertrand comparison, *University Library of Munich* ,108498, https://mpra.ub.uni-muenchen.de/108498

Dragone Davide, Lambertini Luca & Palestini Arsen. (2021). Emission taxation, green innovations and inverted-U aggregate R&D efforts in a linear state differential game, *Research in Economics,* (76), 62-68, https://doi.org/10.1016/j.rie.2021.11.001

Field C Barry & Field K Martha, (7th edition). (2017). *Environmental Economics an Introduction*, McGraw-Hill Education.

Hoel, M. (1991). 'Global environmental problems: the effects of unilateral actions taken by one country', *Journal of Environmental Economics and Management*, 20, pp 55-70, https://doi.org/10.1016/0095-0696(91)90023-C

Kangsik Choi. (2012). Price and Quantity in a unionised mixed duopoly: The cases of substitutes and complements, *Australian Economic Papers*, (51), 1-22, https://doi.org/10.1111/j.1467-8454.2012.00419.x

Kat B, Şahin Ü, Teimourzadeh S., O.B.Tör, E.Voyvoda & A.E.Yeldan. (2024). A new energy-economy-environment modeling framework: Insights from decarbonization of the Turkish power Sector towards net-zero Emission targets, *Energy,* (302), 131760, https://doi.org/10.1016/j.energy.2024.131760

Kellog Ryan & Reguant Mar. (2021). *Energy and environmental markets, industrial organization and regulation*, National Bureau of Economic Research,https://www.nber.org/system/files/working_papers/w29235/w29235.pdf

Kopel Micheal & Putz Maria Eva. (2021). Information sharing in a Cournot–Bertrand duopoly, *Managerial and Decision Economics,* (42), 1645-1655, https://doi.org/10.1002/mde.3348

Matsumoto Akio & Szidarovszky Ferenc. (2021). Controlling non-point source pollution in Cournot oligopolies with hyperbolic demand, *Springer Nature Switzerland*, (1) ,38, https://doi.org/10.1007/s43546-020-00023-8

Miller, Nathan & Osborne, Matthew. (2010). Competition Among Spatially Differentiated Firms: An Empirical Model with an Application to Cement, *EAG Discussion Paper*, No. EAG 10-2,https://hdl.handle.net/10419/202388

Operational Group. (2024). *Abatement techniques.* https://www.operationalgroup.com/air-pollution-abatement-techniques/

Pan Weihua, Cao Hang & Ying Liu.(2023). "Green" innovation, privacy regulation and environmental policy, *Renewable Energy* ,(203) , 245–254, https://doi.org/10.1016/j.renene.2022.12.025

Tremblay Horton Carol & Tremblay J. Victor (2019), Oligopoly games under the Cournot-Bertrand model: a survey, *Journal of Economic Surveys* ,(33), 1555–1577, 10.1111/joes.12336

Yeldan A.Erinç & Voyvoda Ebru.(2015), *Low Carbon Development Pathways and Priorities for Turkey* , WWF-Turkey and Istanbul Policy Center, https://www.wwf.org.tr/?5060/lowcarbonpathwaysforturkey

Wei Zhongjun, Yi Yongxi & Fu Chunyan (2019), Cournot competition and ''green'' innovation under efficiency-improving learning by doing, *Physica A,* 0378-4371, https://doi.org/10.1016/j.physa.2019.121762

APPENDICES

APPENDIX 1

Headlines	Authors	Model type	Results
Controlling non-point source pollution in Cournot oligopolies with hyperbolic demand	Akio Matsumoto Ferenc Szidarovszky	n-firm Cournot oligopoly	İncreasing average abatement technology in the industry will also increases the positivity of individual Cournot output.
Emission taxation, green innovations and inverted-U aggregate R&D efforts in a linear state differential game	David Dragone Luca Lambertini Arsen Palestini	Differential Cournot game	If environmental damage depends solely on emissions and not on industry output, the aggregate green R&D effort exhibits an inverted U-shape.
Oligopoly games and the Cournot Bertrand model : A survey	Carol Horton Tremblay Victor J. Tremblay	Cournot-Bertrand model	Cournot firm will choose a "greener" technology compared to its Bertrand rival. Also, it will produce more, charge less, and yield higher profits compared to the Bertrand firm.
Environmental Technological Choice in a Cournot-Bertrand Model	Elias Asproudis Eleftherios Filippiadis	Cournot-Bertrand model	Bertrand firm will choose a dirtier technology compared to its Cournot rival and when competing in quantities firms choose greener technology than when competing in price. Also, abatement technologies will increase total output in the market and decrease total emissions.
Cournot competition and ''green'' innovation under efficiency-improving learning by doing.	Zhongjun Wei, Yongxi Yi Chunyan Fu	Stackelberg differential game model	Cooperation in R&D of firms competing in the product market forces incentives to invest in green innovation.
"Green" innovation, privacy regulation and environmental policy	Wiehua Pan Hang Cao Ying Liu	Hotelling duopoly	With strong privacy protections, emission taxes should be higher than the case with no privacy protections. In the asymmetric case, the optimal emission tax is highest to achieve emission reductions.
Price and quantity competition in a differentiated duopoly	Nirvikar Singh Xavier Vives	Horizontal product differentiation	Bertrand competition is more efficient than the Cournot competition.

To abate, or not to abate? A strategic approach on green production in Cournot and Bertrand duopolies	Domenica Buccella Luciano Fanti Luca Gori	Pure Cournot & pure Bertrand	They have found that social welfare under abatement is always larger than under no abatement in both cases.

APPENDIX 2

To find Cournot equilibrium $(q_1{}^*, q_2{}^*)$ we need to solve best response functions simultenously:

$$q_1{}^{BR} = q_2{}^{BR}$$

Let's plug $q_2 = \frac{1-q_1-t}{2}$ into the Firm 1's best response function:

$$q_1 = \frac{1-\left(\frac{1-q_1-t}{2}\right)-t}{2} \quad ;$$

$$2q_1 = 1 - \left(\frac{1-q_1-t}{2}\right) - t$$

$$4q_1 = 2 - (1 - q_1 - t) - 2t$$

$$4q_1 = 1 + q_1 - t$$

$$3q_1 = 1 - t \quad ; \; q_1{}^* = \frac{1-t}{3}$$

To continue with firm 2; Let's plug $q_1 = \frac{1-q_2-t}{2}$ into the Firm 2's best response function:

$$q_2 = \frac{1-\left(\frac{1-q_2-t}{2}\right)-t}{2} \quad ;$$

$$2q_2 = 1 - \left(\frac{1-q_2-t}{2}\right) - t$$

$$4q_2 = 2 - (1 - q_2 - t) - 2t$$

$$4q_2 = 1 + q_2 - t$$

$$3q_2 = 1 - t \quad ; \; q_2{}^* = \frac{1-t}{3}$$

APPENDIX 3

Differently, now two firms choose abatement and their optimal emission level, so we need to take optimal emission level, $k_1{}^{opt}$ and $k_2{}^{opt}$ into calculation. Also, we need to adjust PS, CS, GR, ED formulas to abatement situation.

After taking the value of $q_{1,2}{}^{*} = \frac{1-t}{3}$, plugging this into the PS, CS, GR and ED functions:

$$PS^{A} = \pi_1{}^{A} + \pi_2{}^{A}$$

$$CS^{A} = \frac{(q_1+q_2)^2}{2}$$

$$GR^{A} = t.[(q_1 - k_1) + (q_2 - k_2)]$$

$$ED^{A} = \frac{\rho}{2}[(q_1 - k_1) + (q_2 - k_2)]^2$$

$$PS^{A} = \pi_1{}^{A} + \pi_2{}^{A} =$$

$$(1 - q_1 - q_2)q_1 - t(q_1 - k_1) - \frac{zk_1{}^2}{2} + (1 - q_1 - q_2)q_2 - t(q_2 - k_2) - \frac{zk_2{}^2}{2}$$

$$(1-\frac{1-t}{3} - \frac{1-t}{3})\frac{1-t}{3} - t(\frac{1-t}{3} - \frac{t}{z}) - z(\frac{(\frac{t}{z})^2}{2}) + (1-\frac{1-t}{3} - \frac{1-t}{3})\frac{1-t}{3} - t(\frac{1-t}{3} - \frac{t}{z}) - z(\frac{(\frac{t}{z})^2}{2})$$

$$= \frac{2}{9} - \frac{4t}{9} + \frac{2t^2}{9} + \frac{t^2}{z}$$

$$CS^{A} = \frac{(q_1+q_2)^2}{2}$$

$$CS^{A} = \frac{(\frac{1-t}{3}+\frac{1-t}{3})^2}{2}$$

$$CS^{A} = \frac{2}{9}(1-t)^2$$

$$GR^{A} = t.[(q_1 - k_1) + (q_2 - k_2)]$$

$$GR^{A} = t\left(\left[\frac{1-t}{3} - \frac{t}{z}\right] + \left[\frac{1-t}{3} - \frac{t}{z}\right]\right)$$

$$GR^{A} = \frac{2}{3}t\left[1 - \frac{t(3+z)}{z}\right]$$

$$ED^{A} = \frac{\rho}{2}[(q_1 - k_1) + (q_2 - k_2)]^{2}$$

$$ED^{A} = \frac{2}{9}\rho\left[1 - \frac{t(3+z)}{z}\right]^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}}+\mathrm{CS}^{\mathrm{CBNA}}+\mathrm{GR}^{\mathrm{CBNA}}-\mathrm{ED}^{\mathrm{CBNA}}$$

We get;

$$\frac{t^2(9+4z)+2tz(-7+\rho)-2z(-5+\rho)+6t(-3+\rho)}{9z}$$

Taking the first derivative of SW function, we get.

$$\frac{\mathrm{dSW}}{\mathrm{dt}}=\frac{2t(9+4z)+2z(-7+\rho)+6(-3+\rho)}{9z}$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}}=0$ with respect to t, gives us $t^{*CA/A}$:

$$\frac{z(-z+6\rho+2z\rho)}{9z+2z^2+18\rho+12z\rho+2\ ^2\rho}$$

To compare first stage profit functions of two firms, we plug $t*$ into the profit functions:

$$\pi_1{}^{*\mathrm{CA}}=\mathrm{P_iQ_i}\ -\mathrm{t(q_i-k_i)}-\frac{\mathrm{zk_i}^2}{2}$$

$$\pi_2{}^{*\mathrm{CA}}=\mathrm{P_iQ_i}\ -\mathrm{t(q_i-k_i)}-\frac{\mathrm{zk_i}^2}{2}$$

So.

$$\pi_1{}^{*\mathrm{CA}}=\pi_2{}^{*\mathrm{CA}}$$

APPENDIX 4

After taking the value of $q_{1,2}{}^{*} = \frac{1-t}{3}$, plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{A/NA}} = \pi_1{}^{\mathrm{NA}} + \pi_2{}^{\mathrm{A}}$$

$$\mathrm{CS}^{\mathrm{A/NA}} = \frac{(q_1+q_2)^2}{2}$$

$$\mathrm{GR}^{\mathrm{A/NA}} = \mathrm{t}.[(q_1) + (q_2 - k_2)]$$

$$\mathrm{ED}^{\mathrm{A/NA}} = \frac{\rho}{2}[(q_1 + (q_2 - k_2)]^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$$

We get;

$$-\frac{z^2(-4+\rho)-2tz\left(3\rho+z(2+\rho)\right)+t^2\left(9\rho+z^2(8+\rho)+z(9+6\rho)\right)}{18z^2}$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*CA/NA}$:

$$\frac{z(2z+3\rho+z\rho)}{9z+8z^2+9\rho+6z\rho+z^2\rho}$$

Since firm 1 selects not adopting the technology, its profit function takes the form of:

$$\pi_1{}^{*\mathrm{NA}} = \frac{1}{9}(1 - \mathrm{t}^{*})^2$$

Since firm 2 selects adopting the technology, its profit function takes the form of:

$$\pi_2{}^{*\mathrm{A}} = \frac{1}{9}(1 - \mathrm{t}^{*})^2 + \frac{\mathrm{t}^2}{\mathrm{z}}$$

And difference between $\pi_1{}^{*NA}$ and $\pi_2{}^{*A}$ is:

$$\pi_2{}^{*\mathrm{A}} - \pi_1{}^{*\mathrm{NA}} = \frac{\mathrm{t}^{*}}{\mathrm{z}}$$

Since $\mathrm{t}^{*} > 0$, $\pi_2{}^{*\mathrm{A}} - \pi_1{}^{*\mathrm{NA}} > 0$, $\pi_2{}^{*\mathrm{A}} > \pi_1{}^{*\mathrm{NA}}$

APPENDIX 5

To find Bertrand equilibrium (p_1^*, p_2^*) we need to solve best response functions simultenously:

$$p_1^{BR} = p_2^{BR}$$

Let's plug $p_2 = \frac{1-d-t+dp_1}{2}$ into the Firm 1's best response function:

$$P_1^{BR} = \frac{1-d-t+dp_2}{2}$$

$$P_1^{BR} = \frac{1-d-t+d\left(\frac{1-d-t+dp_1}{2}\right)}{2}$$

$$p_1^* = \frac{1-d+t}{2-d}$$

To continue with firm 2; Let's plug $p_1 = \frac{1-d-t+dp_2}{2}$ into the Firm 2's best response function:

$$P_2^{BR} = \frac{1-d-t+dp_1}{2}$$

$$P_2^{BR} = \frac{1-d-t-d\left(\frac{1-d-t+dp_2}{2}\right)}{2}$$

$$p_2^* = \frac{1-d+t}{2-d}$$

So the Bertrand equilibrium is: $\left(\frac{1-d+t}{2-d}, \frac{1-d+t}{2-d}\right)$.

APPENDIX 6

After taking the value of $p_{1,2}{}^{*} = \frac{1+t-d}{2-d}$, plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{BNA} = \pi_1{}^{\mathrm{NA}} + \pi_2{}^{\mathrm{NA}}$$

$$\mathrm{CS}^{BNA} = \frac{(q_1+q_2)^2}{2}$$

$$\mathrm{GR}^{BNA} = \mathrm{t}.(q_1 + q_2)$$

$$\mathrm{ED}^{BNA} = \rho\,(q_1 + q_2)^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$$

We get;

$$\frac{1}{(-2+d)^2}(3 + t(2-8\rho) + t^2(7-4\rho) + d^2(1+2t-4\rho) - 2d(1+t^2-4t(-1+\rho)-4\rho)-4\rho)$$

Taking the first derivative of SW function, we get.

$$\frac{2 + 2d^2 - 2d\big(2t - 4(-1+\rho)\big) + 2t(7-4\rho) - 8\rho}{(-2+d)^2}$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*BNA/BNA}$:

$$\frac{1-4d+d^2+4\mathrm{d}\rho-4\rho}{-7+2d+4\rho}$$

To compare first stage profit functions of two firms, we plug $t*$ into the profit functions:

$$\pi_1{}^{*\mathrm{BNA}} = \frac{1-\mathrm{d}+\mathrm{t}*}{2-\mathrm{d}} * \frac{1-\mathrm{p}_1-\mathrm{d}(1-\mathrm{p}_2)}{(1-\mathrm{d})^2} - \mathrm{t} * \frac{1-\mathrm{p}_1-\mathrm{d}(1-\mathrm{p}_2)}{(1-\mathrm{d})^2}$$

$$\pi_2{}^{*\mathrm{BNA}} = \frac{1-\mathrm{d}+\mathrm{t}*}{2-\mathrm{d}} * \frac{1-\mathrm{p}_2-\mathrm{d}(1-\mathrm{p}_1)}{(1-\mathrm{d})^2} - \mathrm{t} * \frac{1-\mathrm{p}_2-\mathrm{d}(1-\mathrm{p}_1)}{(1-\mathrm{d})^2}$$

$$\pi_1{}^{*\mathrm{BNA}} = \pi_2{}^{*\mathrm{BNA}}$$

APPENDIX 7

After taking the value of $p_{1,2}{}^{*} = \frac{1+t-d}{2-d}$, plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS^A} = \pi_1{}^{\mathrm{A}} + \pi_2{}^{\mathrm{A}}$$

$$\mathrm{CS^A} = \frac{(q_1+q_2)^2}{2}$$

$$\mathrm{GR^A} = \mathrm{t}.[(q_1 - k_1) + (q_2 - k_2)]$$

$$\mathrm{ED^A} = \frac{\rho}{2}[(q_1 - k_1) + (q_2 - k_2)]^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS^{CBA}} + \mathrm{CS^{CBA}} + \mathrm{GR^{CBA}} - \mathrm{ED^{CBA}}$$

We get;

$$-\frac{1}{4(-2+d)^2(-1+d)z^2}\Big(4(-1+d)z^2\big(-3+d(2-4\rho)+2\rho+d^2(-1+2\rho)\big) + t^2\Big(z^2\big(3d^2+d^3+8d(-3+\rho)-8(-2+\rho)\big) + 8(-2+d)^2(-1+d)\rho + 4(2-3d+d^2)z(-2+d+4\rho)\Big) - 4tz\Big(4(-2+d)(-1+d)^2\rho + z\big(2d^3+d(11-8\rho)+4(-1+\rho)+2d^2(-5+2\rho)\big)\Big)\Big)$$

Taking the first derivative of SW function, we get.

$$-\frac{1}{4(-2+d)^2(-1+d)z^2}\Big(2t\Big(z^2\big(3d^2+d^3+8d(-3+\rho)-8(-2+\rho)\big) + 8(-2+d)^2(-1+d)\rho + 4(2-3d+d^2)z(-2+d+4\rho)\Big) - 4z\Big(4(-2+d)(-1+d)^2\rho + z\big(2d^3+d(11-8\rho)+4(-1+\rho)+2d^2(-5+2\rho)\big)\Big)\Big)$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*BA/BA}$:

$$\frac{2z\big(-4z+11dz-10d^2z+2d^3z-8\rho+20d\rho-16\ {}^2\rho+4d^3\rho+4z\rho-8dz\rho+4\ {}^2z\rho\big)}{-16z+32dz-2\ {}^2z+4d^3z+16z^2-24dz^2+3d^2z^2+d^3z^2-32\rho+64d\rho-40\ {}^2\rho+8d^3\rho+32z\rho-48dz\rho+16d^2z\rho-8z^2\rho+8dz^2\rho}$$

To compare first stage profit functions of two firms, we plug $t*$ into the profit functions:

$$\pi_1{}^{*BA} = P_1Q_1 - t*(q_1 - k_1) - \frac{zk_1{}^2}{2}$$

$$\pi_2{}^{*BA} = P_2Q_2 - t*(q_2 - k_2) - \frac{zk_2{}^2}{2}$$

$$\pi_1{}^{*BA} = \pi_2{}^{*BA}$$

APPENDIX 8

After taking the value of $p_{1,2}{}^{*} = \frac{1+t-d}{2-d}$, plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{A/NA}} = \pi_1{}^{\mathrm{NA}} + \pi_2{}^{\mathrm{A}}$$

$$\mathrm{CS}^{\mathrm{A/NA}} = \frac{(q_1+q_2)^2}{2}$$

$$\mathrm{GR}^{\mathrm{A/NA}} = \mathrm{t}.[(q_1) + (q_2 - k_2)]$$

$$\mathrm{ED}^{\mathrm{A/NA}} = \frac{\rho}{2}[(q_1 + (q_2 - k_2)]^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$$

We get;

$$\frac{8tz(z(1+d^2+2d(-2+\rho)-2\rho)+(2-3d+d^2)\rho)+4z^2\left(3+d^2(1-2\rho)-2\rho+d(-2+4\rho)\right)-t^2\left(2(-2+d)^2\rho+4z^2(-7+2d+2\rho)+(-2+d)z(-2+d+8\rho)\right)}{4(-2+d)^2z^2}$$

Taking the first derivative of SW function, we get.

$$\frac{8z(z(1+d^2+2d(-2+\rho)-2\rho)+(2-3d+d^2)\rho)-2t\left(2(-2+d)^2\rho+4z^2(-7+2d+2\rho)+(-2+d)z(-2+d+8\rho)\right)}{4(-2+d)^2z^2}$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*BA/BNA}$:

$$\frac{4z\left(z-4dz+d^2z+2\rho-3d\rho+{}^2\rho-2z\rho+2dz\rho\right)}{4z-4dz+d^2z-28z^2+8dz^2+8\rho-8d\rho+2d^2\rho-16z\rho+8dz\rho+8z^2\rho}$$

To compare first stage profit functions of two firms, we plug $t*$ into the profit functions:

$$\pi_1{}^{*\mathrm{BNA}} = \frac{1-\mathrm{d}+\mathrm{t}*}{2-\mathrm{d}} * \frac{1-\mathrm{p}_1-\mathrm{d}(1-\mathrm{p}_2)}{(1-\mathrm{d})^2} - \mathrm{t} * \frac{1-\mathrm{p}_1-\mathrm{d}(1-\mathrm{p}_2)}{(1-\mathrm{d})^2}$$

$$\pi_2{}^{*\mathrm{BA}} = \frac{1-\mathrm{d}+\mathrm{t}*}{2-\mathrm{d}} * \left[\frac{(1-\mathrm{p}_2-\mathrm{d}(1-\mathrm{p}_1))}{(1-\mathrm{d})^2}\right] - \mathrm{t} * \left[\frac{(1-\mathrm{p}_2-\mathrm{d}(1-\mathrm{p}_1))}{(1-\mathrm{d})^2} - \mathrm{k}_2\right] - \frac{\mathrm{zk}_2{}^2}{2}$$

And difference between $\pi_1{}^{*NA}$ and $\pi_2{}^{*A}$ is:

$$\pi_2{}^{*\mathrm{A}} - \pi_1{}^{*\mathrm{NA}} = \frac{\mathrm{t}^*}{\mathrm{z}}$$

Since $\frac{\mathrm{t}^*}{\mathrm{z}} > 0$; So : $\pi_2{}^{*\mathrm{BA/NA}} > \pi_1{}^{*\mathrm{BA/NA}}$

APPENDIX 9

For firm 1

$$\pi_1{}^{CBNA} = P_1Q_1 - tq_1$$

$$\pi_1{}^{CBNA} = [(1-d) - (1-d^2)q_1 + dp_2]\, q_1 - tq_1$$

When we expand the formula.

$$\pi_1{}^{CBNA} = q_1 - dq_1 - (1-d^2)q_1q_1 + dp_2q_1\text{-}tq_1$$

To find firm 1 ‘s best response function we need to take first derivative with respect to $q_1$of profit function.

$$\frac{d_{\pi_1 CB^{NA}}}{d_{q_1}} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

$$q_1{}^{BR} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

To continue with firm 2, first we need to derive firm 2’s profit function:

For firm 2;

$$\pi_2{}^{CBNA} = P_2Q_2 - tq_2$$

$$\pi_2{}^{CBNA} = P_2(1-p_2-dq_1) - t(a-p_2-dq_1)$$

$$\pi_2{}^{CBNA} = P_2 - p_2{}^2 - P_2dq_1 - t - tp_2 - tdq_1$$

To find firm 2 ‘s best response function we need to take first derivative with respect to $p_2$of profit function.

$$\frac{d_{\pi_2 CB^{NA}}}{d_{p_2}} = \frac{1}{2}(1-t-dq_1)$$

$$p_2{}^{BR} = \frac{1}{2}(1-t-dq_1)$$

To find Cournot Bernard equilibrium $(q_1{}^*, p_2{}^*)$ we need to solve best response functions simultaneously:

$$q_1{}^{BR} = p_2{}^{BR}$$

Let’s plug $p_2 = \frac{1}{2}(1-t-dq_1)$ into the Firm 1’s best response function:

$$q_1{}^{BR} = \frac{1-d-t+dp_2}{2(1-d^2)}$$

$$q_1{}^{BR} = \frac{1-d-t+d\left(\frac{1}{2}(1-t-dq_1)\right)}{2(1-d^2)}$$

$$q_1{}^{*} = \frac{2-d-dt}{4-3d^2}$$

To continue with firm 2; Let's plug $q_1 = \frac{1-d-t+dp_2}{2(1-d^2)}$ into the Firm 2's best response function:

$$p_2{}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

$$p_2{}^{BR} = \frac{1}{2}\left(1 - t - d\left(\frac{2-d-dt}{4-3d^2}\right)\right)$$

$$p_2{}^{*} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$$

Substituting the above points into functions for p_1 and q_2:

$$p_1 = (1-d) - (1-d^2)q_1 + dp_2$$

$$p_1 = (1-d) - (1-d^2)\frac{2-d-2t-dt}{4-3d^2} + d\frac{(2-d-d^2)+(2+d-2d^2)t}{4-3d^2}$$

$$p_1{}^{*} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2}$$

And

$$q_2 = 1 - p_2 - dq_1$$

$$q_2 = 1 - \frac{(2-d-d^2)+(2+d-2d^2)t}{4-3d^2} - d\frac{2-d-2t-dt}{4-3d^2}$$

$$q_2{}^{*} = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

APPENDIX 10

After taking the value of $q_1{}^* = \frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2}$, $p_2{}^* = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$ plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{CBNA}} = \pi_1{}^{\mathrm{CBNA}} + \pi_2{}^{\mathrm{CBNA}}$$

$$\pi_1{}^{\mathrm{CBNA}} = \mathrm{P_1Q_1} - \mathrm{tq_1}$$

$$\pi_1{}^{\mathrm{CBNA}} = = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2} \cdot \frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} - \mathrm{t}\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2}$$

$$\pi_2{}^{\mathrm{CBNA}} = \mathrm{P_2Q_2} - \mathrm{tq_2}$$

$$\pi_2{}^{\mathrm{CBNA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2} \cdot \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

$$\mathrm{CS}^{\mathrm{CBNA}} = \mathrm{d}.\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)^2$$

$$\mathrm{GR}^{\mathrm{CBNA}} = \mathrm{t}.\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)$$

$$\mathrm{ED}^{\mathrm{CBNA}} = \rho\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)\Big)^2$$

Applying formula with these values:

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$$

We get:

$$\frac{1}{(4-3d^2)^2}(d^5(1+t)^2 - \mathrm{dt}^2\rho^2 + d^3(2+\mathrm{dt}-11t+\mathrm{dt}t-4t^2-4\rho^2-4t\rho^2) + \mathrm{dt}(-2+4(2+t)\rho^2) - 4(-2+4\rho^2+4t\rho^2+t^2(1+\rho^2)) - d^4(1+t)(-4+\rho^2+t(2+\rho^2)) + d(8+\mathrm{dt}^2+4t^2+16\rho^2+2t(7+4\rho^2) - \mathrm{dt}(7+3t+4\rho^2)) + d^2(-22+6\mathrm{dt}+4\rho^2 - 2\mathrm{dt}(1+t)\rho^2 + t^2(6+4\rho^2) + t(-5+12\rho^2)))$$

Taking first derivative of SW function:

$$\frac{\mathrm{dSW}}{\mathrm{dt}} = \frac{1}{(4-3d^2)^2}(2d^5(1+t) + 4\mathrm{dt}\rho^2 + d^3(-11+\mathrm{dt}-8t-4\rho^2) - d^4(1+t)(2+\rho^2) - 4(4\rho^2+2t(1+\rho^2)) - d^4(-4+\rho^2+t(2+\rho^2)) + d^2(-5+12\rho^2-2\mathrm{dt}\rho^2+2t(6+4\rho^2)) + d(-3\mathrm{dt}+8t+2(7+4\rho^2)))$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $\mathrm{t}^{*\mathrm{CBNA}}$:

$$\mathrm{t}^{*\mathrm{CBNA}} = \frac{(-2d^5-4(-4+\mathrm{dt})\rho^2+d(-14+3\mathrm{dt}-8\rho^2)+2d^4(-1+\rho^2)+d^3(11-\mathrm{dt}+4\rho^2)+d^2(5+2(-6+\mathrm{dt})\rho^2))}{(2(-2+d^2)(-2d+d^3+2(1+\rho^2)-d^2(2+\rho^2)))}$$

APPENDIX 11

To compare first stage profit functions of two firms, we plug t^* into the profit functions:

$$\pi_1{}^{*\mathrm{CBNA}} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2} \cdot \frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \mathrm{t}^* \frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2}$$

And

$$\pi_2{}^{*\mathrm{CBNA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2} \cdot \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}^* \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

And the difference between $\pi_1{}^{*CBNA}$ and $\pi_2{}^{*CBNA}$:

$$\frac{1}{(4-3d^2)^2}\big(4d(-1+t)t + 12t^2 - d^2t(4+15t) + d^3(2+5t-3t^2) + 2d^4(-1+t+2t^2)\big)$$

Since $t > 0$, d>0 :

$$\frac{\big(4d(-1+t)t+12t^2-d^2t(4+15t)+d^3\big(2+5t-3t^2\big)+2d^4\big(-1+t+2t^2\big)\big)}{(4-3d^2)^2} > 0$$

So,

$$\pi_1{}^{*\mathrm{CBNA}} > \pi_2{}^{*\mathrm{CBNA}}$$

APPENDIX 12

So after their decision of abatement level and output, profit functions of firm 1 and firm 2 takes the following form:

$$\pi_i{}^{CBA} = P_i Q_i \ - t(q_i - k_i) - \frac{z{k_i}^2}{2}$$

$$\pi_1{}^{CBA} = [(1-d) - (1-d^2)q_1 + dp_2]q_1 - t(q_1 - k_1) - \frac{z({k_1}^2)}{2}$$

When we expand the functions, we get:

$$\pi_1{}^{CBA} = q_1 - dq_1 - q_1 q_1 - d^2 q_1 q_1 + dp_2 q_1 \text{-} tq_1 + t\,k_1 - \frac{z({k_1}^2)}{2}$$

To find firm 1's best response function and optimum abatement level, we take first derivative of profit functions with respect to $q_1$and k_1.

$$\frac{d_{\pi_1{}^{BA}}}{d_{q_1}} = \ 1 - d - t + dp_2 - 2q_1 - 2d^2 q_1$$

$${q_1}^{BR} = \frac{1 - d - t + dp_2}{2(1-d^2)}$$

$$\frac{d_{\pi_1{}^{BA}}}{d_{k_1}} = \ t - zk_1 \ ; {k_1}^{CBopt} = \frac{t}{z}$$

To continue with firm 2, first we need to derive firm 2's profit function:

For firm 2:

$$\pi_2{}^{CBA} = P_2 Q_2 - t(q_2 - k_2) - \frac{z({k_2}^2)}{2}$$

$$\pi_2{}^{CBA} = P_2 - {p_2}^2 - P_2 dq_1 \ - t - tp_2 - tdq_1 + tk_2 - \frac{z({k_2}^2)}{2}$$

To find firm 2's best response function and optimum abatement level, we take first derivative of profit functions with respect to $q_2$and k_2.

$$\frac{d_{\pi_2{}^{CBA}}}{d_{p_2}} = 1 - t - 2p_2 - dq_1$$

$${p_2}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

$$\frac{d_{\pi_2}{}^{CBA}}{d_{k_2}} = t - zk_2 \;\; ; {k_2}^{CBopt} = \frac{t}{z}$$

To find Cournot Bernard equilibrium $({q_1}^*, {p_2}^*)$ we need to solve best response functions simultaneously:

$${q_1}^{BR} = {p_2}^{BR}$$

Let's plug $p_2 = \frac{1}{2}(1 - t - dq_1)$ into the Firm 1's best response function:

$${q_1}^{BR} = \frac{1-d-t+dp_2}{2(-1+d^2)}$$

$${q_1}^{BR} = \frac{1-d-t+d\left(\frac{1}{2}(1-t-dq_1)\right)}{2(-1+d^2)}$$

$${q_1}^{*} = \frac{2-d-d}{4-3d^2}$$

To continue with firm 2; Let's plug $q_1 = \frac{1-d-t+dp_2}{2(1-d^2)}$ into the Firm 2's best response function:

$${p_2}^{BR} = \frac{1}{2}(1 - t - dq_1)$$

$${p_2}^{BR} = \frac{1}{2}\left(1 - t - d\frac{1-d-t+dp_2}{2(1-d^2)}\right)$$

$${p_2}^{*} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$$

So, the Cournot- Bertrand equilibrium is: $\left(\frac{2-d-dt}{4-3d^2}, \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\right)$

APPENDIX 13

After taking the value of $q_1{}^* = \frac{2-d-dt}{4-3d^2}$, $p_2{}^* = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$ and

$p_1{}^* = -\frac{(-1+d^2)(-2+d+d\ )}{-4+3d^2}$, $q_2{}^* = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$:

plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{CBA}} = \pi_1{}^{\mathrm{CBA}} + \pi_2{}^{\mathrm{CBA}}$$

$$\pi_1{}^{\mathrm{CBA}} = \mathrm{p_1q_1} - \mathrm{t(q_1 - k_1)} - \frac{\mathrm{z}\left(\mathrm{k_1}^2\right)}{2}$$

$$\pi_1{}^{\mathrm{CBA}} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2} \cdot \frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \mathrm{t}\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

And

$$\pi_2{}^{\mathrm{CBA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2} \cdot \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}\left(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

Also.

$$\mathrm{CS}^{\mathrm{CBA}} = \mathrm{d}.\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)^2$$

$$\mathrm{GR}^{\mathrm{CBA}} = \mathrm{t}.\left[(\mathrm{q_1 - k_1}) + (\mathrm{q_2 - k_2})\right]$$

$$\mathrm{t}\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}\right)$$

$$\mathrm{ED}^{\mathrm{CBA}} = \frac{\rho}{2}\left[(\mathrm{q_1 - k_1}) + (\mathrm{q_2 - k_2})\right]^2$$

$$\frac{\rho}{2} \cdot \left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}\right)^2$$

By plogging these values into the; $\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$

We get;

$$\frac{1}{2(4-3d^2)^2z^2}\Big(2d^2(1+t)(-2+d+d^2+2t-2d^2t)z^2$$
$$+z^2\Big(2d^5+2d^3(1+\mathrm{dt}-2\rho)-2d^2(20+\mathrm{dt}(-6+\rho)-2\rho)-d^4(-6+\rho)$$
$$-(-4+\mathrm{dt})(4+(-4+\mathrm{dt})\rho)+2d(8-7\mathrm{dt}+\mathrm{dt}^2+8\rho-2\mathrm{dt}\rho)\Big)$$
$$+t^2\Big(-4(4-3d^2)^2\rho+z^2\Big(8d-8d^3+2d^5-d^4\rho-4(2+\rho)+4d^2(2+\rho)\Big)$$
$$+2(-4+3d^2)z\big(4-4\rho+d^2(-3+2\rho)\big)\Big)$$
$$+2tz\Big(2d^5z+2(-4+\mathrm{dt})(-4+z)\rho+d^4(-z(-3+\rho)+6\rho)$$
$$+d\big(-16\rho+z(14-3\mathrm{dt}+4\rho)\big)-d^2\big(32\rho-6\mathrm{dt}\rho+z(5+(-6+\mathrm{dt})\rho)\big)$$
$$+d^3\left(12\rho+z\big(\mathrm{dt}-2(6+\rho)\big)\right)\Big)\Big)$$

Taking the first derivative of SW function, we get.

$$\frac{\mathrm{dSW}}{\mathrm{dt}}=\frac{1}{2(4-3d^2)^2z^2}(2d^2(2-2d^2)(1+t)z^2+2d^2(-2+d+d^2+2t-2d^2t)z^2+2t(-4(4-3d^2)^2\rho$$
$$+z^2(8d-8d^3+2d^5-d^4\rho-4(2+\rho)+4d^2(2+\rho))+2(-4+3d^2)z(4-4\rho$$
$$+d^2(-3+2\rho)))+2z(2d^5z+2(-4+\mathrm{dt})(-4+z)\rho+d^4(-z(-3+\rho)+6\rho)$$
$$+d(-16\rho+z(14-3\mathrm{dt}+4\rho))-d^2(32\rho-6\mathrm{dt}\rho+z(5+(-6+\mathrm{dt})\rho))+d^3(12\rho$$
$$+z(\mathrm{dt}-2(6+\rho)))))$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}}=0$ with respect to t, gives us $\mathrm{t}^{*\mathrm{CBA}}$:

$\mathrm{t}^{*\mathrm{CBA}}=$

$$\frac{-((z(2d^5z+2(-4+\mathrm{dt})(-4+z)\rho+d^4(-z(-2+\rho)+6\rho)+d^3(z(-11+\mathrm{dt}-2\rho)+12\rho)+d(-16\rho+\ (14-3\mathrm{dt}+4\rho))-d^2(32\rho-6\mathrm{dt}\rho+z(5+(-6+\mathrm{dt})\rho))))}{(-4(4-3d^2)^2\rho+(-2+d^2)z^2(-4d+2d^3+2(2+\rho)-d^2(4+\rho))+2(-4+3d^2)z(4-4\rho+d^2(-3+2\rho)}$$

APPENDIX 14

To compare first stage profit functions of two firms, we plug t^* into the profit functions:

$$\pi_1{}^{*\mathrm{CBA}} = -\frac{(-1+d^2)(-2+d+d\ )}{-4+3d^2}.\frac{2-d-dt}{4-3\mathrm{d}^2} - \mathrm{t}^*.\left(\frac{2-d-dt}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - z.\frac{(\frac{\mathrm{t}}{\mathrm{z}})^2}{2}$$

And

$$\pi_2{}^{*\mathrm{CBA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2} * \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}^*\left(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - z.\frac{(\frac{\mathrm{t}}{\mathrm{z}})^2}{2}$$

$$\pi_1{}^{*\mathrm{CBA}} - \pi_2{}^{*\mathrm{CBA}}$$

=

$$\frac{1}{(4-3d^2)^2}(12t^2 + \mathrm{dt}(-2+4t) - d^3(-2+\mathrm{dt}-3t+\mathrm{dt}t) + d(\mathrm{dt}-2t+\mathrm{dt}t) + d^4(-2+3t+5t^2) - d^2(\mathrm{dt}(-2+3t)+t(5+16t)))$$

Since z>0, t>0 and d>0;

$$\pi_1{}^{*\mathrm{CBA}} - \pi_2{}^{*\mathrm{CBA}} > 0$$

So;

$$\pi_1{}^{*CBA} > \pi_2{}^{*CBA}$$

APPENDIX 15

After taking the value of $q_1{}^* = \frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2}$, $p_2{}^* = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$ and

$p_1{}^* = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2}$, $q_2{}^* = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$ plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{CBA/CNA}} = \pi_1{}^{\mathrm{CBNA}} + \pi_2{}^{\mathrm{CBA}}$$

$$\pi_1{}^{\mathrm{CBNA}} = \mathrm{P_1Q_1} - \mathrm{tq_1}$$

$$\pi_1{}^{\mathrm{CBNA}} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2}.\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} - \mathrm{t}\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2}$$

And

$$\pi_2{}^{\mathrm{CBA}} = \mathrm{P_2Q_2} - \mathrm{t}(\mathrm{q_2} - \mathrm{k_2}) - \frac{\mathrm{z}(\mathrm{k_2}{}^2)}{2}$$

$$\pi_2{}^{\mathrm{CBA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}.\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

Also

$$\mathrm{CS}^{\mathrm{CBA/CBNA}} = \mathrm{d}.\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)^2$$

$$\mathrm{GR}^{\mathrm{CBA/CBNA}} = \mathrm{t}.[(\mathrm{q_1}) + (\mathrm{q_2} - \mathrm{k_2})]$$

$$= \mathrm{t}.\left[\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \left(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}}\right)\right]$$

$$\mathrm{ED}^{\mathrm{BA/BNA}} = \frac{\rho}{2}\,[(\mathrm{q_1} + (\mathrm{q_2} - \mathrm{k_2})]^{\ 2}$$

$$\frac{\rho}{2}\left[\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \frac{\mathrm{t}}{\mathrm{z}})\right]^{\ 2}$$

By plogging these values into the.

$$\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$$

We get ;

$$-\frac{1}{2(4-3d^2)^2z^2}(-2d(2d + \mathrm{dt} + d^2(1+t) - 2(2+t))^2z^2 + z^2(d^4\rho - 4d^2(-3 + \mathrm{dt} + \rho) + 2d^3(-6 + \mathrm{dt} + 2\rho) + 4(-4 + \mathrm{dt} + 4\rho) - 2d(-8 + \mathrm{dt} + 8\rho)) + t^2((4-3d^2)^2\rho - (-4+3d^2)z(4 - 4\rho + 2d\rho + d^2(-3+2\rho)) + (-1 + d)z^2(-8d - 4(2+\rho) + d^3(4+\rho) + d^2(4+3\rho))) + 2tz(8(-2+z)\rho +$$

$$d^4(-3\rho + z(2+\rho)) + d^3(-6\rho + z(-1+\mathrm{dt}+3\rho)) + d^2(16\rho - z(3+4\rho)) + d(8\rho - z(-2+\mathrm{dt}+8\rho))))$$

Taking the first derivative of SW function, we get.

$$\frac{\mathrm{dSW}}{\mathrm{dt}} = -\frac{1}{2(4-3d^2)^2 z^2}(-4d(-2+d^2)(2d+\mathrm{dt}+d^2(1+t)-2(2+t))z^2 + 2t((4-3d^2)^2\rho - (-4 + 3d^2)z(4-4\rho+2d\rho+d^2(-3+2\rho)) + (-1+d)z^2(-8d-4(2+\rho)+d^3(4+\rho) + d^2(4+3\rho))) + 2z(8(-2+z)\rho + d^4(-3\rho+z(2+\rho)) + d^3(-6\rho+z(-1+\mathrm{dt}+3\rho)) + d^2(16\rho - z(3+4\rho)) + d(8\rho - z(-2+\mathrm{dt}+8\rho))))$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*CBNA/A}$:

$$\frac{-((z(2d^5 z - 8(-2+z)\rho + d^4(-z(-2+\rho)+3\rho) + d^3(z(-11+\mathrm{dt}-3\rho)+6\rho) + d^2(-16\rho + (-5+4\rho)) + d(-8\rho + z(14-3\mathrm{dt}+8\rho))))}{(-(4-3d^2)^2\rho + z^2(2d^5 - 4(2+\rho) + 4d(2+\rho) + 3d^2(4+\rho) - 2d^3(4+\rho) - d^4(4+\rho)) + (-4+3d^2)z(4-4\rho+2d\rho+{}^2(-3+2\rho))))}$$

APPENDIX 16

To compare first stage profit functions of two firms, we plug $t*$ into the profit functions:

$$\pi_1{}^{*\mathrm{CBNA/NA}} = -\frac{(-1+d^2)(-2+d+d)}{-4+3d^2}.\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \mathrm{t}\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2}$$

And

$$\pi_2{}^{*\mathrm{CBNA/A}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}.\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - \mathrm{t}(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\text{-}\frac{\mathrm{t}}{\mathrm{z}}) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

So,

$$\pi_1{}^{*\mathrm{CBNA/A}} - \pi_2{}^{*\mathrm{CBNA/A}} =$$

$$\frac{1}{2(4-3d^2)^2z}(-2(-1+d)(2d^3-2\mathrm{dt}-d\mathrm{dt}+d^2\mathrm{dt})z+2d(-2-5d+3d^3-d^2(-3+\mathrm{dt})+\mathrm{dt})tz+t^2(8dz-6d^3z+8(-2+3z)-8d^2(-3+4z)+d^4(-9+10z))$$

Since d>0, t>0 and z>0; $\pi_1{}^{*\mathrm{CBNA/A}} - \pi_2{}^{*\mathrm{CBNA/A}} < 0$, so :

$$\pi_2{}^{*\mathrm{CBNA/A}} > \pi_1{}^{*\mathrm{CBNA/A}}$$

APPENDIX 17

After taking the value of $q_1{}^* = \frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2}$, $p_2{}^* = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}$ and

$p_1{}^* = -\frac{(-1+d^2)(-2+d+d\ )}{-4+3d^2}$, $q_2{}^* = \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$ plugging this into the PS, CS, GR and ED functions:

$$\mathrm{PS}^{\mathrm{CBA/CBNA}} = \pi_1{}^{\mathrm{CBA}} + \pi_2{}^{\mathrm{CBNA}}$$

$$\pi_1{}^{\mathrm{CBA}} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2} \cdot \frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} - \mathrm{t}\left(\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

And

$$\pi_2{}^{\mathrm{CBNA}} = \mathrm{P_2Q_2} - \mathrm{tq_2}$$

$$\pi_2{}^{\mathrm{CBNA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2} \cdot \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - t. \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

Also;

$$\mathrm{CS}^{\mathrm{CBA/CBNA}} = \mathrm{d}.\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)^2$$

$$\mathrm{GR}^{\mathrm{CBA/CBNA}} = \mathrm{t}.[(\mathrm{q_1} - \mathrm{k_1}) + (\mathrm{q_2})]$$

$$= \mathrm{t}.\left[\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}} + \left(\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\right)\right]$$

$$\mathrm{ED}^{\mathrm{BA/BNA}} = \frac{\rho}{2}\ [(\mathrm{q_1} - \mathrm{k_1} + (\mathrm{q_2})]^{\ 2}$$

$$\frac{\rho}{2}\left[\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}} + \frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}\ )\right]^2$$

By plogging these values into the; $\mathrm{SW}=\mathrm{PS}^{\mathrm{CBNA}} + \mathrm{CS}^{\mathrm{CBNA}} + \mathrm{GR}^{\mathrm{CBNA}} - \mathrm{ED}^{\mathrm{CBNA}}$

$\mathrm{SW}=\frac{1}{2(4-3d^2)^2z^2}(z^2(2d^5 + 2d^3(2 + \mathrm{dt} - 2\rho) - d^4(-8 + \rho) + 4d^2(-11 + 3\mathrm{dt} + \rho) - 4(-4 + \mathrm{dt} + 4\rho) + 2d(8 - 7\mathrm{dt} + \mathrm{dt}^2 + 8\rho)) + t^2(-(4 - 3d^2)^2\rho + z^2(2d^5 - 4(2 + \rho) + 4d(2 + \rho) + 3d^2(4 + \rho) - 2d^3(4 + \rho) - d^4(4 + \rho)) + (-4 + 3d^2)z(4 - 4\rho + 2d\rho + d^2(-3 + 2\rho))) + 2tz(2d^5z - 8(-2 + z)\rho + d^4(-z(-2 + \rho) + 3\rho) + d^3(z(-11 + \mathrm{dt} - 3\rho) + 6\rho) + d^2(-16\rho + z(-5 + 4\rho)) + d(-8\rho + z(14 - 3\mathrm{dt} + 8\rho))))$

Taking the first derivative of SW function, we get.

$$\frac{dSW}{dt} = \frac{1}{2(4-3d^2)^2 z^2}(2t(-(4-3d^2)^2\rho + z^2(2d^5 - 4(2+\rho) + 4d(2+\rho) + 3d^2(4+\rho) - 2d^3(4+\rho) - d^4(4+\rho)) + (-4+3d^2)z(4-4\rho+2d\rho+d^2(-3+2\rho))) + 2z(2d^5 z - 8(-2+z)\rho + d^4(-z(-2+\rho)+3\rho) + d^3(z(-11+\mathrm{dt}-3\rho)+6\rho) + d^2(-16\rho+z(-5+4\rho)) + d(-8\rho+z(14-3\mathrm{dt}+8\rho))))$$

Solving for $\frac{\mathrm{dSW}}{\mathrm{dt}} = 0$ with respect to t, gives us $t^{*CBNA/NA}$:

$$\mathrm{t}^{*\mathrm{CBA/NA}} = -(\Big(z(14dz - 5d^2z - 11d^3z + 2d^4z + 2d^5z - 3d\mathrm{dtz} + d^3\mathrm{dtz} + 16\rho - 8d\rho - 16d^2\rho + 6d^3\rho + 3d^4\rho - 8z\rho + 8dz\rho + 4d^2z\rho - 3d^3z\rho - d^4z\rho)\Big) / (-16z + 24d^2z - 9d^4z - 8z^2 + 8dz^2 + 12d^2z^2 - 8d^3z^2 - 4d^4z^2 + 2d^5z^2 - 16\rho + 24d^2\rho - 9d^4\rho + 16z\rho - 8dz\rho - 20d^2z\rho + 6d^3z\rho + 6d^4z\rho - 4z^2\rho + 4dz^2\rho + 3d^2z^2\rho - 2d^3z^2\rho - d^4z^2\rho))$$

APPENDIX 18

To compare first stage profit functions of two firms, we plug t^* into the profit functions:

$$\pi_1{}^{*\mathrm{CBA/NA}} = -\frac{(-1+d^2)(-2+d+dt)}{-4+3d^2}\cdot\frac{2-\mathrm{d}-\mathrm{d}}{4-3\mathrm{d}^2} - \mathrm{t}\left(\frac{2-\mathrm{d}-\mathrm{dt}}{4-3\mathrm{d}^2} - \frac{\mathrm{t}}{\mathrm{z}}\right) - \frac{\mathrm{z}\left(\frac{\mathrm{t}^2}{\mathrm{z}}\right)}{2}$$

And

$$\pi_2{}^{\mathrm{CBNA}} = \frac{2+2d^2t-d-2t-d^2}{4-3d^2}\cdot\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2} - t.\frac{d-2(1+t)+d^2(1+t)}{-4+3d^2}$$

The difference between $\pi_1{}^{*\mathrm{CBA/NA}} - \text{and } \pi_2{}^{*\mathrm{CBA/NA}}$:

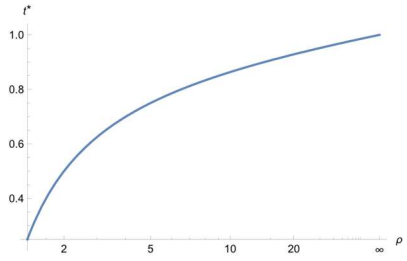
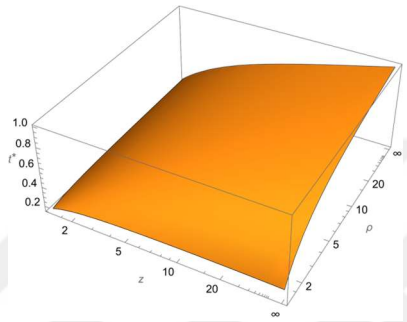
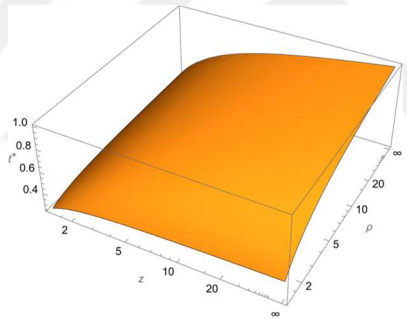
$$\frac{1}{2(4-3d^2)^2z}\Big(-2(-1+d)(2d^3-2\mathrm{dt}-d\mathrm{dt}+d^2\mathrm{dt})z + 2d(-2-5d+3d^3-d^2(-3+\mathrm{dt})+\mathrm{dt})tz + t^2\big(8dz-6d^3z+8(2+3z)-8d^2(3+4z)+d^4(9+10z)\big)\Big)$$

Since d>0, $\pi_1{}^{*\mathrm{CBA/NA}} - \pi_2{}^{*\mathrm{CBA/NA}} > 0$

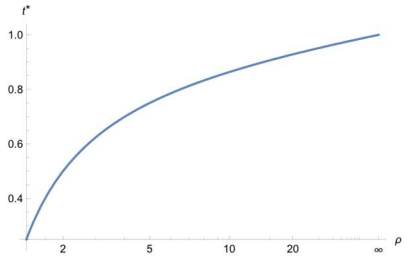
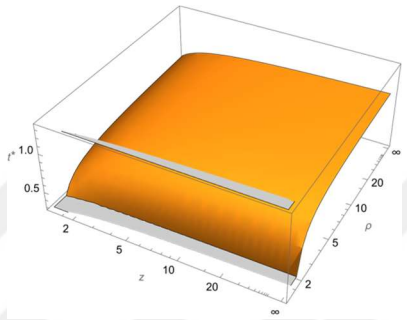
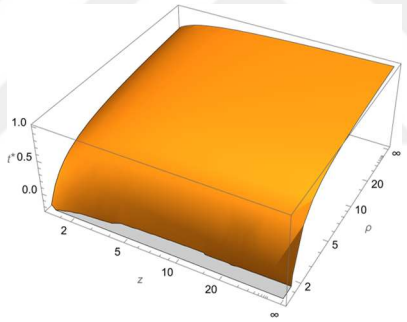
So:

$$\pi_1{}^{*\mathrm{CBA/NA}} > \pi_2{}^{*\mathrm{CBA/NA}}$$

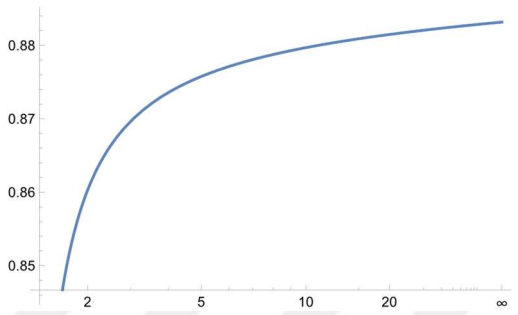
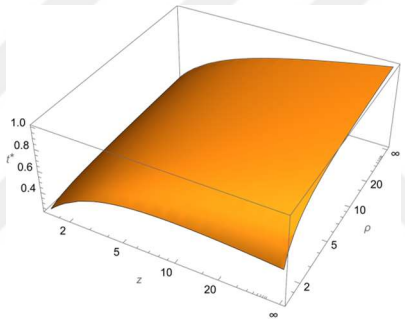
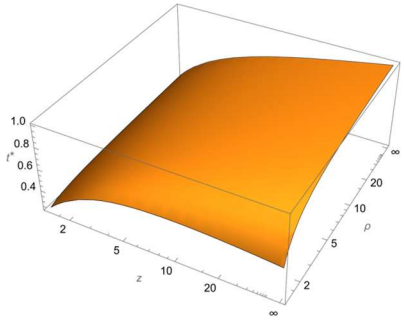
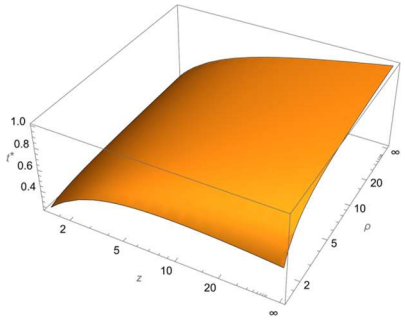
APPENDIX 19

Cournot	Variables	Graphs	Relationship
Not Abate / Not Abate	$t^{*} and\ \rho$		$\rho(\uparrow); t^{*}(\uparrow)$ Positive relationship
Abate / Not Abate	$t^{*}, z\ and\ \rho$		$\rho(\uparrow); t^{*}(\uparrow)$ $z(\uparrow); t^{*}(\uparrow)$ Positive relationship
Abate / Abate	$t^{*}, z\ and\ \rho$		$\rho(\uparrow); t^{*}(\uparrow)$ $z(\uparrow); t^{*}(\uparrow)$ Positive relationship

APPENDIX 20

Bertrand	Variables	Graphs	Relationship
Not Abate / Not Abate	$t^*, d \text{ and } \rho$		$\rho(\uparrow); t^*(\uparrow)$ Positive relationship
Abate / Not Abate	$t^*, d, \rho \text{ and } z$		$\rho(\uparrow); t^*(\uparrow)$ $z(\uparrow); t^*(\uparrow)$ Positive relationship
Abate / Abate	$t^*, d, \rho \text{ and } z$		$\rho(\uparrow); t^*(\uparrow)$ $z(\uparrow); t^*(\uparrow)$ Positive relationship

APPENDIX 21

Cournot - Bertrand	Variables	Graphs	Relationship
Not Abate / Not Abate	$t^*, d \text{ and } \rho$ When d=0,5		$\rho(\uparrow); t^*(\uparrow)$ Positive relationship
Abate / Not Abate	$t^*, d, \rho \text{ and } z$ When d=0,5		$\rho(\uparrow); t^*(\uparrow)$ $z(\uparrow); t^*(\uparrow)$ Positive relationship
Not Abate / Abate	$t^*, d, \rho \text{ and } z$ When d=0,5		$\rho(\uparrow); t^*(\uparrow)$ $z(\uparrow); t^*(\uparrow)$ Positive relationship
Abate / Abate	$t^*, d, \rho \text{ and } z$ When d=0,5		$\rho(\uparrow); t^*(\uparrow)$ $z(\uparrow); t^*(\uparrow)$ Positive relationship

APPENDIX 22

Since we have;

$$\mathrm{SW}^{CA*} = -\frac{t^2(9+4z)+2tz\,(-7+\rho)-2z\,(-5+\rho)+6t\,(-3+\rho)}{9z};$$

$$\mathrm{SW}^{BA*} = \frac{1}{4(-2+d)^2(-1+d)z^2}\Big(4(-1+d)z^2\big(-3+d(2-4\rho)+2\rho+d^2(-1+2\rho)\big)+t^2\Big(z^2\big(3d^2+d^3+8d(-3+\rho)-8(-2+\rho)\big)+8(-2+d)^2(-1+d)\rho+4(2-3d+d^2)z(-2+d+4\rho)\Big)-4tz\Big(4(-2+d)(-1+d)^2\rho+z\big(2d^3+d(11-8\rho)+4(-1+\rho)+2d^2(-5+2\rho)\big)\Big)\Big);$$

$$\begin{aligned}\mathrm{SW}^{CBA*} = \frac{1}{2(4-3d^2)^2z^2}\Big(&2d^2(1+t)(-2+d+d^2+2t-2d^2t)z^2\\&+z^2\big(2d^5+2d^3(1+\mathrm{dt}-2\rho)-2d^2(20+\mathrm{dt}(-6+\rho)-2\rho)-d^4(-6+\rho)\\&-(-4+\mathrm{dt})(4+(-4+\mathrm{dt})\rho)+2d(8-7\mathrm{dt}+\mathrm{dt}^2+8\rho-2\mathrm{dt}\rho)\big)\\&+t^2\Big(-4(4-3d^2)^2\rho+z^2\big(8d-8d^3+2d^5-d^4\rho-4(2+\rho)+4d^2(2+\rho)\big)\\&+2(-4+3d^2)z\big(4-4\rho+d^2(-3+2\rho)\big)\Big)\\&+2tz\Big(2d^5z+2(-4+\mathrm{dt})(-4+z)\rho+d^4(-z(-3+\rho)+6\rho)\\&+d\big(-16\rho+z(14-3\mathrm{dt}+4\rho)\big)-d^2\big(32\rho-6\mathrm{dt}\rho+z(5+(-6+\mathrm{dt})\rho)\big)\\&+d^3\Big(12\rho+z\big(\mathrm{dt}-2(6+\rho)\big)\Big)\Big)\Big)\end{aligned}$$

For d=1/2, t=1/2 z=10 and ρ :

$\mathrm{SW}^{CA*} = \frac{49}{120}, \mathrm{SW}^{BA*} = \frac{4339}{720}, \mathrm{SW}^{CBA*} = \frac{12421}{3380}$

So,

$$\mathrm{SW}^{BA*} > \mathrm{SW}^{CBA*} > \mathrm{SW}^{CA*}$$