END4400 – System Dynamics

Week 6 – 13/4/2021

Assist. Prof. Mert Edalı
medali@yildiz.edu.tr

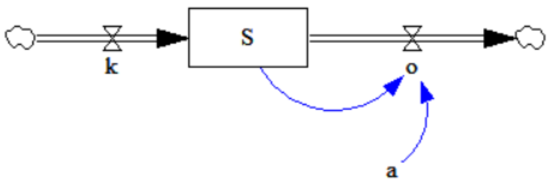
Outline

- Basic Stock – Flow Dynamics
- Coupling a Negative and a Positive Feedback Loop
  - Vensim
- Goal-Seeking Behavior

- Week 5 slides updated!

Basic Stock – Flow Dynamics

- coupling a constant inflow and a negative feedback loop
  - stock flow diagram



  - differential equation

$$\frac{dS}{dt} = k - o$$

  - plug in all variables and rewrite the differential equation

$$\frac{dS}{dt} = k - a * S$$

Basic Stock – Flow Dynamics

- analytical solution → not very straightforward (but doable)
  - change of variables

$$\frac{dS}{k - a * S} = dt$$

$$k - a * S = u$$

$$-a * dS = du$$

$$dS = \frac{-du}{a}$$

Plug in this to the differential equation

$$-\frac{1}{a} * \frac{du}{u} = dt$$

$$\int -\frac{1}{a} * \frac{du}{u} = \int dt$$

$$-\frac{1}{a} * \int \frac{du}{u} = \int dt$$

$$-\frac{1}{a} * lnu = t + C$$

$$lnu = -a * t + C$$

$$\ln(k - a * S) = -a * t + C$$

$$e^{(-a*t+C)} = k - a * S$$

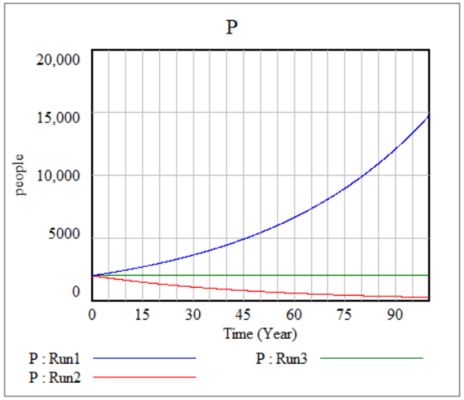
$$S(t) = \frac{k}{a} - C * \frac{e^{-a*t}}{a}$$

$$C = k - a * S(0)$$

$$S(t) = \frac{k}{a} - \frac{e^{-a*t} * \left(k - a * S(0)\right)}{a}$$

Basic Stock – Flow Dynamics

- homework:
  - please build the stock-flow diagram and experiment with the different values of *a*
  - try to explore the possible dynamics that can be generated by the model
  - similar to the population model with births and deaths, can we write down specific conditions for the values of *a*?
  - *hint*: try to discover the equilibrium case first, then consider other possible cases
  - try to generate a plot like this



Basic Stock – Flow Dynamics

- let us try to solve the homework
  - the differential equation (all the variables plugged in)

$$\frac{dS}{dt} = k - a * S$$

  - equilibrium case (net flow = 0)

$$k - a * S = 0$$

$$k = a * S$$

$$a = \frac{k}{S}$$

Recall:
S(0) = 100
k = 15

$$a = \frac{k}{S} = \frac{15}{100} = 0.15$$

Basic Stock – Flow Dynamics

- look for the case where net flow > 0

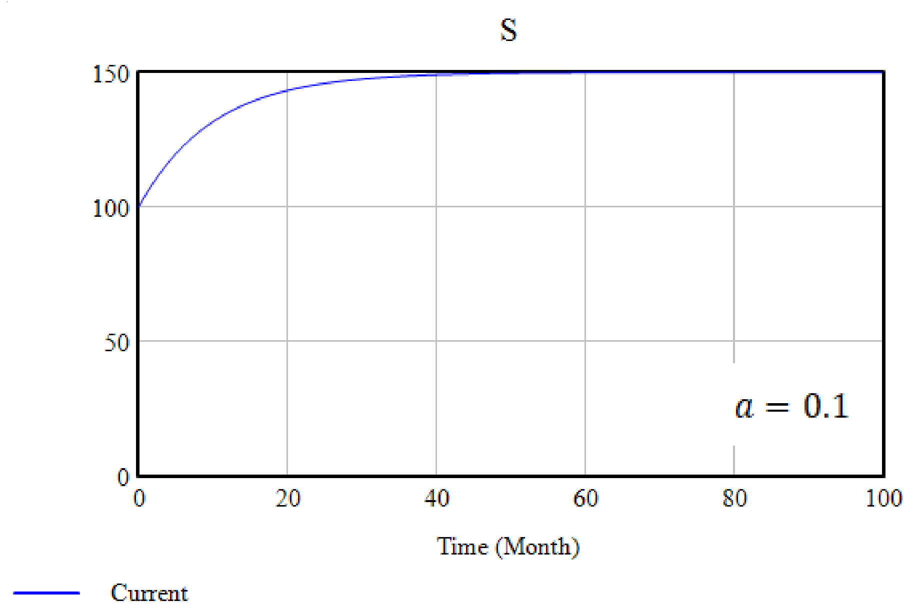
$$k - a * S > 0$$

$$k > a * S$$

$$a < \frac{k}{S}$$

$$a < \frac{15}{100} = 0.15$$

Recall:
S(0) = 100
k = 15



$a = 0.1$

Basic Stock – Flow Dynamics

- look for the case where net flow < 0

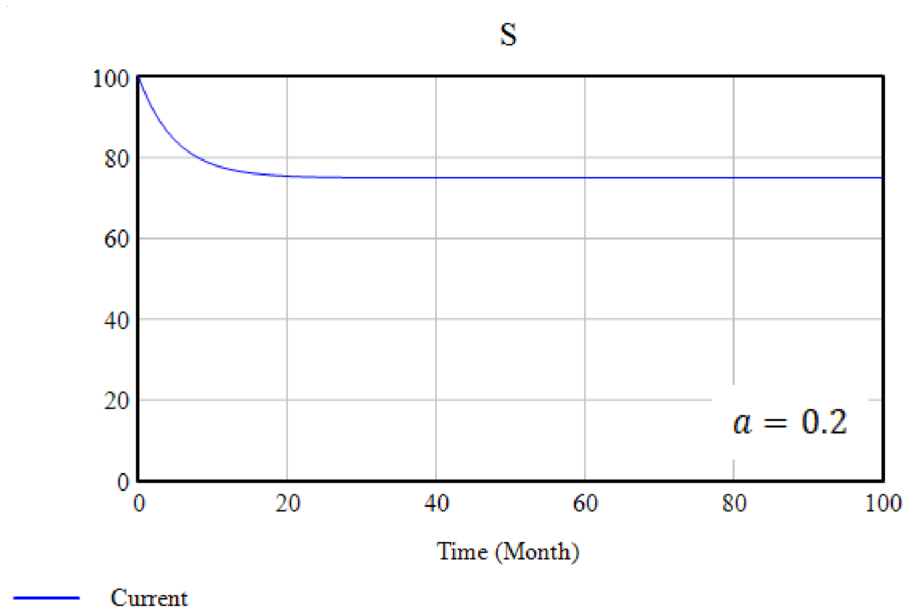
$$k - a * S < 0$$

$$k < a * S$$

$$a > \frac{k}{S}$$

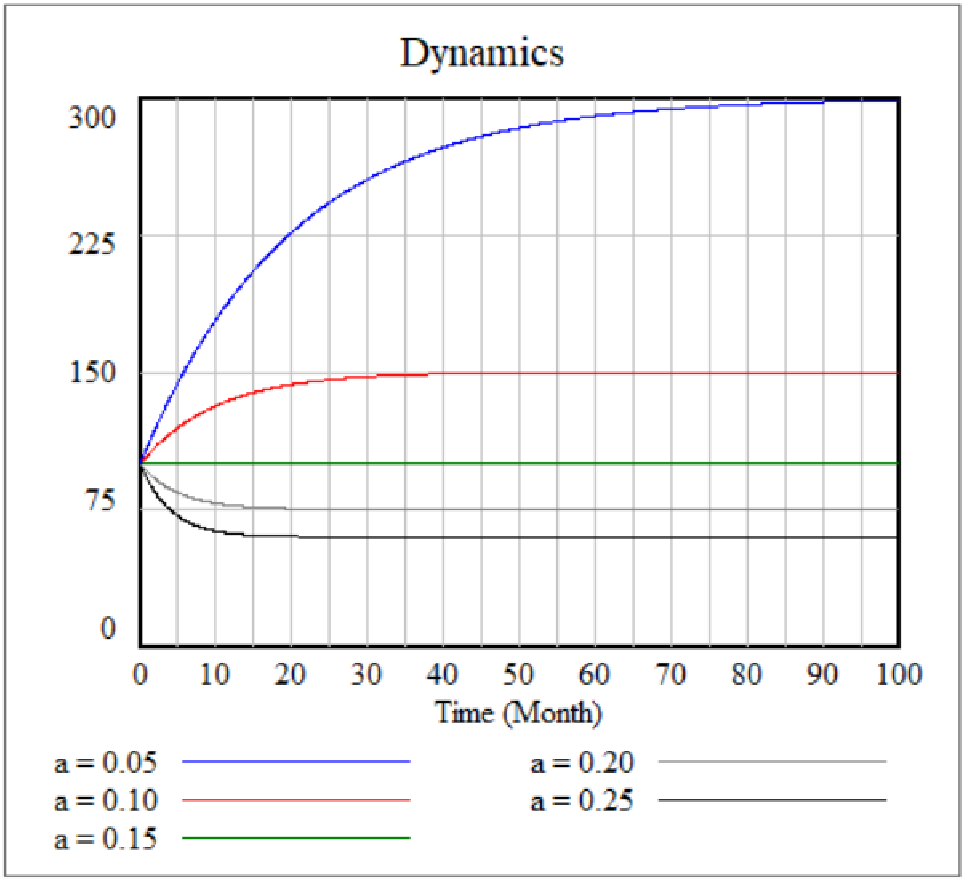
$$a > \frac{15}{100} = 0.15$$

Recall:
S(0) = 100
k = 15



Basic Stock – Flow Dynamics

- resulting dynamics with respect to different values of *a*

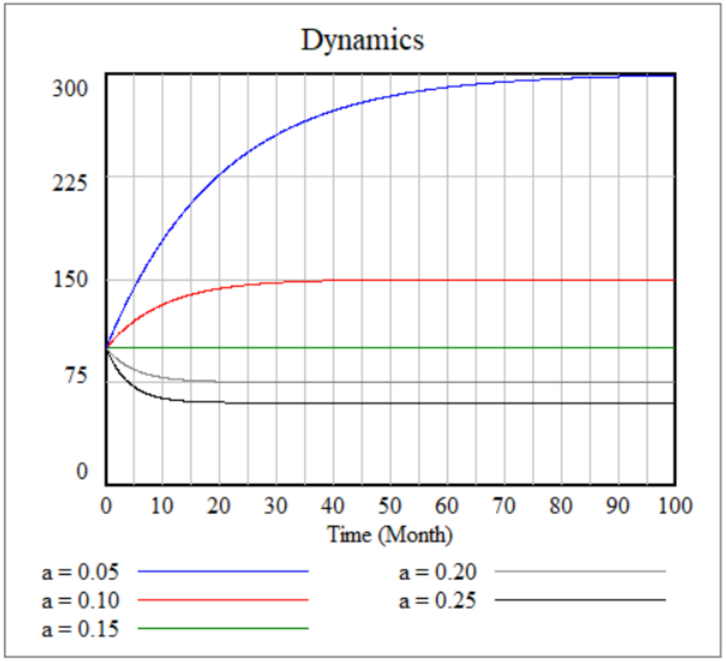


Basic Stock – Flow Dynamics

- why do all these trajectories converge to some specific values?
- can we determine where they are going to converge without running the model?
- consider the solution of the differential equation:

$$S(t) = \frac{k}{a} - \frac{e^{-a*t} * (k - a * S(0))}{a}$$

- what is the behavior of the solution as t → ∞?



Basic Stock – Flow Dynamics

- try to analyze the potential behavior as t → ∞

$$S(t) = \frac{k}{a} - \frac{e^{-a*t} * (k - a * S(0))}{a}$$

converges to 0 as t → ∞

$$S(t) = \frac{k}{a} - \frac{e^{-a*t} * (k - a * S(0))}{a}$$

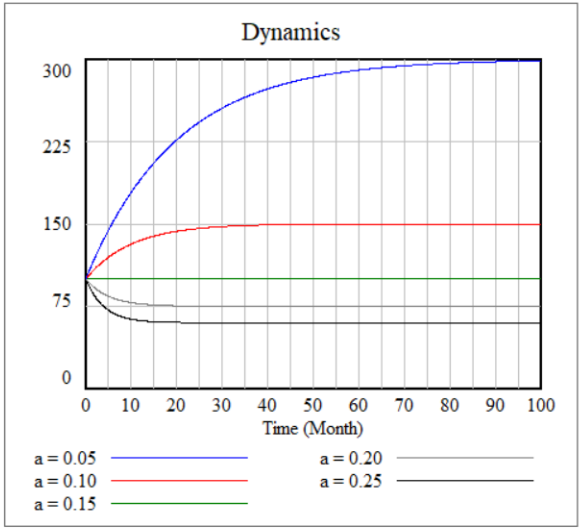
converges to 0 as t → ∞

$$\lim_{t\to\infty} S(t) = \frac{k}{a}$$

Basic Stock – Flow Dynamics

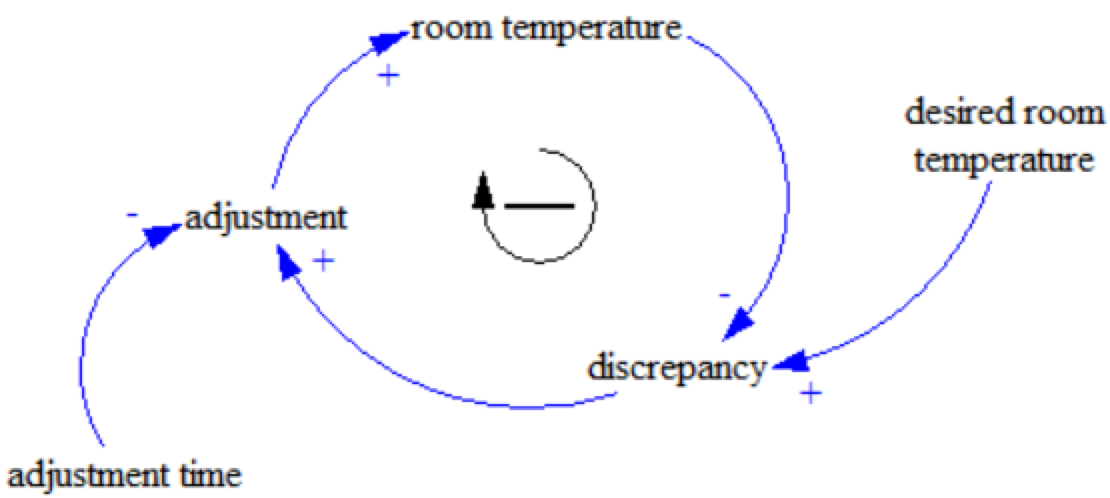
- consider
  - S(0) = 100
  - k = 15
  - a = 0.05

$$\lim_{t \to \infty} S(t) = \frac{k}{a}$$ → it will converge to 300



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- final causal loop diagram



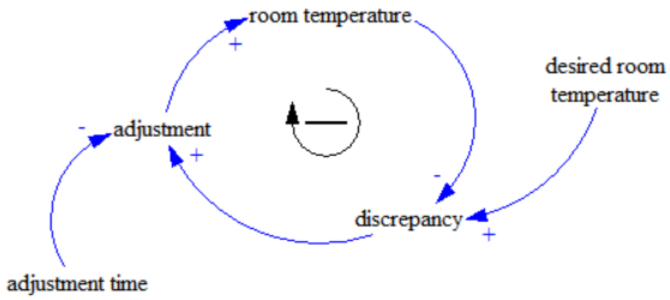
Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

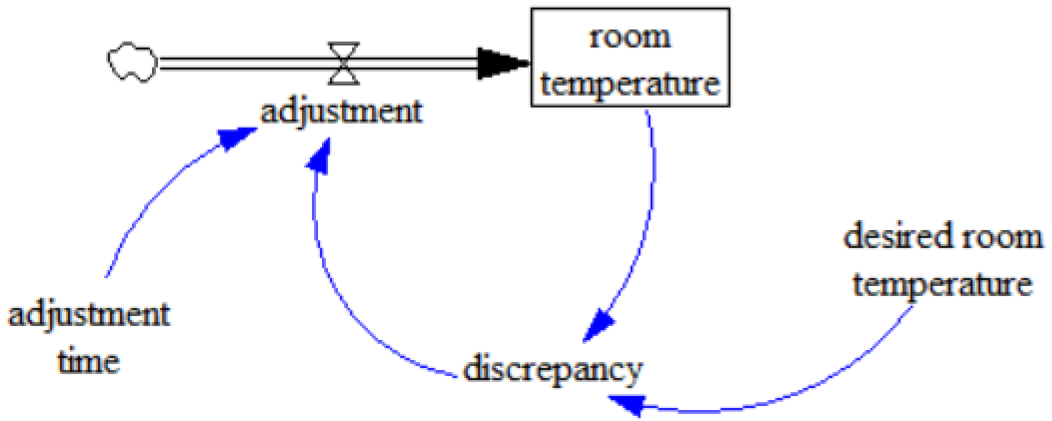
- drawing the stock-flow diagram
- stock variable(s)?
  - what are the accumulations in the model?
    - "room temperature"
- flow(s)?
  - how system states change?
    - "adjustment"
- all the remaining variables are auxiliary variables

"room temperature"
"desired room temperature"
"discrepancy"
"adjustment"
"adjustment time"

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- the stock-flow diagram





Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- before inserting the equations for the stock-flow diagram, we need to specify appropriate equations for *discrepancy* and *adjustment*
- *adjustment*
  - enables us to make "adjustments" on the room temperature
  - recall:
    - adjustments are not instantaneous!
    - unit consistency: flows describe how stocks change **over time**
  - its unit should be °C/time

$$adjustment = \frac{discrepancy}{time}$$

→ at each time step, you can "adjust" only a fraction of the discrepancy

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

$$adjustment = \frac{discrepancy}{time}$$

- *discrepancy*
  - the difference between room temperature and desired room temperature

$$(room\ temperature) - (desired\ room\ temperature)$$

or

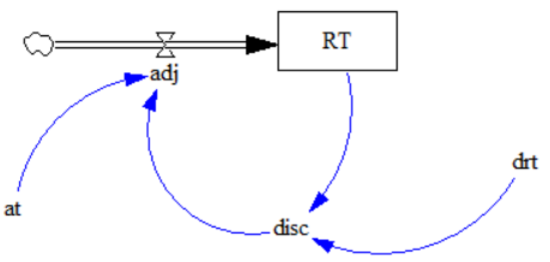
$$(desired\ room\ temperature) - (room\ temperature)$$

  - recall: adjustment is an inflow!
    - when drt > rt → adjustment > 0
    - when drt < rt → adjustment < 0

$$(desired\ room\ temperature) - (room\ temperature)$$

$$adjustment = \frac{discrepancy}{time} = \frac{(desired\ room\ temperature) - (room\ temperature)}{time}$$

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)



- model equations
  - differential equation

$$\frac{d(RT)}{dt} = adj$$

  - approximate integral equation

$$RT(t + dt) = RT(t) + adj(t) \times dt$$

  - plug in all variables and rewrite the differential equation

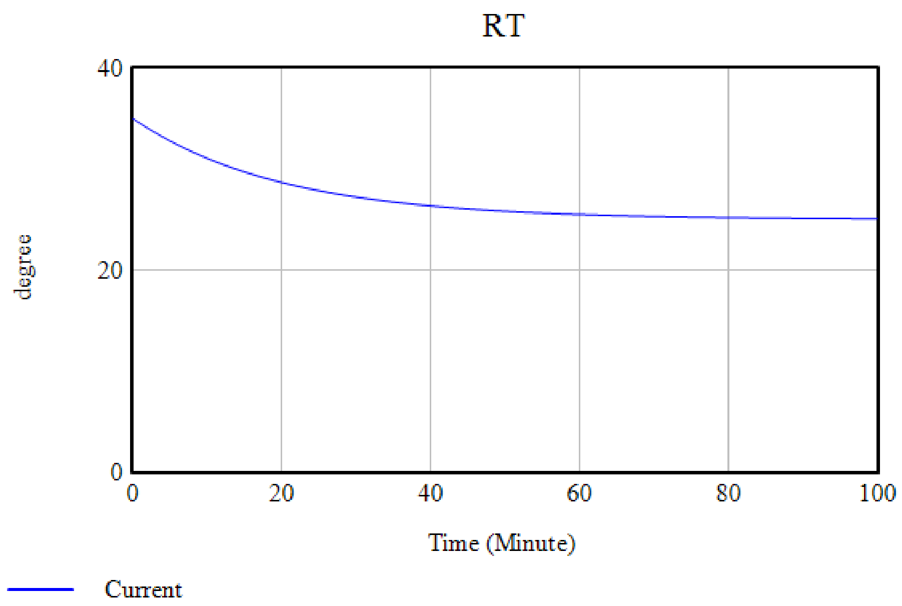
$$\frac{d(RT)}{dt} = \frac{disc}{at}$$

$$\Downarrow$$

$$\frac{d(RT)}{dt} = \frac{drt - RT}{at}$$ ⇒ analytical solution? → homework

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

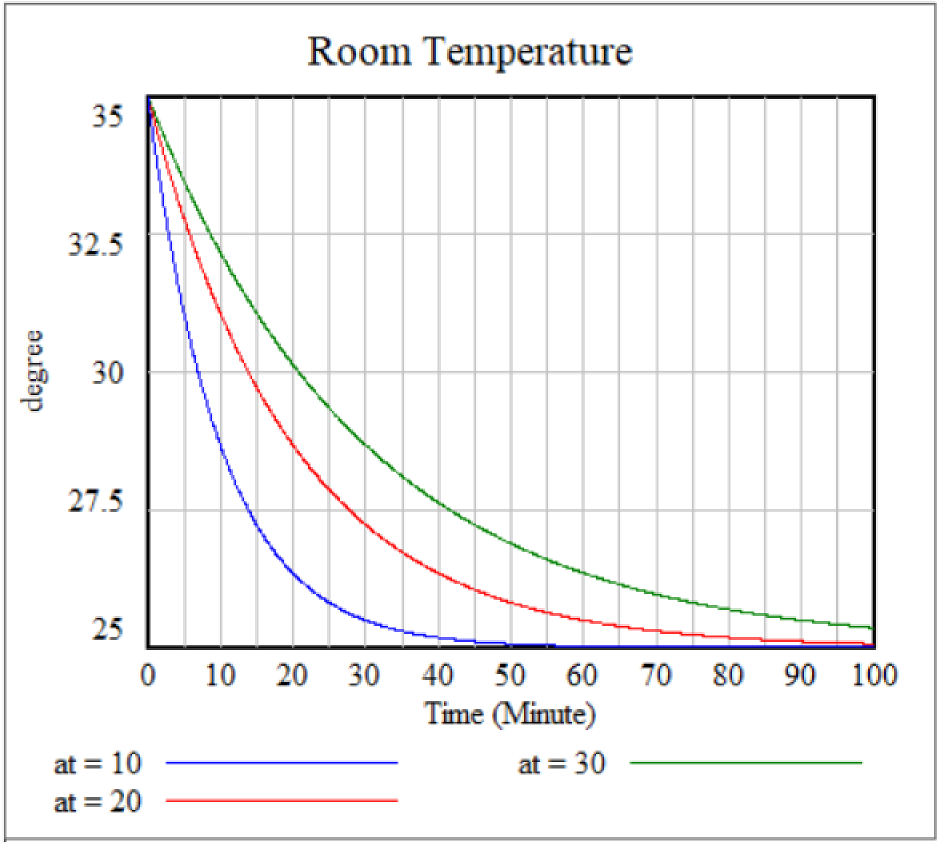
- our model is capable of modeling both heating and cooling
- case 1 (summer):
  - RT(0) = 35 °C
  - drt = 25 °C
  - at = 20 minutes



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- case 1 (summer):
  - $RT(0) = 35\ ^{\circ}C$
  - $drt = 25\ ^{\circ}C$
- consider different adjustment time values:
  - at = 10 minutes
  - at = 20 minutes
  - at = 30 minutes
- what kind of behavior do you expect to see (with respect to different values of *at*)?

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

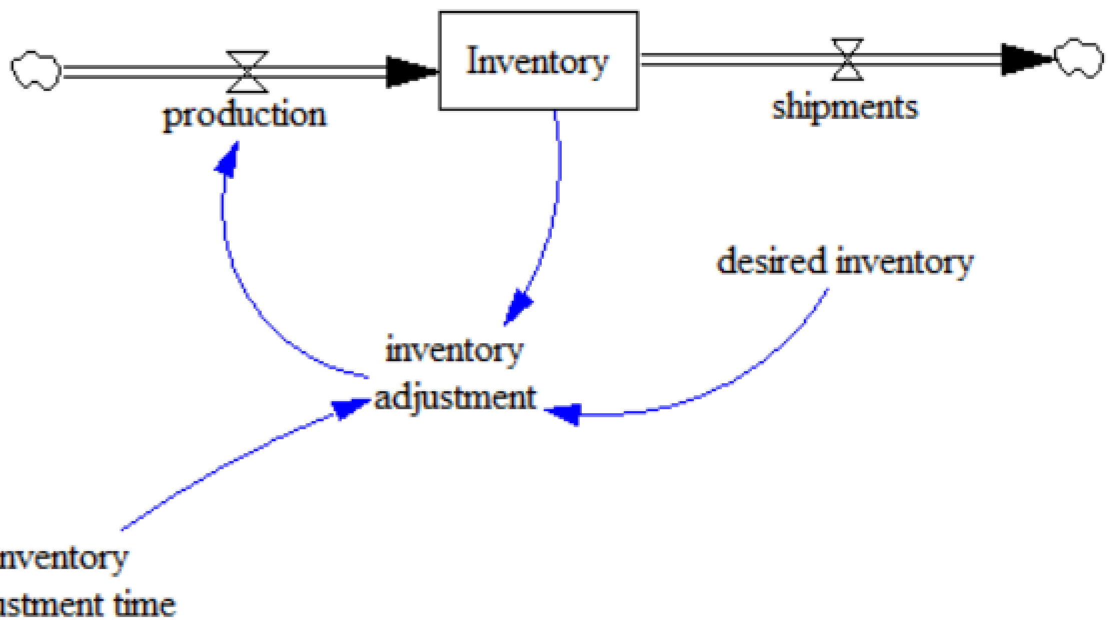
- case 2 (winter):
  - RT(0) = 10 °C
  - drt = 25 °C
- consider different adjustment time values:
  - at = 10 minutes
  - at = 20 minutes
  - at = 30 minutes
- homework:
  - simulate this in Vensim to obtain a similar graph for the values of *at* given above

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- let us consider another problem in this context
  - a firm keeps the inventory of a certain product
  - the firm has a desired inventory level
  - the inventory is changed by two flows
    - production
    - shipments
  - the firm uses a decision rule to maintain the desired inventory level
    - aim to close the gap between the desired and actual inventory level in 7 days

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- the stock-flow diagram is similar to a goal-seeking structure
  - however, there is an additional outflow → shipments



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- differential equation

$$\frac{d(Inventory)}{dt} = production - shipments$$

- approximate integral equation

$$Inventory(t+dt) = Inventory(t) + \left(production(t) - shipments(t)\right) \times dt$$

- plug in all variables and rewrite the differential equation

$$\frac{d(Inventory)}{dt} = inventory\ adjustment - shipments$$

$$\Downarrow$$

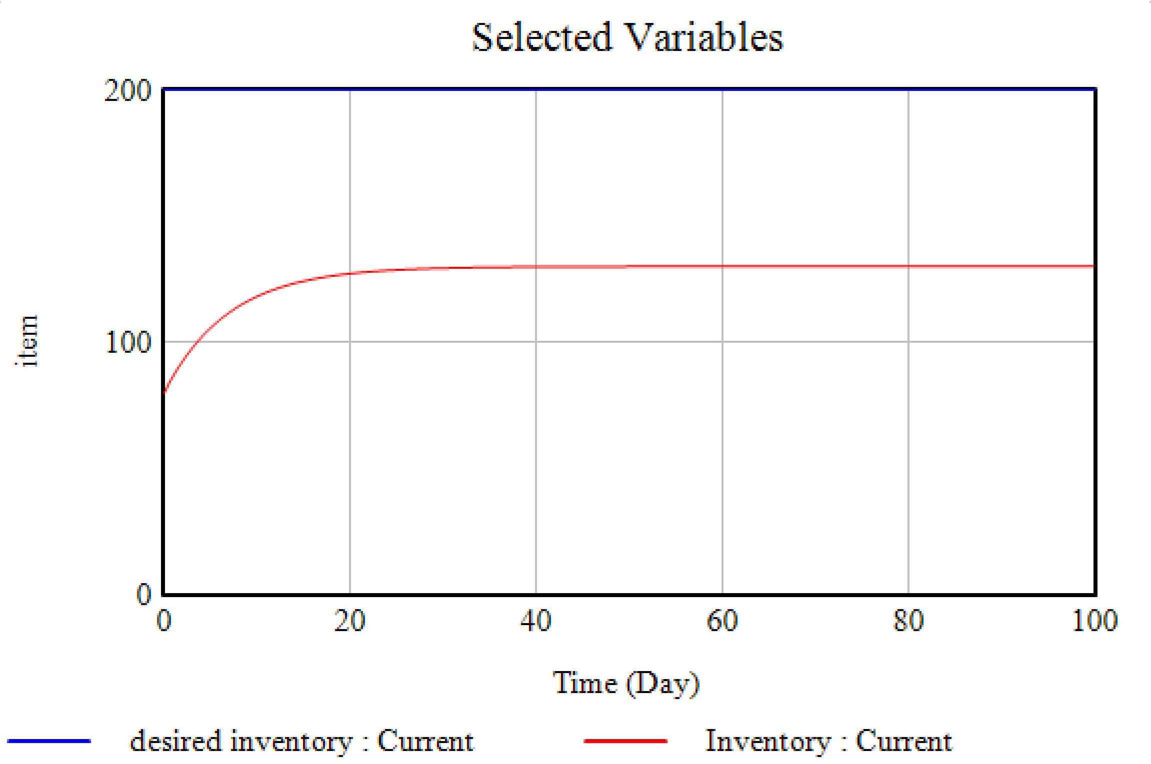
$$\frac{d(Inventory)}{dt} = \frac{desired\ inventory - inventory}{inventory\ adjustment\ time} - shipments$$

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- simulate the model with the following parameters:
  - inventory(0) = 80 items
  - shipments = 10 items/day
  - desired inventory = 200 items
  - inventory adjustment time = 7 days
- what kind of behavior do we expect to see?
  - we still have a negative feedback loop (goal-seeking structure)
  - can inventory reach its desired level?

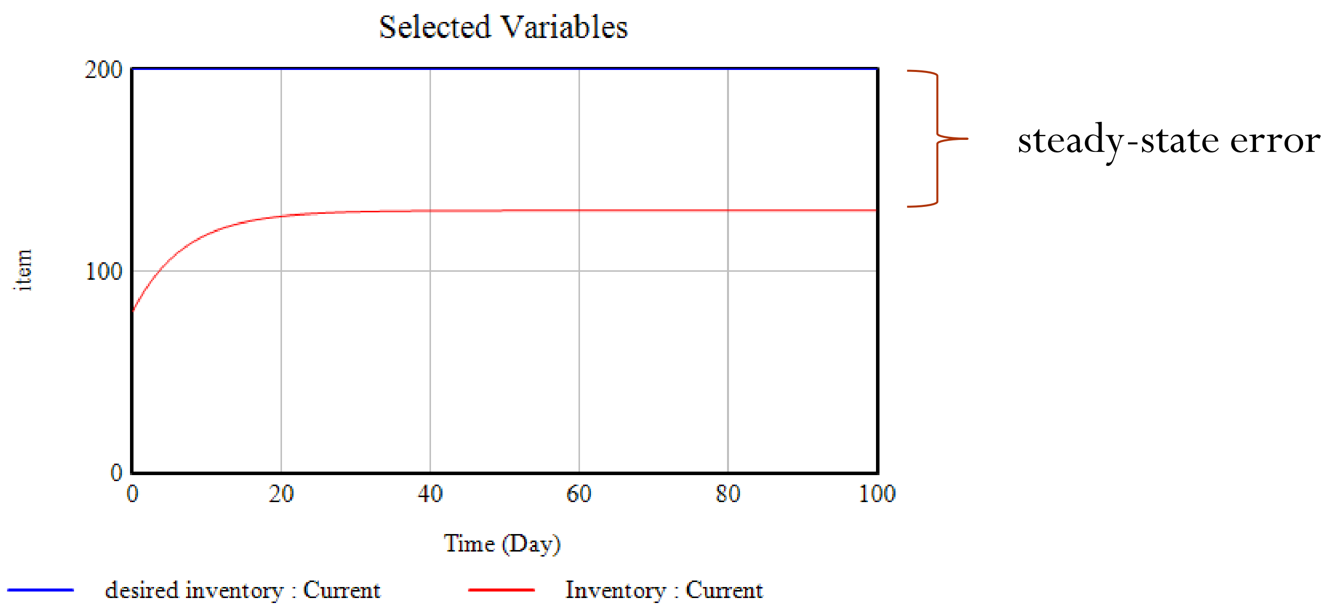
Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- the answer is **no**!
- let us see the behavior of the state variable vs. desired inventory



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

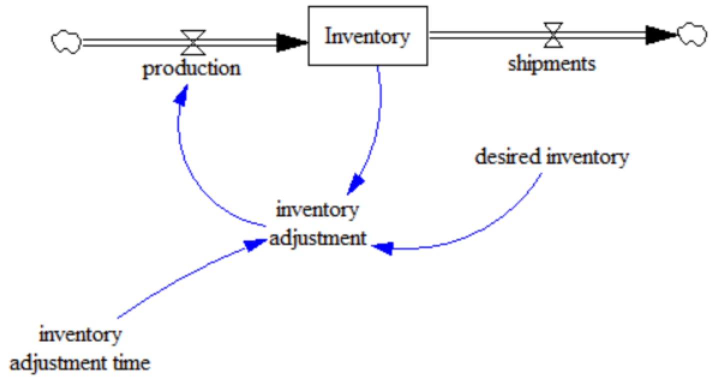
- there is a constant gap between the desired and actual level of the inventory



- what are the possible reasons to observe the steady-state error?

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- the main reason
  - we are ignoring the outflow (shipments) during production!

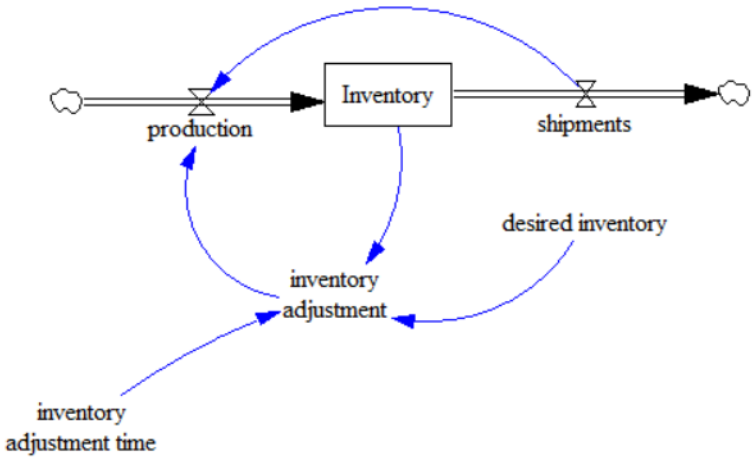


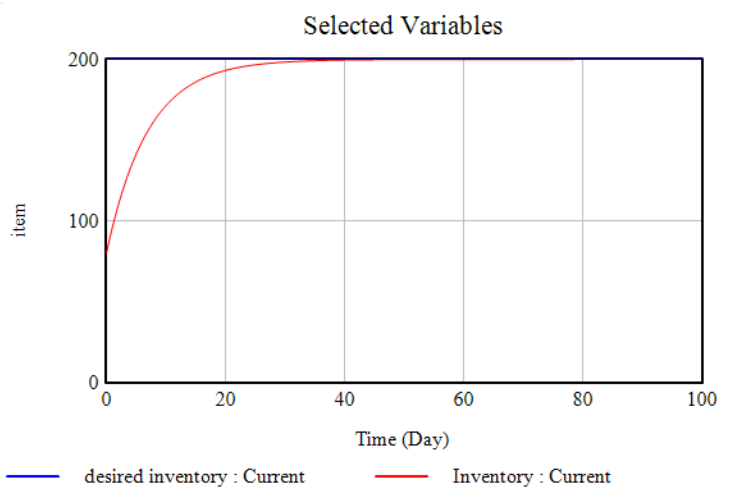
  - production depends only on the level of inventory
- we need to consider the shipments in the production

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- first solution: include shipments in the production

$$production = inventory\ adjustment + shipments$$





- is this formulation realistic?
  - recall the argument about flows!

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- second solution: forming an expectation about shipments
  - instead of directly incorporating the quantity of shipments in the production, we can use *expected shipments*
  - "expectation formation"
    - a kind of forecasting process
    - a kind of negative feedback loop structure
  - we will talk about this later (towards the end of the semester)

Basic Stock – Flow Dynamics

- References

Barlas, Y. "System Dynamics: Systemic Feedback Modeling for Policy Analysis" in Knowledge for Sustainable Development - An Insight into the Encyclopedia of Life Support Systems, UNESCO-EOLSS Publishers, Paris, Oxford, UK. 2002, pp.1131-1175.

Sterman, J. Business Dynamics. Systems Thinking and Modeling for a Complex World. McGraw-Hill, U.S.A., 2000.