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Dynamic equality of opportunity

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What are the long-term effects of policies intended to equalize opportunities among different social classes of children? To find out, we study the stationary states of an intergenerational model where adults are either White or Blue collar employees. Both adults and the state invest in their children's education. Our analysis indicates that the major obstacle to equalizing opportunities in the long-run is private educational investment. Next we examine economies where only the state invests in education, motivated by the Nordic experience. In a majority of these economies, no child lags behind regarding future prospects, a theoretical result confirmed by simulations.

INTRODUCTION

While inequality of income is a ubiquitous phenomenon, it does not necessarily imply injustice. The equality-of-opportunity ethic maintains that differences in income can be just to the extent that they can be attributed to differences in effort, a factor for which individuals can be held responsible. In contrast, if income inequality is due to factors for which individuals should not be held responsible, then the inequality is unjust.

As a particular example, suppose that some children are expected to have lower incomes as adults due to their lacking sufficient education. If those children are less educated because they had inferior educational resources, or home environments less supportive of education, the resulting skill differences are not, at least obviously, due to a lack of effort, and may be unjust. The poor educational outcomes of some children, to the extent they are caused by paucity of family resources, are not the responsibility of the child.

Note how dynamics is an inherent part of the story in this example. Inequality of income among adults induces inequality of opportunity among children if poor parents cannot provide sufficient resources for the education of their own children or if they lack social connections that rich parents possess. This will manifest itself in reproducing income inequality when the children become adults whose children will, hence, also face unequal opportunities. Therefore, inequality of opportunity and inequality of income potentially feed each other over generations.

Equality of opportunity has emerged, in the last twenty years, as an attractive alternative to welfarist approaches to social choice and welfare economics (see Roemer (1998, 2012), Fleurbaey (2008), Roemer and Trannoy (in press)) . As well as a burgeoning literature on the theory of equal opportunity, there is now a growing empirical literature (see, for example, Paes de Barro et al. (2009), Brunello and Checchi (2007), Peragine (2004)). Both theoretical and empirical literatures overwhelmingly take a static approach. In the theoretical case, this means that the focus is upon defining what the optimal opportunity-equalizing policy is at a moment in time, ignoring the dynamic issue we raise here.

Here, we study an economy with successive generations. Each generation comprises a continuum of households and each household consists of a parent and child. We postulate that a parent is either a White collar professional or a Blue collar worker. At each generation, the state decides how much to invest in the education of children from these two social backgrounds. The expenses of the state are financed by a linear income tax. Given the state's policy, each parent privately invests in her own child's education to maximize a weighted sum of her own after-tax income and the expected after-tax income of her child. The objective of the state is to equalize opportunities for children, which means investing in the education of children whose expected future prospects lag behind due to factors for which no child can be held responsible (in our case, whether her parent is a White or Blue collar employee).

These decisions by the state and parents produce the next generation's distribution of White and Blue collar adults, who solve their optimization problems under the new circumstances. The process continues indefinitely.

We want to understand the stationary states of this dynamic process[1]. In particular, what do the stationary states look like that are optimal from the equal-opportunity point of view? There are three possible kinds of stationary state: laissez-faire, moderate, and ideal. If the government collects no taxes and does not invest in education then the solution is laissez-faire. In a moderate solution, the government intervenes but the gap between the two types of children, when they enter the labor market, does not fully disappear. In an ideal solution, no child lags behind in the labor market due to her parent's type (Blue or White), and all children have identical future prospects in expected terms. In this case, the state has been able fully to compensate Blue collar children for the relative disadvantage inherent in their background, whose nature will be made precise below.

We conduct both simulations and theoretical analysis. In our simulations, laissez-faire turns out to be the most frequent kind of solution, and the least frequent solution is the ideal one. The theoretical analysis explains this regularity. We show that for a large set of parameter vectors, White collar parents react to state investment in Blue collar children by increasing private investment in their own children, undoing the effect of (well-intended) state policy. Therefore, the state, in a plurality of cases, finds it optimal not to invest in Blue collar children. If the impact of education on productivity is sufficiently high then the solution is moderate, meaning the gap between children's opportunities is reduced, but still persists. The ideal solution is observed only when the efficacy of investment in education is low.

These results point to a perpetual inequality among children due to their parental backgrounds when private supplements to public education are available, even if the state's objective is providing equality of opportunity for everyone. Thus, inspired by the Nordic experience in education, where there is negligible privately funded education, we ask what happens if there is only public investment in education. In this case, the ideal solution holds in a clear majority in all solutions in the simulations. We also prove that the solution should indeed be ideal, which ensures no child's chances are inhibited by his/her parent's type except when the impact of education on future outcomes is very low.

We proceed by specifying the model, explaining the optimization problem, reporting our simulations, and deriving the theoretical results.

I. THE ENVIRONMENT

We consider an infinite horizon dynamic economy with successive non-overlapping generations. A generation comprises a continuum of households. Each household consists of a parent and a child. Both parents and the state invest in children's education. Children from White collar backgrounds possess an advantage over children from Blue collar backgrounds in the labor market.

In formal terms, each parent is either a White collar (W) professional or a Blue collar (B) employee. The pre-tax income of a parent with a J type occupation is y_J, for $J \in \{B, W\}$. The income y_J is a function of the education that the J type parent had during her childhood.

The sum of private and public investments in a child of type J is

$$z_J = i_J + s_J$$

for any child whose parent is of type J . Here $i_J \geq 0$ and $s_J \geq 0$ are the private and public investments, respectively, in the child's education. Public investment in education is financed by an affine income tax at rate t. The budget constraint of the state shall be discussed below.

Write $\xi = (t, s_B, s_W, i_B, i_W)$ for the set of all endogenous variables of the model. Consider a child whose parent is of type J. We hypothesize that the child's income in her adulthood will be either

$$y_B(\xi) = \theta_B + \beta z_J$$

if the child becomes a B worker, or

$$y_W(\xi) = \theta_W + \beta z_J$$

if the child becomes a W professional. Note that the vector of endogenous variables $\xi = (t, s_B, s_W, i_B, i_W)$ gives us the total investment in each type of child, z_J . The parameters (θ_B, θ_W) are exogenously given basic-level wages for both types of employee. They correspond to the income of each type of employee in the case of no investment in education. We assume $\theta_B < \theta_W$. Moreover, β is the marginal impact of investment in education on income with certainty.

Note that two adults of the same type (e.g. two white collar professionals) may have different incomes. This happens when the past investment in the education of these two adults differed. Thus, not all adults of the same type earn the same income.

Whether a child will become a Blue or a White collar employee in adulthood is a stochastic event that is determined by the endogenous vector ξ. More formally, the probability that a child, whose parent has a J type job, will become a W adult is

$$\pi_J(\xi), \quad J \in \{B, W\}.$$

A parametric form of π_J will be assumed presently. Each J parent's standard of living is

$$u_J(\xi|\bar{\xi}) = y_J(\bar{\xi})(1 - t) - i_J$$

where ξ is the present generation's endogenous variable vector and $\overline{\xi}$ is the past generation's endogenous variable vector. Recall that the present adult's income y_J is a function of the past generation's $\overline{\xi}$. Let ξ^* denote the future generation's endogenous variables. Hence, by definition, the expected standard of living of the child is

$$E_J(\xi^*|\xi) = \pi_J(\xi)u_W(\xi^*|\xi) + \Big(1 - \pi_J(\xi)\Big)u_B(\xi^*|\xi).$$

Each J parent solves

(1) $$\max_{i_J} \; u_J\big(\xi|\overline{\xi}\big) + \phi E_J(\xi^*|\xi)$$

where ϕ represents the welfare weight on the child's standard of living from the parent's viewpoint. (To do this, the parent must know ξ^*; this will be the case in the stationary state.) Parents care about both themselves and their children. Note that we take the expectation of the child's material payoff (i.e. standard of living) because whether she will be a Blue collar or a White collar employee is uncertain, and may be interpreted as due to luck or to effort in the job market.

Today's optimal i_J is typically a function of future ξ^*. (That is, a parent must contemplate her child's future investment in *her* child, to determine the child's expected standard of living.) However, optimal i_J today does not depend on past $\overline{\xi}$.

Lemma 1 *Optimal i_J of Eq* (1) *does not depend upon past endogenous $\overline{\xi}$.*

Proof. Define

$$U_J = u_J\big(\xi|\overline{\xi}\big) + \phi E_J(\xi^*|\xi).$$

It is easy to see that $\overline{\xi}$ does not appear in

(2) $$\frac{dU_J}{di_J} = -1 + \phi\frac{dE_J(\xi^*|\xi)}{di_J},$$

establishing the claim. ■

Lemma 1, though obvious, is crucial, because it ensures that the number of different income levels does not increase over time but stays bounded. In fact, at any given generation, there can be at most four different income

levels[2]. Since optimal i_J does not depend upon past $\overline{\xi}$, it follows that optimal i_J is identical among all J type families although two J type adults' incomes may be different, as can be readily seen from Eq (2). As a consequence, $z_J = i_J + s_J$ is identical among all J type families. In other words, both z_B and z_W are well-defined at each generation. Now recall that the income of a B worker is either $\theta_B + \beta z_B$ or $\theta_B + \beta z_W$. Thus there are at most two levels of income of B workers. The same argument applies to W professionals, and so there are at most four different income levels.

Given that z_J is identical among all J type families, we postulate that

(3) $$\pi_W(\xi) = \frac{1}{1+e^{-x}} \text{ and } \pi_B(\xi) = \frac{a}{1+e^{x}}$$

where $x = b\frac{z_W - z_B}{\theta_W - \theta_B}$, $a \in (0,1)$ and $b > 0$. Note that the probability of becoming a white collar professional depends on $\Delta z = z_W - z_B$ for all children - the difference in investment in the education of the two types of child. The role of Δz in π_J is to capture the competitive edge that is provided by education. For instance, think of education as a signaling device in the labor market where young potential employees compete for white collar positions. Employers deem better education as a signal of competence for the tasks required from white collar professionals. However, the strength of education as a signaling device is negatively related to $\Delta\theta = \theta_W - \theta_B$. That is to say, more resource must be invested in education to become a white collar employee as the relative advantage of being a white collar employee increases. The parameter $b > 0$ represents the efficacy of investment in education.

The fact that $a < 1$ models the idea that Blue collar children are disadvantaged compared to their While collar counterparts. This can be interpreted as due to network effects. Bewley (1999) estimates that 30 to 60 percent of jobs were found through friends or relatives. Corak (2013) reports findings that show that the sons of very high earning fathers take first jobs in their father's firm much more frequently than the sons of other fathers do. In the present setting, our interpretation is that White collar parents help their children to find white collar jobs using their high-level social connections. A blue collar parent lacks this advantageous network. Chetty et. al (2015) report that, in an experiment in which poor children moved to better neighborhoods, their income increased 31% compared to children who did not move.

As for the population dynamics, write f_J^τ for the fraction of J type employees in generation τ. Of course, $f_W^\tau + f_B^\tau = 1$. Normalize the population mass to unity. The per capita mean income in generation τ is

$$\mu(\xi) = f_W^\tau \theta_W + f_B^\tau \theta_B + \beta(f_W^{\tau-1} z_W + f_B^{\tau-1} z_B)$$

where ξ represents the endogenous variables in generation $\tau - 1$. The budget constraint of the state at generation τ is

$$t\mu(\xi) = f_W^\tau s_W + f_B^\tau s_B.$$

Population fractions evolve according to

$$f_W^{\tau+1} = \pi_B f_B^\tau + \pi_W f_W^\tau$$

while $f_B^{\tau+1}$ can be deduced from the identity $f_B^{\tau+1} = 1 - f_W^{\tau+1}$. In the stationary state

$$f_J^\tau = f_J^{\tau+1} \text{ for each } J \in \{B, W\}.$$

Thus, in the stationary state we have:

$$f_W^* = \pi_B f_B^* + \pi_W f_W^*$$

or

$$f_W^*(1 - \pi_W) = \pi_B f_B^* = \pi_B(1 - f_W^*)$$

or $f_W^* = a(1 - f_W^*)$ which implies

$$f_W^* = \frac{a}{1+a},\ f_B^* = \frac{1}{1+a}.$$

In other words, the two types of job occur in fixed proportions in all stationary states.

II. STATIONARY STATES AND EQUALITY OF OPPORTUNITY POLICY

Stationary states

A concern with the long-run suggests that we seek a policy whose induced stationary state is best from the viewpoint of the social objective – in our case, equalizing opportunities. First we define a stationary state, and then we define what 'equalizing opportunities' means.

A stationary state is a state policy (t, s_W, s_B) and private investment decisions of the households (i_W, i_B) that are constant over time and induce a stationary distribution of job types and income. In particular, in a stationary state, investment decisions taken by the households are constant over time, when each expects the policy to remain fixed.

In formal terms, a stationary state is a vector of $\xi^* = (t^*, s_B^*, s_W^*, i_B^*, i_W^*)$ such that

$$t^*\mu(\xi^*) = f_W^* s_W^* + f_B^* s_B^*$$

and i_J^* is a solution in i_J to

$$\max_{i_J} \; u_J(\xi'|\xi^*) + \phi E_J(\xi'|\xi^*)$$

(4) subject to

$$\xi' = (t^*, s_B^*, s_W^*, i_B, i_W^*) \text{ if } J = B$$

$$\xi' = (t^*, s_B^*, s_W^*, i_B^*, i_W) \text{ if } J = W$$

for each J.

Of course, stationary-state analysis cannot be used to study a growing economy. If $\beta \geq 1$ then any arbitrarily high (s_W^*, s_B^*) is financially feasible given that $t^* \in (0,1)$ is chosen accordingly. Therefore, we assume $\beta \in (0,1)$ to ensure that no tax rate can finance arbitrarily high expenditures by the state. Without such a restriction on β, stationary-state analysis cannot be conducted, and we would need an endogenous growth model to analyze balanced growth paths. This would, of course, take us far from our objective: understanding the dynamic trade-offs involved in the equality of opportunity paradigm.

Let Ξ denote the set of all stationary states, ξ^*. Observe that any stationary state ξ^*, by definition, is both incentive-compatible from the standpoint of households and financially feasible from the standpoint of the state. Hence, it only remains to specify the social objective. This will determine which stationary state should be chosen from all possible elements of Ξ.

Equality of Opportunity

In our model, the clear circumstance demanding compensation is the parental status of the parent as *B* or *W*, and hence also the parental investments in the education of their children. However, one must also specify what lies behind the stochastic nature of occupational assignment – namely, the probabilities π_W and π_B. Is the fact that some children of a given type become W adults and some B adults due to differential effort, or to other circumstances that are unnamed?

We take the position that all accomplishments and behavior of children be classified as due to circumstances: that is, children should be held responsible for nothing until an 'age of consent' is reached (perhaps 14 , 16, or 18 years of age). This does not mean one should refrain from punishing and rewarding children for kinds of behavior, actions that can instill a sense of responsibility that will help the individual as an adult. Therefore, it would be inconsistent for us to interpret the stochastic element in the model as due to differential effort of *children*. Instead, we think of adult incomes as permanent incomes, and interpret the stochastic element as due to differential effort among adults. Accordingly, we define the EOp policy as one that seeks to erase the differential *expected* incomes of adults coming from different social backgrounds. We use Van de gaer's (1993) version of the EOp objective, which can be summarized as the minimum of means across types. The socially optimal stationary state is therefore the solution of the program:

$$\max_{\xi^* \in \Xi} \min[\bar{E}_B(\xi^*), \bar{E}_W(\xi^*)] \tag{5}$$

where $E_J(\xi^*|\xi^*) = \bar{E}_J(\xi^*)$, for $J = B, W$.

We note in passing what is a common feature in thinking about equality of opportunity, that the effort of one person may well become a circumstance for another. Thus, an adult may have become a W adult through hard work, but her high status becomes a circumstance for her child. Adults have no right, according to this ethic, to pass on special advantage to their child by virtue of their own effort. A similar remark applies, for example, to inheritance. It is not inconsistent to view some wealth as justly acquired yet forbid or highly restrict transferring it to one's children[3].

Types of optimal stationary state

At any solution ξ^* of the EOp program, either $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$ or not. In the former case, all children are equally well-off in expected terms, independent of their parent's type. We call this an *ideal* solution. Otherwise,

equality of opportunity is incomplete at the optimum. If this inequality is associated with $t^* = 0$ then we call the solution *laissez-faire*. When the solution is laissez-faire then the state does nothing to support the disadvantaged children. Of course, inequality between $\overline{E}_W(\xi^*)$ and $\overline{E}_B(\xi^*)$ can coexist with $t^* > 0$. In this case, we call the solution *moderate.* At a moderate solution the state actively intervenes but cannot fully eradicate the inequality between children from different backgrounds.

In sum, the three different possible regimes are listed in Table 1.

<TABLE 1 HERE>

III. ANALYSIS

In this section we begin our analysis by conducting numerical simulations. Certain patterns will emerge in simulating and calibrating the model. Theoretical explanations will follow.

Numerical Analysis

The first question we address is the distribution of the three cases as defined in Table 1 in the space of parameters. We first use numerical simulation to study this question. The vector of exogenous variables is $e = (a, b, \phi, \beta, \theta_W, \theta_B)$. We ran simulations by drawing 1000 random samples of e, calculating the solution to the EOp program as defined in (5) for each case, and finally counting the number of laissez-faire, moderate and ideal regimes in these 1000 solutions.

In the first numerical simulation, the support of the parameters is set to be relatively wide. To be specific, random samples are drawn as follows: $(a, \phi, \beta) \in (0,1)^3$, $\theta_W \in (0,100)$, $\theta_B \in (0, \theta_W)$, and $b \in (0,50)$. All parameters are assumed to be distributed uniformly on their supports. We obtain the distribution of solutions reported in Table 2.

<TABLE 2 HERE>

Laissez-faire policy has the highest frequency, and is followed by the moderate regime. Ideal solutions comprise the lowest frequency. Moreover, as one would expect, $\overline{E}_W(\xi^*) \geq \overline{E}_B(\xi^*)$ in all cases. Nonethless, $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$ occurs in only 29.5% of the cases. Therefore, in a clear majority of the cases $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$: children of white collar parents are better-off than the children of blue collar parents in 70.5% of the cases.

We also report the averages of certain variables of interest in this simulation, in Table 3. These variables are the tax rate, the probability of becoming a W professional, the ratio of expected future incomes, and how much is invested in education, as a proportion of income, according to type.

<TABLE 3 HERE>

It is noteworthy that the averages of t at moderate and ideal solutions are similar. In contrast, laissez-faire and moderate solutions exhibit a similarity in average π_W. Of course, $\pi_B = a(1 - \pi_W)$ can be used to retrieve the average value of π_B at each solution. The average ratios of $\overline{E}_W$ to $\overline{E}_B$ at laissez-faire and moderate solutions are around 2 while the same ratio is, by definition, exactly 1 at any ideal solution.

In order to see whether there is an observable regularity, we also ran a simulation that admits a visual, geometric relation between parameters and the solution to the EOp program. See Figure 1a and 1b.

<FIGURE 1.A AND 1.B HERE>

The random samples for the simulations in Figure 1a-b are drawn as follows. First, we fix $a = 0.56, \theta_W = 100, \theta_B = 58$ by calibrating these parameters to the US data. Our calibration method, and the data are explained in Appendix. Since b and β are hardly observed in real life, $(b, \beta) \in (0,50) \times (0,1)$ are randomly chosen. As for φ, we impose 0.4 and 0.9 respectively. Our aim is to see the impact of φ on the results as clearly as possible. The number of the random samples is 1000 for each figure.

We see the following regularities. If b is high then we observe no ideal solutions, i.e. $\overline{E}_W - \overline{E}_B > 0$ is always true. If, however, β is high then there is no laissez-faire solution, i.e. optimal EOp taxation is always positive. Finally, comparing these two figures shows that the number of ideal solutions decreases as φ increases.

Finally, we investigate the possible conflict between opportunity-egalitarian and utilitarian ethics. The utilitarian would choose the stationary state that maximizes income per capita, which is the same thing as total expected income per capita in our model. Critics of equality of opportunity in educational policy often say that if taken too far, the *average* skills of the population will suffer. Some say that ambitious parents will cease investing so much in their children, if they see the effect being undone by state subsidies to the disadvantaged. To study this, we chose the vectors where the EOp solution was ideal. There are 295 parameter vectors that satisfy this criterion.

We then restricted the tax rate to be zero for these economies, and calculated the stationary state: in other words, that stationary state in which only private investment in education occurs. We denote the zero-tax rate stationary states as ones of 'mandatory laissez-faire.' Table 4 presents the comparison of the ideal EOp stationary state and the mandatory-laissez-faire stationary state.

<TABLE 4 HERE>

First, we see that there is no conflict between utilitarian and equal-opportunity goals here: that is, average income is higher in the ideal EOp solutions than in the mandatory laissez-faire solutions. Secondly, total investment in education is much higher in the ideal EOp solutions. It is true that *W* parents are dissuaded from investing in their children in the ideal EOp solution (crowding out), but the state actually invests more in *W* children in the ideal EOp solution than the *W* parents invest in them in the mandatory laissez-faire solution. Unsurprisingly, *W* children fare somewhat worse in the ideal EOp solution than in the mandatory laissez-faire solution, but the improvement in the expected incomes of the *B* children more than compensates for this, from the social (i.e., utilitarian) viewpoint.

We next study how many of these regularities we can deduce from the model.

Theoretical analysis

In this sub-section we attempt to explain the regularities observed in the numerical simulations. Let us first focus on the pervasiveness of $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$. We have:

Theorem 2 *Let ξ^* solve the EOp program. If $i_W^* > 0$ then $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$.*

Proof. Suppose $\xi^* = (t^*, s_B^*, s_W^*, i_B^*, i_W^*)$ solves the EOp program and $i_W^* > 0$. The first step of the proof demonstrates that $\pi_W \geq 1/2$.

To see this, consider the first order optimality condition for $i_W^* > 0$

$$\frac{dU_W}{di_W} = -1 + \phi \frac{d\overline{E}_W(\xi^*)}{di_W} = 0$$

which is equivalent to

$$-1 + \phi \left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta \theta} + (1-t^*)\beta \right) = 0 \quad (6)$$

where $\Delta\theta = \theta_W - \theta_B$, and all relevant variables are, of course, evaluated at ξ^*. However, $\Delta u = u_W - u_B > 0$ for otherwise

$$-1 + \phi\left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta} + (1-t^*)\beta\right) \leq -1 + \phi(1-t^*)\beta < 0.$$

The second order condition of the W- parent's optimization problem is

$$\phi b^2(1-2\pi_W)\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta^2} \leq 0.$$

Since $\Delta u > 0$, it follows that $1 - 2\pi_W \leq 0$ as claimed. However,

$$\overline{E}_W(\xi^*) - \overline{E}_B(\xi^*) = \big(\pi_W - a(1-\pi_W)\big)\Delta u + \beta\Delta z(1-t^*).$$

Hence if $\Delta u > 0$ and $\pi_W > a/(1+a)$ and $\Delta z = z_W - z_B > 0$ then $\overline{E}_W(\xi^*) - \overline{E}_B(\xi^*) > 0$. But we showed that $i_W^* > 0$ implies $\pi_W > 1/2$ and $\Delta u > 0$. (See the definition of π_W to see that $\pi_W > 1/2$ implies $z_W - z_B > 0$.) Note that the maximum value of $a/(1+a)$ is always less than 1/2. Conclude that $i_W^* > 0$ implies $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$.■

Theorem 2 explains why $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ is such a common case. An interior solution to the W-parent's maximization problem is sufficient for incomplete equality of opportunity: $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$.

We next show that if b is sufficiently high then $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ (as in Figures 1a-b).

Theorem 3 *Let ξ^* solve the EOp program. If b is sufficiently high then $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$.*

Proof. Suppose that $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$ where ξ^* is the solution to the EOp program. Then, from Theorem 2, $i_W^* = 0$, and so:

(7) $$\overline{E}_W(\xi^*) - \overline{E}_B(\xi^*) = \big(\pi_W - a(1-\pi_W)\big)\Delta\theta(1-t^*) + \beta\Delta z(1-t^*).$$

It follows that

(8) $$a(1-\pi_W) - \pi_W = \beta\frac{\Delta z}{\Delta\theta}.$$

But the definition of π_W implies

(9) $$\frac{\Delta z}{\Delta\theta} = \frac{1}{b}\log\left(\frac{\pi_W}{1-\pi_W}\right).$$

Substituting $\Delta z/\Delta\theta$ in Eq (8) into Eq (9) gives

$$a(1-\pi_W)-\pi_W-\frac{\beta}{b}\log\left(\frac{\pi_W}{1-\pi_W}\right)=0.$$

Deduce that $\pi_W \to a/(1+a)\ as\ b\to\infty$. As a consequence, $dU_W/di_W\to\infty$ as $b\to\infty$ since

$$\frac{dU_W}{di_W}=-1+\phi(b\pi_W(1-\pi_W)+\beta)(1-t^*)$$

Therefore, $\frac{dU_W}{di_W}>0$ is implied by sufficiently high b, contradicting $i_W^*=0$. ■

Large *b* means that the impact of investment in education on the probability of becoming a *W* adult is high. In such a case *W*- parents find it rational to invest positive amounts. Hence, it follows that an ideal solution to the EOp program cannot occur if *b* is high enough.

Note that Theorem 2 can be rephrased as follows: $i_W^*=0$ is a necessary condition for complete equality of opportunity among children from different backgrounds. Under what conditions can we be sure that $i_W^*=0$? The answer turns out to be pertinent to another regularity in the simulations.

Proposition 4 *Suppose* $\phi\left(\frac{b}{4}+\beta\right)<1$. *Then* $i_W^*=0$.

Proof. Recall that the first order optimality condition of W parents is

$$-1+\phi\left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta}+(1-t^*)\beta\right)\leq 0 \text{ (with equality if } i_W^*>0).$$

Note that $\pi_W(1-\pi_W)\leq\frac{1}{4}\text{and}(1-t^*)\beta\leq 1$. Now we shall show that $i_B^*=0$ at any ξ^*. Consider the problem of the B-parent whose first order condition satisfies

$$\frac{dU_B}{di_B}=-1+\phi\left(ab\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta}+(1-t^*)\beta\right)$$

$$<-1+\phi\left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta}+(1-t^*)\beta\right)=\frac{dU_W}{di_W}\leq 0.$$

Deduce that $i_B^*=0$. Therefore $\Delta u=\Delta\theta(1-t^*)-(i_W^*-i_B^*)\leq\Delta\theta$. As a consequence,

$$-1+\phi\left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta}+(1-t^*)\beta\right)\leq$$

$$-1 + \phi\left(\frac{b}{4} + \beta\right) < 0$$

where the last inequality follows from the hypothesis of the proposition. Conclude that $i_W^* = 0$. ■

Hence, a small value of ϕ ensures zero private investment by W parents, which is intuitively unsurprising. Our first theorem states that this is a necessary condition for ξ^* to be an ideal solution. Now we can explain why the number of ideal solutions in the simulations decreases when ϕ increases as can be easily observed from Figures 1a-b. When ϕ is very low then W-parents do not care much about their children and make no investment in them. Therefore, private investment in education cannot impede state policies designed to implement $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$.

Another clear regularity in the simulations is that there is no exception to the inequality $\overline{E}_W(\xi^*) \geq \overline{E}_B(\xi^*)$. The next theorem shows why.

Theorem 5 *Let ξ^* solve the EOp program. Then generically $\overline{E}_W(\xi^*) \geq \overline{E}_B(\xi^*)$.*

Proof. We need to consider only the cases in which $i_W^* = 0$ because when $i_W^* > 0$, $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ has already been proved.

Suppose that the claim were false. Consider a solution at which $\overline{E}_W(\xi^*) < \overline{E}_B(\xi^*)$. We know $i_B^* = 0$ as we proved above, and, by hypothesis, $i_W^* = 0$. Thus, $\Delta u = \Delta\theta(1 - t^*)$. Now suppose $s_B^* > 0$. Then foregoing a small part of s_B^* to finance a small increase in s_W^* while keeping t^* constant would generically not affect private investment decisions. But it would increase

$$\overline{E}_W(\xi^*) = \pi_W u_W + (1 - \pi_W)u_B = \pi_W \Delta u + u_B$$

because Δu is positive, and π_W is monotonically increasing in s_W^* and decreasing in s_B^*. But then increasing the expected standard of living of the worse-off would be feasible. Conclude that $s_B^* > 0$ cannot happen if $\overline{E}_W(\xi^*) < \overline{E}_B(\xi^*)$. Therefore, $\xi^* = (t^*, s_B^*, s_W^*, i_B^*, i_W^*) = (t^*, 0, s_W^*, 0, 0)$,

which ensures that $\pi_W > 1/2 > a/(1 + a)$. But $\pi_W > a/(1 + a)$ implies

$$\overline{E}_W(\xi^*) - \overline{E}_B(\xi^*) = \big(\pi_W - a(1 - \pi_W)\big)\Delta u + \beta z_W(1 - t^*) > 0. \blacksquare$$

Next we address what is arguably the most significant regularity in the simulations. We see that if β is high enough then the solution is never laissez-faire in the simulations. That is, if β is sufficiently high then $t^* > 0$.

Theorem 6 *If β is sufficiently large then $t^* > 0$ at any solution to the EOp program, ξ^*. More specifically, (i) $\beta > \frac{\theta_B}{a\theta_W+\theta_B}$ implies $t^* > 0$ when $i_W^* > 0$, and (ii) $\beta > \frac{a\theta_W+(1-a)\theta_B}{a\theta_W+\theta_B}$ implies $t^* > 0$ when $i_W^* = 0$.*

Proof. Let $\xi = (t, s_B, s_W, i_B, i_W) \in \Xi$ be an arbitrary stationary state. We shall show that if β is sufficiently large and $t = 0$ at ξ then ξ cannot solve the EOp program because a very small increase in t increases the value of $\overline{E}_B$. This is sufficient to prove the claim, since $\overline{E}_B < \overline{E}_W$ at any stationary state ξ if $t = 0$.

In particular, holding s_W constant, the impact of an infinitesimal change in t on $\overline{E}_B$ is

$$\frac{d\overline{E}_B}{dt} = \frac{\partial\overline{E}_B}{\partial t} + \frac{\partial\overline{E}_B}{\partial s_B}\frac{\partial s_B}{\partial t} + \frac{\partial\overline{E}_B}{\partial i_W}\frac{\partial i_W}{\partial t}. \tag{10}$$

Now compute that:

$$\frac{\partial\overline{E}_B}{\partial t} = -a(1-\pi_W)\Delta\theta - (\theta_B + \beta z_B)$$

$$\frac{\partial\overline{E}_B}{\partial s_B} = ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} + \beta(1-t) \tag{11}$$

$$\frac{\partial\overline{E}_B}{\partial i_W} = -ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} - a(1-\pi)$$

where $\Delta\theta = \theta_W - \theta_B$.

Now we derive $\frac{\partial s_B}{\partial t}$ and $\frac{\partial i_W}{\partial t}$. Given t, the pair (s_B, i_W) is a solution to the following nonlinear equation system:

$$t\big(a\theta_W + \theta_B + \beta(az_W + z_B)\big) - as_W - s_B = 0$$

$$-1 + \phi\left(b\pi_W(1-\pi_W)\frac{\Delta u}{\Delta\theta} + (1-t)\beta\right) = 0.$$

The first equation is the budget constraint and the second equation is the optimality condition of W parents, which stipulates an interior solution. The case of boundary solution, i.e. $i_W^* = 0$, is addressed at the end of the proof. Now write this equation system as

$$F(s_B, i_W, t) = 0.$$

By the implicit function theorem,

$$\begin{bmatrix} \frac{\partial s_B}{\partial t} \\ \frac{\partial i_W}{\partial t} \end{bmatrix} = -\begin{bmatrix} D_{s_B}F \\ D_{i_W}F \end{bmatrix}^{-1} \cdot D_t F$$

where D is the differentiation operator. Note that

$$\begin{bmatrix} D_{s_B}F \\ D_{i_W}F \end{bmatrix} = \begin{bmatrix} \beta t - 1 & at\beta \\ -\phi\pi_W(1-\pi_W)\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 & \phi\pi_W(1-\pi_W)\left(\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 - \frac{b}{\Delta\theta}\right) \end{bmatrix}$$

and

$$D_t F = \begin{bmatrix} \mu(1+a) \\ -\phi(b\pi_W(1-\pi_W)+\beta) \end{bmatrix}.$$

Since $t = 0$ it follows that

$$\begin{bmatrix} D_{s_B}F \\ D_{i_W}F \end{bmatrix}^{-1} = -\frac{1}{\phi\pi_W(1-\pi_W)\left(\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 - \frac{b}{\Delta\theta}\right)} \begin{bmatrix} \phi\pi_W(1-\pi_W)\left(\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 - \frac{b}{\Delta\theta}\right) & 0 \\ \phi\pi_W(1-\pi_W)\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 & -1 \end{bmatrix}$$

which implies that

(12) $$\frac{\partial s_B}{\partial t} = \mu(1+a)$$

$$\frac{\partial i_W}{\partial t} = -\frac{\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 \mu(1+a) + \frac{\beta}{\left(\phi\pi_W(1-\pi_W)\right)} + b}{\frac{b}{\Delta\theta} - \left(\frac{b}{\Delta\theta}\right)^2 \Delta u(1-2\pi_W)}$$

at a stationary state ξ such that $t = 0$. Using (11) and (12) to expand (10) we obtain

$$\frac{d\overline{E}_B}{dt} = -a(1-\pi_W)(\theta_W - \theta_B) - \theta_B + \left(ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} + \beta\right)\mu(1+a)$$

$$+\left(ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} + a(1-\pi_W)\right)\frac{\Delta u(1-2\pi_W)\left(\frac{b}{\Delta\theta}\right)^2 \mu(1+a) + \frac{\beta}{\left(\phi\pi_W(1-\pi_W)\right)} + b}{\frac{b}{\Delta\theta} - \left(\frac{b}{\Delta\theta}\right)^2 \Delta u(1-2\pi_W)}.$$

Deduce that

$$\frac{d\overline{E}_B}{dt} = -a(1-\pi_W)\Delta\theta - \theta_B + \left(ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} + \beta\right)\mu(1+a)$$

$$+a(1-\pi_W)\left(1+\pi_W b\frac{\Delta u}{\Delta\theta}\right)\left(\frac{\frac{\beta}{(\phi\pi_W(1-\pi_W))}+b}{\frac{b}{\Delta\theta}-\left(\frac{b}{\Delta\theta}\right)^2\Delta u(1-2\pi_W)} - \mu(1+a)\left(1-\frac{\frac{b}{\Delta\theta}}{\frac{b}{\Delta\theta}-\Delta u\left(\frac{b}{\Delta\theta}\right)^2(1-2\pi_W)}\right)\right)$$

$$= -a(1-\pi_W)\Delta\theta - \theta_B + \big(\beta - a(1-\pi_W)\big)\mu(1+a)$$

$$+a(1-\pi_W)\left(1+\pi_W b\frac{\Delta u}{\Delta\theta}\right)\frac{\frac{\beta}{(\phi\pi_W(1-\pi_W))}+\mu(1+a)\frac{b}{\Delta\theta}+b}{\frac{b}{\Delta\theta}-\left(\frac{b}{\Delta\theta}\right)^2\Delta u(1-2\pi_W)}.$$

Since $\pi_W > -(1-2\pi_W)$:

$$\frac{d\overline{E}_B}{dt} > -a(1-\pi_W)\Delta\theta - \theta_B + \big(\beta - a(1-\pi_W)\big)\mu(1+a) + a(1-\pi_W)\big(\Delta\theta + \beta + \mu(1+a)\big)$$

$$> -\theta_B + \beta\mu(1+a) \geq -\theta_B + \beta(a\theta_W + \theta_B)$$

which implies $\frac{d\overline{E}_B}{dt} > 0$ if $\beta > \frac{\theta_B}{a\theta_W+\theta_B}$ and if $i_W^* > 0$.

As for the case in which $i_W^* = 0$, one can show that

$$\frac{d\overline{E}_B}{dt} = -a(1-\pi_W)(\theta_W - \theta_B) - \theta_B + \left(ab(1-\pi_W)\pi_W\frac{\Delta u}{\Delta\theta} + \beta\right)\mu(1+a)$$

$$\geq -a(1-\pi_W)(\theta_W - \theta_B) - \theta_B + \beta\mu(1+a)$$

$$\geq -a(\theta_W - \theta_B) - \theta_B + \beta\mu(1+a)$$

which implies $\frac{d\overline{E}_B}{dt} > 0$ if $\beta > \frac{a\theta_W+(1-a)\theta_B}{a\theta_W+\theta_B}$.■

Empirical discussion

What can be inferred about reality from this theoretical discussion? We argue that –when confronted with data-- our model points to the moderate solution as a more likely outcome if the policy makers possess an equal-opportunity ethic.

First, let us see why the ideal solution is a less likely outcome. The ideal solution can occur only if *W*-parents do not privately invest in their children. (Theorem 2). Nonetheless, relatively affluent and well-educated parents typically invest in their children's education in real life. This basic empirical observation means the ideal solution will not occur; it can occur only if the government forbids private investment in education or a social norm prevents it. We shall discuss provision of education only in public schools in the next section.

Second, laissez-faire is a less likely outcome in real life. As we shall see now, relatively low values of β discourage the laissez-faire solution by creating ample incentive for active taxation. Here we invoke Theorem 6 and some basic data. Recall that

$$\beta > \frac{1}{a\theta_W/\theta_B+1}$$

ensures active taxation for equality of opportunity, $t^* > 0$ (Theorem 6). Using OECD data, we calibrate this threshold level of β given by $1/(a\theta_W/\theta_B + 1)$ for selected countries. Our method of calibration and the data are explained in Appendix.

<TABLE 5 HERE>

As can be seen in Table 5, the average threshold of β is 0.53: $\beta > 0.53$ implies that, on average, active taxation is optimal for policy makers whose objective is equality of opportunity. The highest calibrated value of the threshold of β is 0.6 for Sweden, Canada, and Japan.

But is the actual value of β greater than 0.6? Although we cannot derive a precise value for β, a rough picture can be obtained as follows. According to Psacharopoulos and Patrinos (2004), private returns to education in OECD countries vary between 11% and %13. Therefore, in the current setting, the marginal impact of private investment on expected income is

$$\frac{d\bar{E}_W}{di_W} \approx 1.12.$$

In words, $1 investment in education implies $1.12 increase in expected income. However, this rate of return is not risk-free. The uncertainty in this rate of change stems from the fact that whether a child will be a *W* or *B* collar worker is a random event in our model. The increase in income due to education that does not depend on this risky outcome is represented by the parameter β. Since, however, education is typically not a short-run investment, its

return presumably involves low risk, which would be the case with high β. Hence, we deem that $\beta > 0.6$ is plausible. If this is the case, active taxation is optimal in the EOp program for all the countries in Table 5. Of course, further data and deeper empirical analysis would contribute to the discussion because we cannot determine the exact value of β. We believe that it is not far-fetched to think that β is close to 1 if education is indeed a relatively safe investment of parents for their children in the long-run.

In conclusion, the moderate solution seems to be a more likely outcome if the objective of the policy makers is to equalize opportunities when our model is confronted with data.

IV. ONLY PUBLIC SCHOOLS

Our analysis suggests that private investment in education is a major obstacle to obtaining equality of opportunity among children from different backgrounds. In particular, we proved that $i_W^* > 0$ implies that $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$. Moreover, sufficiently high b also implies $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$. Finally, the number of cases in which $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$ decreases as ϕ increases in our simulations. This numerical result is related to the theoretical fact that that if ϕ is low enough then $i_W^* = 0$. Nevertheless, we find low ϕ an empirically implausible condition: parents do value their children's welfare quite highly in real life.

Therefore, we now posit that the state forbids private investment in education, or that a social ethos prevents it, which may the historical explanation of the Nordic practice. Now we seek a solution to the following program in ξ^*:

$$\max_{\xi^*} \ \min\left[\overline{E}_B(\xi^*), \overline{E}_W(\xi^*)\right]$$

$$\text{s.t. } t^*\mu(\xi^*) = f_W^* s_W^* + f_B^* s_B^* \tag{13}$$

$$i_J^* = 0, J = B, W$$

$$\xi^* \in [0,1] \times \mathbb{R}_+^4.$$

Let us call (1(the *Nordic EOp program* . The interpretation is that the state aims at equalizing opportunities by investing in education in an economy where, for some reason, we constrain $i_W = i_B = 0$.

We can calculate the distribution of laissez-faire, moderate and ideal solutions to the Nordic EOp program by means of numerical simulations. Random samples are again drawn as follows: $(a, \phi, \beta) \in (0,1)^3$ and $\in (0,50)$, $\theta_W \in (0,100)$ and $\theta_B \in (0, \theta_W)$ while all parameters are assumed to be distributed uniformly on these supports. We report the distribution of regimes in Table 6.

<TABLE 6 HERE>

When the incentive compatibility constraints are dropped we see a dramatic change in the results. The ideal solution comprises 80.4% of all solutions. The cases in which $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ drop to 19.6% when there is only investment in education by the state.

To understand the reason behind this change in the distribution of solutions, let us run another simulation that can be visualized. We fix $a = 0.56, \phi = 0.9, \theta_B = 58, \theta_W = 100$. These are the calibrated values of these parameters to the US data. We draw 1000 samples of $\beta \in (0,1)$ and $b \in (0,50)$.

<FIGURE 2 HERE>

In an overwhelming majority of the cases, the solution to the Nordic EOp program is ideal. More specifically, we observe that if b and/or β is sufficiently high then the solution induces $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$. Indeed, this visual regularity in our simulations is verified by a theoretical result.

Theorem 7 *If b or β is sufficiently large then the solution to the Nordic EOp ξ^* is ideal: $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$.*

Proof. We first claim that $\overline{E}_W(\xi^*) \geq \overline{E}_B(\xi^*)$ at any solution ξ^* to the Nordic EOp program. Assume the claim were false. Then children of W adults are worse off at some ξ^*. There are two cases: $s_B^* > 0$ or $s_B^* = 0$. In the former case, a small decrease in s_B^* to increase s_W^* while keeping t^* constant would increase

$$\overline{E}_W(\xi^*) = \pi_W u_W + (1 - \pi_W) u_B = \pi_W \Delta u + u_B$$

since $\Delta u = (\theta_W - \theta_B)(1 - t^*) > 0$ and π_W is monotonically increasing in s_W^* and decreasing in s_B^*. But then increasing the expected standard of living of the worse-off would be feasible. Conclude that $s_B^* > 0$ cannot happen if $\overline{E}_W(\xi^*) < \overline{E}_B(\xi^*)$. Therefore, $s_B^* = 0$ which implies

$$\xi^* = (t^*, s_B^*, s_W^*, i_B^*, i_W^*) = (t^*, 0, s_W^*, 0, 0)$$

so that $\pi_W > 1/2$. But $1/2 > a/(1+a)$ implies $\pi_W > a/(1+a)$. Therefore, conclude that $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ as we proved earlier. This contradiction demonstrates the claim.

Now assume there is a solution ξ^* such that $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$. Consider the impact of a small change in s_B^* on $\overline{E}_B$ while keeping s_W^* fixed:

$$\frac{d\overline{E}_B}{ds_B} = \frac{\partial \overline{E}_B}{\partial s_B} + \frac{\partial \overline{E}_B}{\partial t}\frac{\partial t}{\partial s_B}$$

where

$$\frac{\partial \overline{E}_B}{\partial s_B} = ab(1-\pi_W)\pi_W \frac{\Delta u}{\Delta \theta} + \beta(1-t)$$

$$\frac{\partial \overline{E}_B}{\partial t} = -a(1-\pi_W)\Delta\theta - (\theta_B + \beta z_B)$$

$$\frac{\partial t}{\partial s_B} = \frac{1-\beta t}{\mu}.$$

Of course, $\partial t / \partial s_B$ is derived invoking the budget constraint. Hence,

$$\frac{d\overline{E}_B}{ds_B} = ab(1-\pi_W)\pi_W \frac{\Delta u}{\Delta \theta} + \beta(1-t) - \left(a(1-\pi_W)\Delta\theta + (\theta_B + \beta s_B^*)\right)\frac{1-\beta t}{\mu}$$

$$> ab(1-\pi_W)\pi_W \frac{\Delta u}{\Delta \theta} + \beta(1-t) - (1-\beta t) \geq ab(1-\pi_W)\pi_W - (1-\beta).$$

Obviously, if b or β is sufficiently high then $ab(1-\pi_W)\pi_W - (1-\beta) > 0$ which implies that $d\overline{E}_B/ds_B > 0$. Therefore, ξ^* such that $\overline{E}_W(\xi^*) > \overline{E}_B(\xi^*)$ cannot be a solution to the Nordic EOp if b or β is large enough. Conclude that if b or β is large enough then $\overline{E}_W(\xi^*) = \overline{E}_B(\xi^*)$ at any solution to the Nordic EOp. ■

V. CRITIQUE AND CONCLUSION

Critique may be made of our assumption that the state differentiates its expenditures by the type of parental occupation rather than by the income of the parent. It is theoretically possible that some blue collar workers earn more than some white collar workers, if much more is invested in the former's education than in the latter's. Would it be more defensible, then, for the state to target children according to parental *income* rather than *occupation*?

Our response is that we must be interested in each child's expected standard of living, not how rich her parents were *per se*. And the expected standard of living of all children from blue collar families is the same regardless of how rich their parents are. The key assumption is that social networks, whose effects are captured in the parameter *a*, are more tied to the 'type' of occupation than to the income of the worker.

The effort expended by young workers determines whether they do well in the lottery to acquire white collar jobs; it is represented only in reduced form in our model. The fact that the probability of a Blue collar child's accessing a white collar job is, ceteris paribus, less than the probability of a White collar child's accessing a white collar job (that is, that $a < 1$) is the key circumstance that policy seeks to redress. Because this is a characteristic of a type it is appropriately called a circumstance rather than something due to differential efforts of white and blue collar children.

We do not claim to have established definitively that private financing of schools should be abolished. First, we have not considered possible virtues of privately financed schools. Second, we have worked with a model that is merely an example. Nevertheless, due to the importance of equalizing opportunities for children through educational policy, and the stark results of our model with respect to the possibility of equalizing opportunities with different financing practices, we believe the results should not be ignored.

Finally, we are fully aware that a political analysis of how policy evolves, when not in a stationary state, would be very useful[4]. Nevertheless, our analysis can be viewed as putting an upper bound on a political-economic analysis. With private investment in education, we have shown that in many cases *at best*, the stationary state may be laissez-faire: the stationary state to which a political-economic dynamic process converges (if any) can only be worse. Without privately financed education, the history of actual Nordic education and intergenerational mobility (for the latter, see Corak (2013)) is, however, auspicious.

APPENDIX

This appendix explains how we calibrate the threshold level for β given by World Indicators of Skills for Employment, a dataset publicly available on the website of the OECD, provides the frequency of high skilled employees. In our notation, this datum corresponds to

$$\frac{a}{1+a}.$$

The ratio of the average salary of highly skilled employees to the average wage is also provided by the same dataset. This datum corresponds to $\frac{a\theta_W+\theta_W}{a\theta_W+\theta_B}$. We summarize in Table 7.

<TABLE 7 HERE>

Routine calculations with these data yield the desired ratio $1/(a\theta_w/\theta_B+1)$ as given in Table 5. In a sithese numbers imply

$$a = 0.56 \text{ and } \frac{\theta_B}{\theta_W} = 0.58$$

for the US. Normalization by setting $\theta_W = 100$ gives $\theta_B = 58$. These give us the numbers used in the simulation presented on page 14.

ACKNOWLEDGMENTS

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NOTES

1. There is no political-economy element in this paper, which could determine how state policy is chosen at each date. It is purely a welfare analysis.
2. Without this kind of structure, the number of income levels would become infinite over time, and the analysis of stationary states would become much more difficult.
3. One might conjecture this view leaves the justly approved relationship between parent and child rather sterile. For a discussion of what one of us believes parents *can* pass on to their children, see Roemer (2012).
4. See Roemer (2006) for one such attempt.

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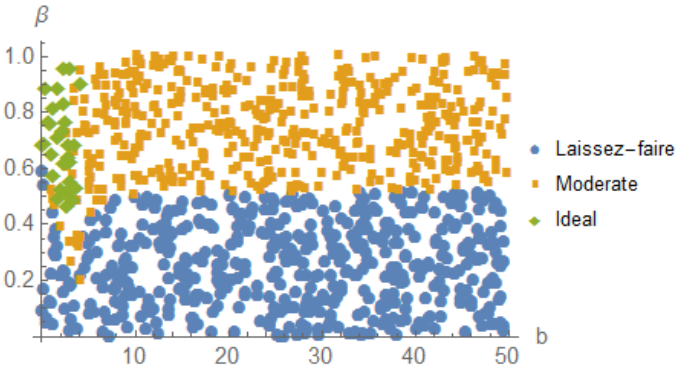


FIGURE 1a: Distribution of all solutions when $\phi = 0.4$

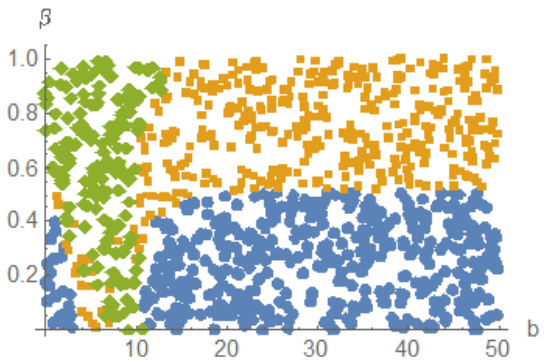


FIGURE 1b: Distribution of all solutions when $\phi = 0.9$

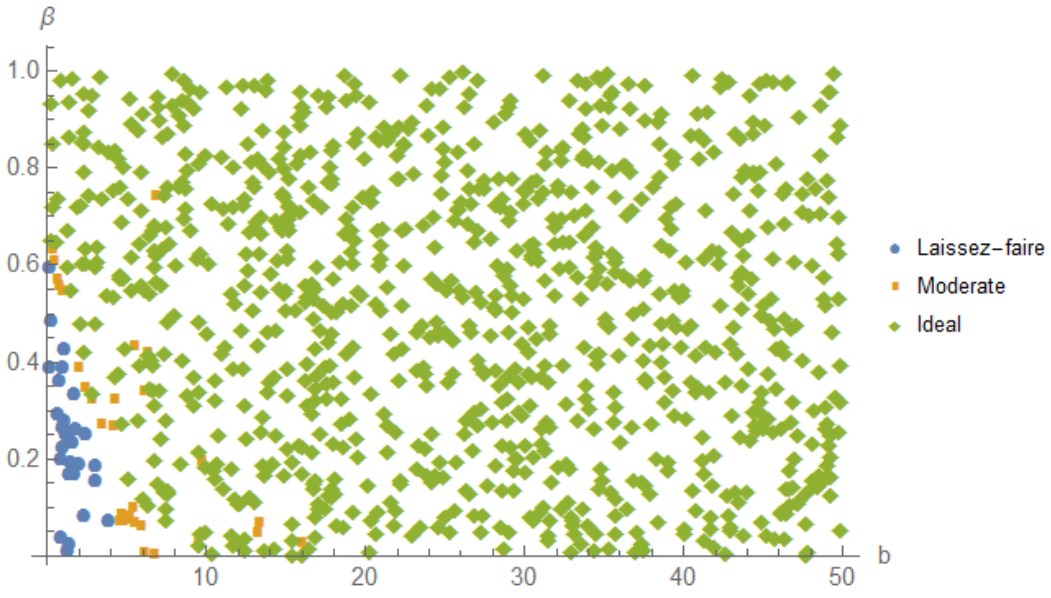


FIGURE2. Distribution of all solutions when some variables are fixed.

TABLE 1

ALL POSSIBLE SOLUTIONS TO PROGRAM (5)

	Laissez-faire	Moderate	Ideal
t	= 0	> 0	> 0
$\left\|\overline{E}_W(\xi^*) - \overline{E}_B(\xi^*)\right\|$	> 0	> 0	= 0

TABLE 2

PERCENT SOLUTIONS OF THE EOP PROGRAM OF THREE TYPES

Laissez-faire	Moderate	Ideal
37.7	32.8	29.5

TABLE 3

AVAREGES OF CERTAIN ENDOGENOUS VARIABLES

	t	π_W	$\overline{E}_W/\overline{E}_B$	$f_W z_W/\mu$	$f_B z_B/\mu$
Laissez-faire	0	0.85	2.18	0.15	0
Moderate	0.13	0.83	1.83	0.08	0.12
Ideal	0.11	0.36	1	0.04	0.17

TABLE 4

STATISTICS FOR ECONOMIES WHERE IDEAL EOP IS OPTIMAL, BUT RESTRICTED TO BEING LAISSEZ-FAIRE

	i_W	s_W	i_B	s_B	μ	E_W	E_B
Ideal EOp	0	3.85	0	5.4	30.93	25.52	25.52
Mandatory Laissez-faire	1.95	0	0	0	28.06	37.15	23.46

TABLE 5

CALIBRATION OF THE THRESHOLD OF β.

	Average	Sweden	Norway	Canada	UK	US	Germany	France	Japan
Threshold of β	0.53	0.6	0.45	0.6	0.5	0.5	0.48	0.52	0.6

TABLE 6

PERCENTAGES OF ALL SOLUTIONS WITHOUT PRIVATE INVESTMENT, NORDIC PROGRAM

Laissez-faire	Moderate	Ideal
10.2	9.4	80.4

TABLE 7

DATA OF SELECTED COUNTRIES, SOURCE: WORLD INDICATORS OF SKILLS FOR EMPLOYMENT, OECD WEB SITE.

	Sweden	Norway	Canada	UK	US	Germany	France	Japan
$\frac{a}{1+a}$	0.37	0.39	0.45	0.33	0.36	0.32	0.38	0.31
$\frac{a\theta_W + \theta_W}{a\theta_W + \theta_B}$	1.1	1.2	1.21	1.21	1.37	1.63	1.26	1.26