

YILDIZ TEKNİK ÜNİVERSİTESİ

ELEKTRİK MÜHENDİSLİĞİ BÖLÜMÜ

DEVRE TEORİSİ

Ders Notu

**AC VE DC DEVRELERDE
YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ**

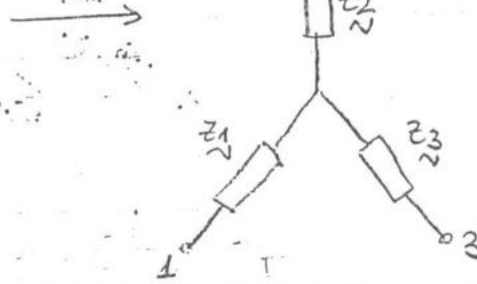
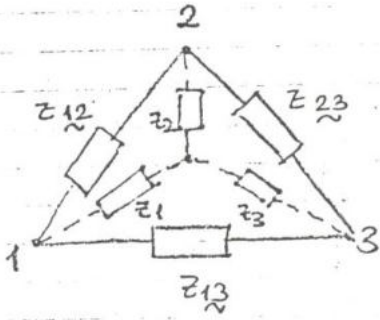
Doç. Dr. Recep YUMURTACI

YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ

Δ : Üçgen bağlantı

λ : Yıldız //

① $\Delta \rightarrow \lambda$ Dönüşümü



z_{12}, z_{23}, z_{13} biliniyor.

Bunların λ bağlı eşdeğerleri:

$z_1 = ?$ $z_2 = ?$ $z_3 = ?$

$$z_1 = \frac{z_{12} \cdot z_{13}}{z_{12} + z_{13} + z_{23}}$$

$$z_2 = \frac{z_{12} \cdot z_{23}}{z_{12} + z_{13} + z_{23}}$$

$$z_3 = \frac{z_{13} \cdot z_{23}}{z_{12} + z_{13} + z_{23}}$$

Özel durum

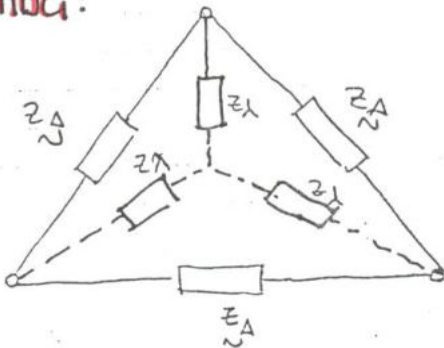
$z_{12} = z_{13} = z_{23} = z$ olsun.

$$z_1 = \frac{z \cdot z}{3z} = \frac{z}{3} \quad \boxed{z_1 = \frac{z}{3}}$$

$$\boxed{z_2 = \frac{z}{3}}$$

$$\boxed{z_3 = \frac{z}{3}}$$

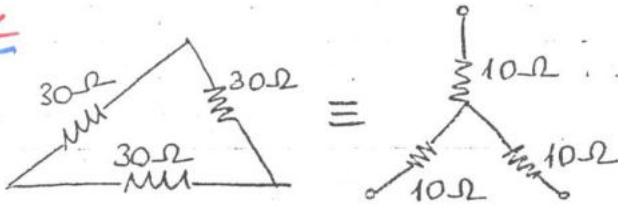
Sonuç:



$$\boxed{z_{\lambda} = \frac{z_{\Delta}}{3}}$$

Not: Bu formüller direnç ve reaktanslar içinde geçerlidir.

Örnek



$R_{\Delta} = 30 \Omega$

$R_{\lambda} = \frac{R_{\Delta}}{3} = 10 \Omega$

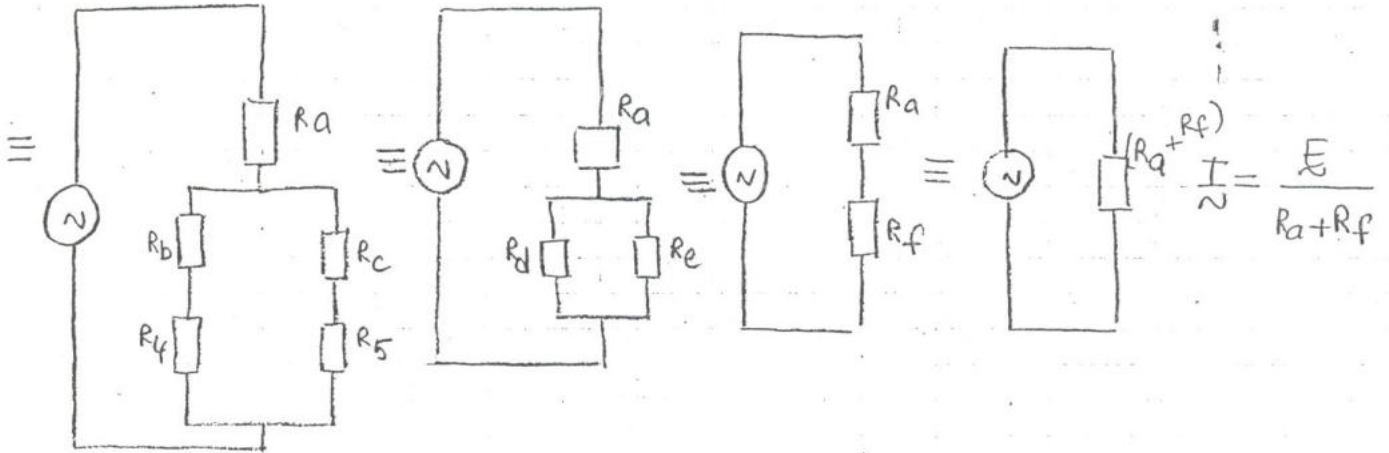
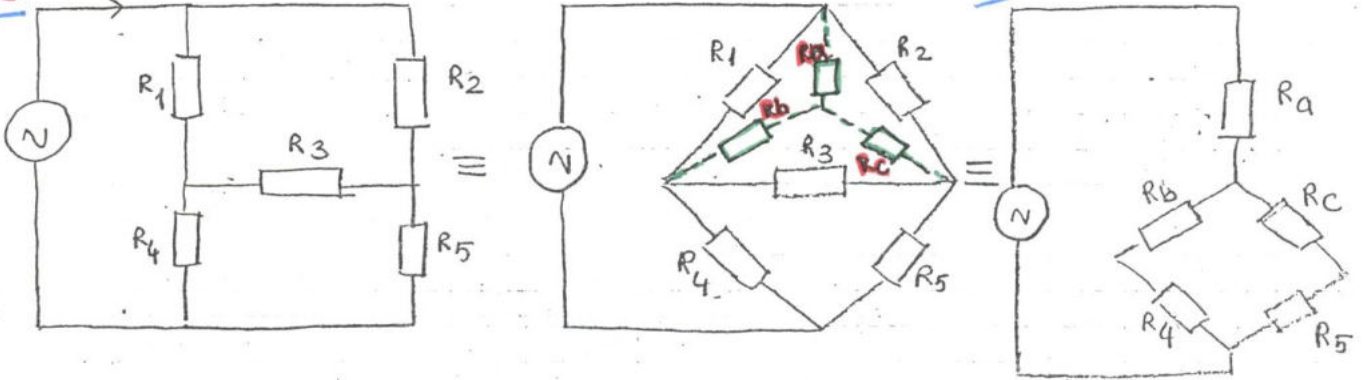
$z_{\Delta} = 9 \angle 60^{\circ}$ olsun.

$z_{\lambda} = \frac{z_{\Delta}}{3} = \frac{9}{3} \angle 60^{\circ} = 3 \angle 60^{\circ} \Omega$

Δ bağlı R_1, R_2, R_3 'de λ 'a dönüştürmek

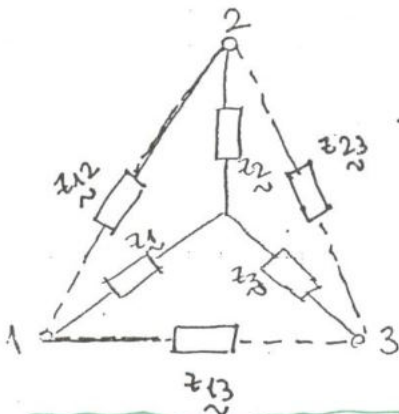
Örnek

$z = ?$

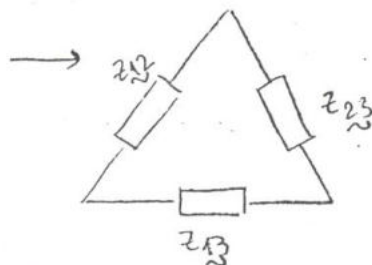


$\frac{I}{2} = \frac{\epsilon}{R_a + R_f}$

2) $\lambda \rightarrow \Delta$ Dönüşümü



z_1, z_2, z_3 biliniyor $z_{12} = ?$, $z_{23} = ?$, $z_{13} = ?$

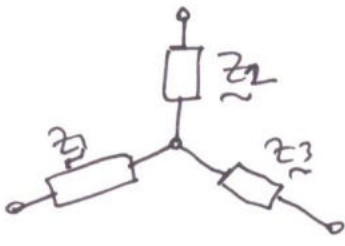


$z_{12} = \frac{z_1 \cdot z_2 + z_1 \cdot z_3 + z_2 \cdot z_3}{z_3}$

$z_{13} = \frac{z_1 \cdot z_3 + z_1 \cdot z_2 + z_2 \cdot z_3}{z_2}$

$z_{23} = \frac{z_1 \cdot z_2 + z_1 \cdot z_3 + z_2 \cdot z_3}{z_1}$

$\lambda \rightarrow \Delta$ Dönüşümünde Özel Durum :



$\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3 = \underline{Z}_\lambda$ olsun (Empedanslar dengeli)

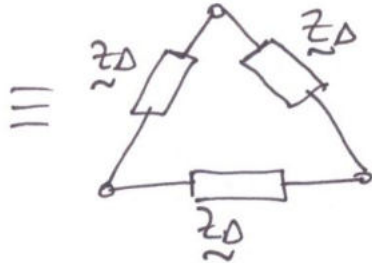
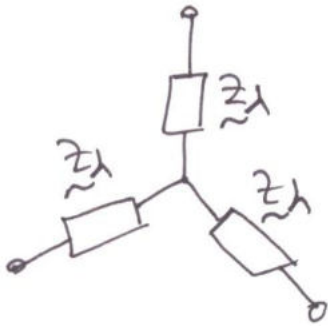
$$\underline{Z}_{12} = \frac{3 \cdot \underline{Z}_\lambda \cdot \underline{Z}_\lambda}{\underline{Z}_\lambda} = 3 \cdot \underline{Z}_\lambda$$

$$\underline{Z}_{23} = \frac{3 \cdot \underline{Z}_\lambda \cdot \underline{Z}_\lambda}{\underline{Z}_\lambda} = 3 \cdot \underline{Z}_\lambda$$

$$\underline{Z}_{13} = \frac{3 \cdot \underline{Z}_\lambda \cdot \underline{Z}_\lambda}{\underline{Z}_\lambda} = 3 \cdot \underline{Z}_\lambda$$

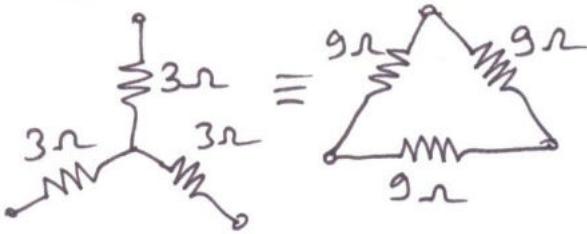
$$\left. \begin{aligned} \underline{Z}_{12} &= 3 \cdot \underline{Z}_\lambda \\ \underline{Z}_{23} &= 3 \cdot \underline{Z}_\lambda \\ \underline{Z}_{13} &= 3 \cdot \underline{Z}_\lambda \end{aligned} \right\}$$

Sonuç : $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3 = \underline{Z}_\lambda$ ise $\underline{Z}_{12} = \underline{Z}_{23} = \underline{Z}_{13} = \underline{Z}_\Delta = 3 \cdot \underline{Z}_\lambda$ olur

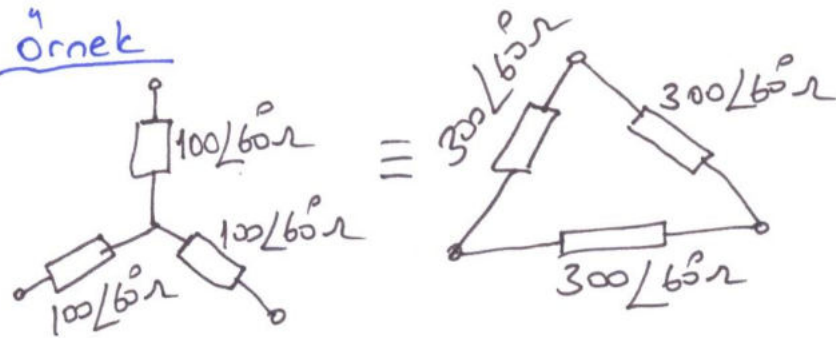


$$\underline{Z}_\Delta = 3 \cdot \underline{Z}_\lambda$$

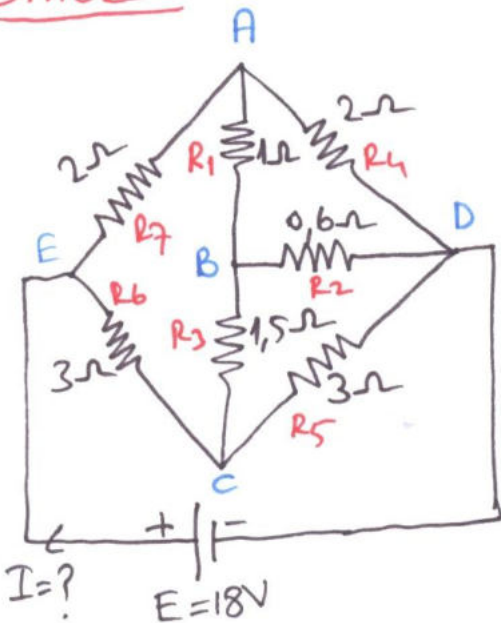
Örnek



Örnek



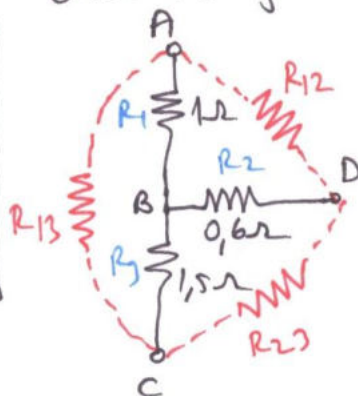
Örnek



Şekildeki devrede kaynaktan sekilen akımı hesaplayınız

Gözüm

Önce λ bağlı R_1, R_2, R_3 'ü Δ 'e dönüştürelim



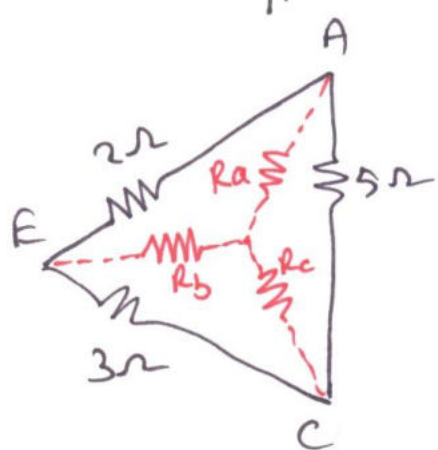
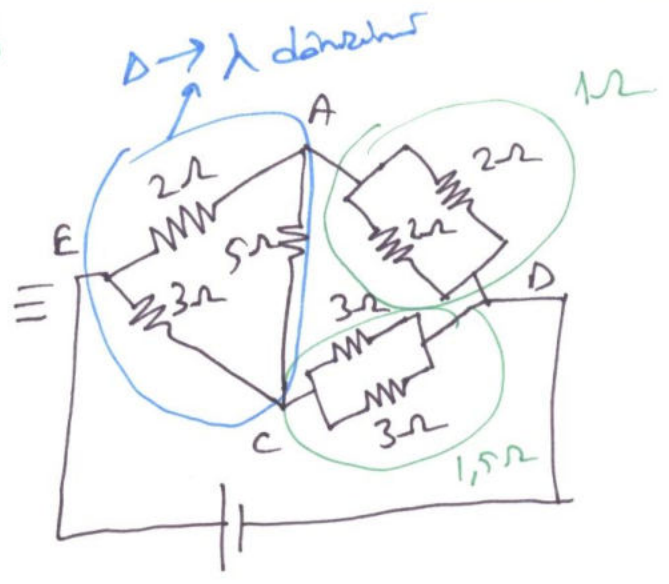
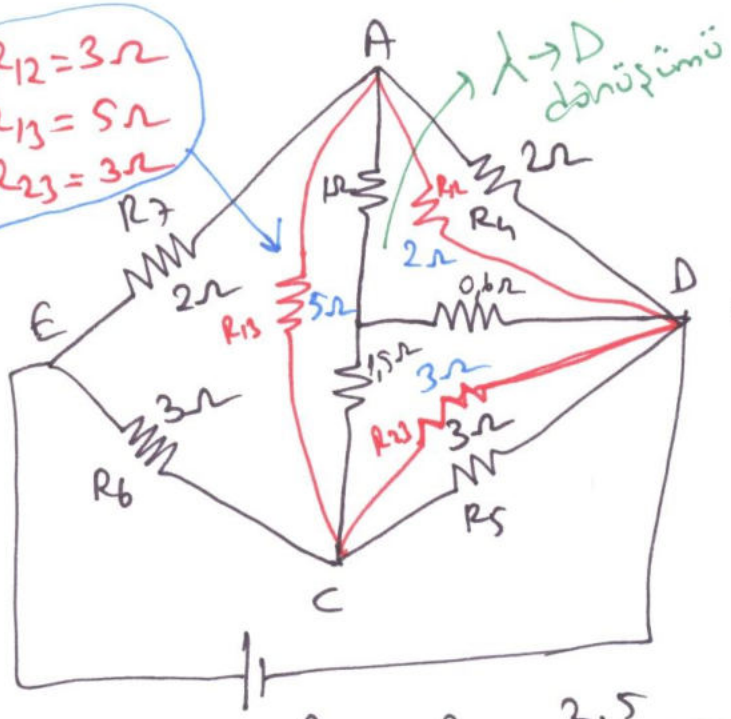
$$(R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3) = 1 \cdot 0,6 + 1,5 + 0,6 \cdot 1,5 = 3 \Omega^2$$

$$R_{12} = \frac{3}{R_3} = \frac{3}{1,5} = 2 \Omega$$

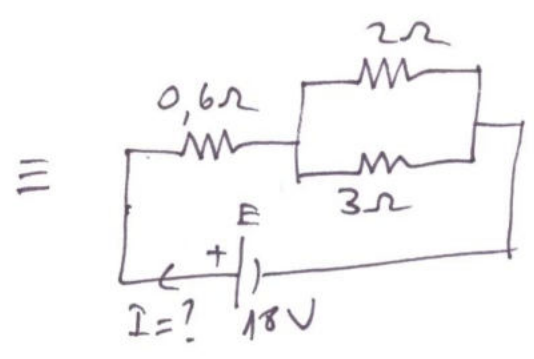
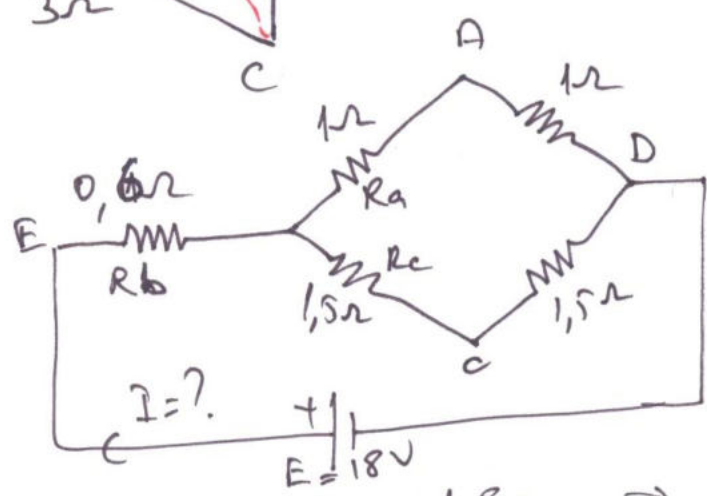
$$R_{13} = \frac{3}{R_2} = \frac{3}{0,6} = 5 \Omega$$

$$R_{23} = \frac{3}{R_1} = \frac{3}{1} = 3 \Omega$$

$R_{12} = 3\Omega$
 $R_{13} = 5\Omega$
 $R_{23} = 3\Omega$



$R_a = \frac{2 \cdot 5}{2 + 3 + 5} = 1\Omega$
 $R_b = \frac{2 \cdot 3}{2 + 3 + 5} = 0,6\Omega$
 $R_c = \frac{3 \cdot 5}{2 + 3 + 5} = 1,5\Omega$



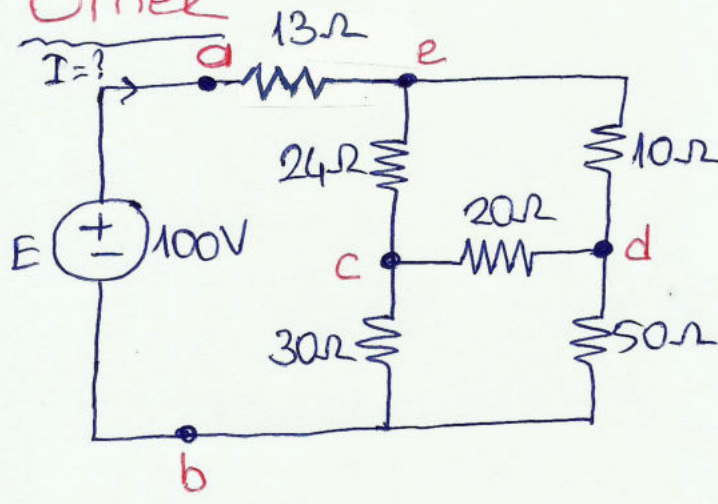
$R_T = 0,6 + \frac{2 \cdot 3}{2 + 3} = 1,8\Omega \Rightarrow$

$I = \frac{E}{R_T} = \frac{18}{1,8} = 10A$

$I = 10A$

DC DEVRELERDE YILDIZ-LİĞEN (Δ - Δ) DÖNÜŞÜMLERİ

Örnek

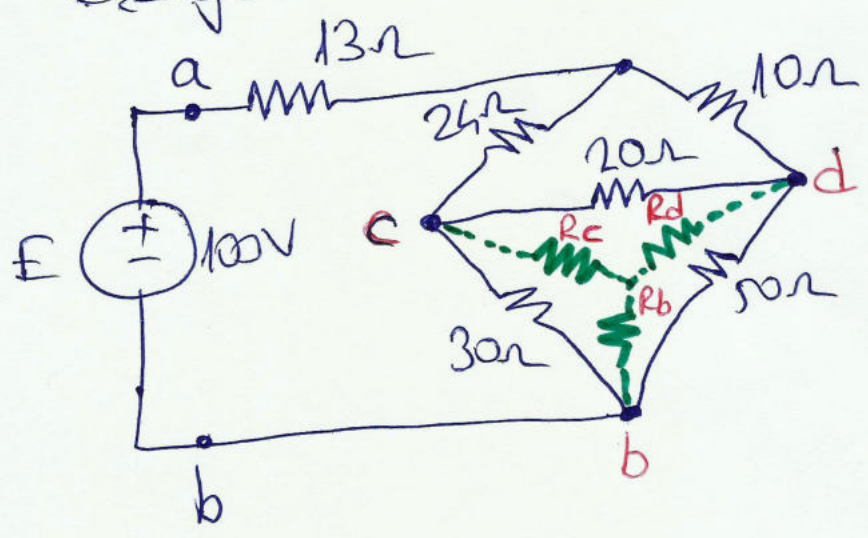


- a) $R_{ab} = ?$ ($\Delta \rightarrow \lambda$ dönüşümüyle bulunur)
- b) $I = ?$
- c) 13Ω 'luk direncin gücü ve kWh olarak bir ayda harcandığı elektrik enerjisi hesaplanır
- d) R_{ab} direncini λ bacağındaki $20\Omega, 10\Omega$ ve 50Ω dirençlerini Δ 'e çevirerek $\Delta \rightarrow \lambda$ dönüşümüyle bulunur

Gözüm

- a) a-b uçları arasındaki eşdeğer direnci bulmak için aşağıdaki λ - Δ dönüşümleri seçeneklerinden birisini kullanabiliriz
 - Δ bağlı olan $20\Omega, 30\Omega$ ve 50Ω 'u λ 'a çevirmek
 - " " " " $24\Omega, 20\Omega$ ve 10Ω 'u " "
 - λ bacağı olan $20\Omega, 10\Omega$ ve 50Ω 'u Δ 'e çevirmek
 - λ " " " $24\Omega, 20\Omega$ ve 30Ω 'u " "

Biz Δ bağlı olan alttaki $20\Omega, 30\Omega$ ve 50Ω 'luk dirençlere $\Delta \rightarrow \lambda$ dönüşümü uygulayıp bunları λ eşdeğerlerini bulalım:

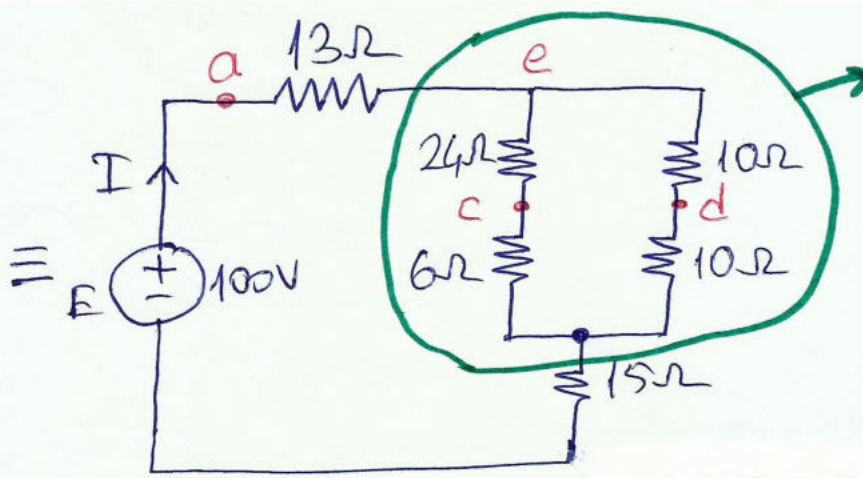


$$\Sigma R_{\Delta} = 20 + 30 + 50 = 100\Omega$$

$$R_c = \frac{30 \cdot 20}{100} = 6\Omega$$

$$R_d = \frac{20 \cdot 50}{100} = 10\Omega$$

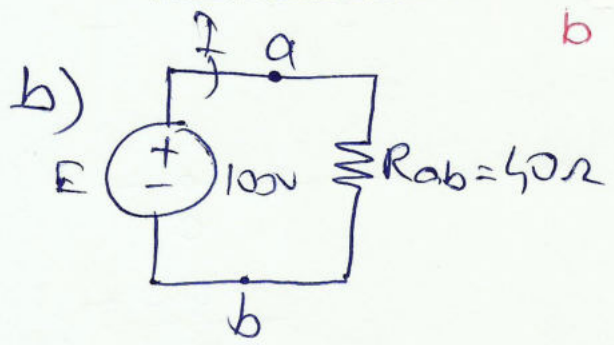
$$R_b = \frac{30 \cdot 50}{100} = 15\Omega$$



$$R_{e1} = \frac{30 \cdot 20}{30 + 20} = 12 \Omega \quad (2)$$

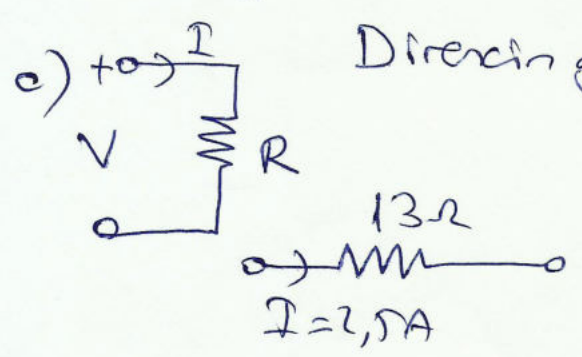
$$R_{ab} = 13 + 12 + 15$$

$$R_{ab} = 40 \Omega$$



$$I = \frac{E}{R_{ab}} = \frac{100}{40}$$

$$I = 2,5 A$$



Direksi ng ω : $P = V \cdot I = (R \cdot I) I \Rightarrow P = RI^2$

$$P = V \cdot I = V \cdot \left(\frac{V}{R}\right) \Rightarrow P = \frac{V^2}{R}$$

$$P = RI^2 = 13 \cdot 2,5^2$$

$$P = 81,25 W$$

Energi (kWh) oloak

$$E = P \cdot t$$

\downarrow kWh \downarrow kW \rightarrow h

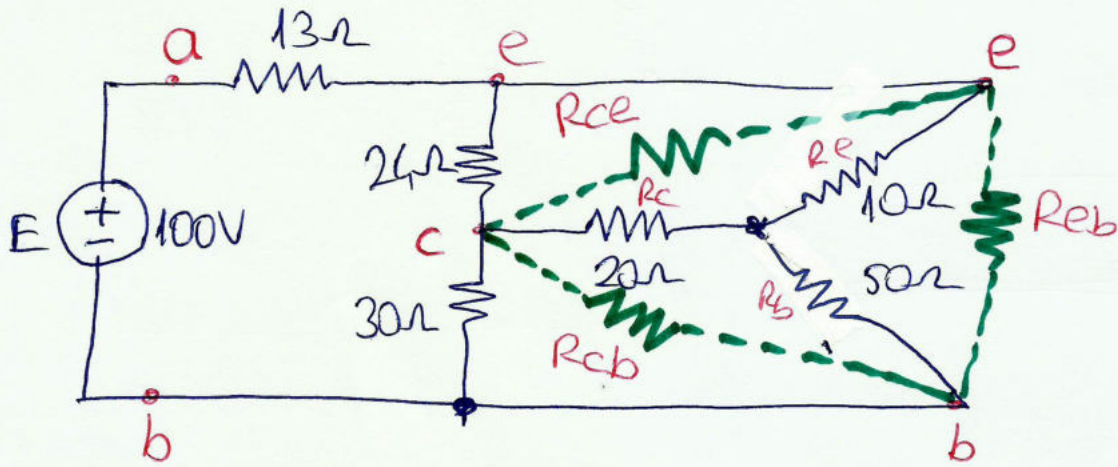
$$P = 81,25 W = 81,25 \cdot 10^{-3} kW$$

$$t = 1 \text{ day} = 30 \cdot 24 = 720 \text{ h (saat)}$$

$$E = P \cdot t = 81,25 \cdot 10^{-3} \cdot 720$$

$$E = 58,5 kWh$$

d) R_{ab} 'ın $\Delta \rightarrow \Delta$ dönüşümüyle bulunması



İkizetli üç dirençli bir grupların toplama:

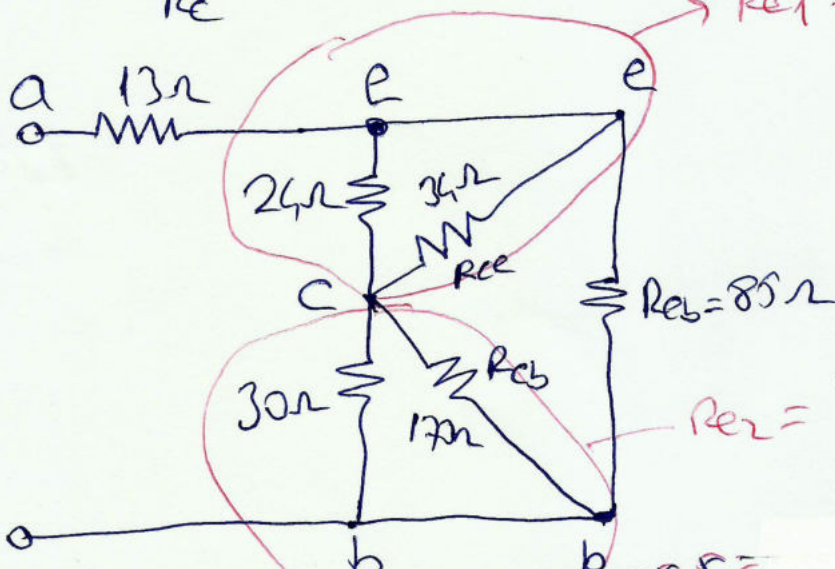
$$R_c \cdot R_b + R_c \cdot R_e + R_b \cdot R_e = 20 \cdot 50 + 20 \cdot 10 + 50 \cdot 10 = 1700$$

$$R_{ce} = \frac{R_c \cdot R_b + R_c \cdot R_e + R_b \cdot R_e}{R_b} = \frac{1700}{50} = 34 \Omega$$

$$R_{cb} = \frac{1700}{R_e} = \frac{1700}{10} = 170 \Omega$$

$$R_{eb} = \frac{1700}{R_c} = \frac{1700}{20} = 85 \Omega$$

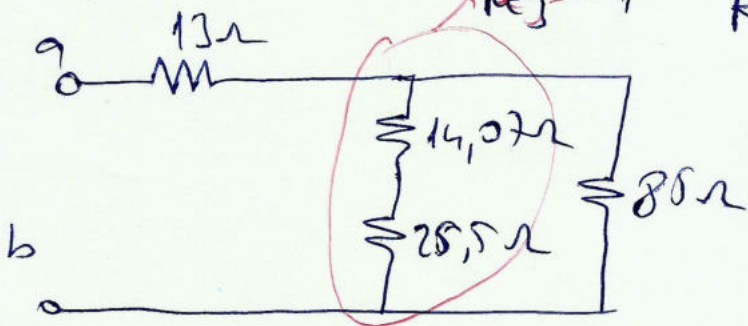
$$R_{e1} = \frac{24 \cdot 34}{24 + 34} = 14,07 \Omega$$



$$R_{e2} = \frac{30 \cdot 170}{30 + 170} = 25,5 \Omega$$

$$R_{e3} = 14,07 + 25,5 = 39,57 \Omega$$

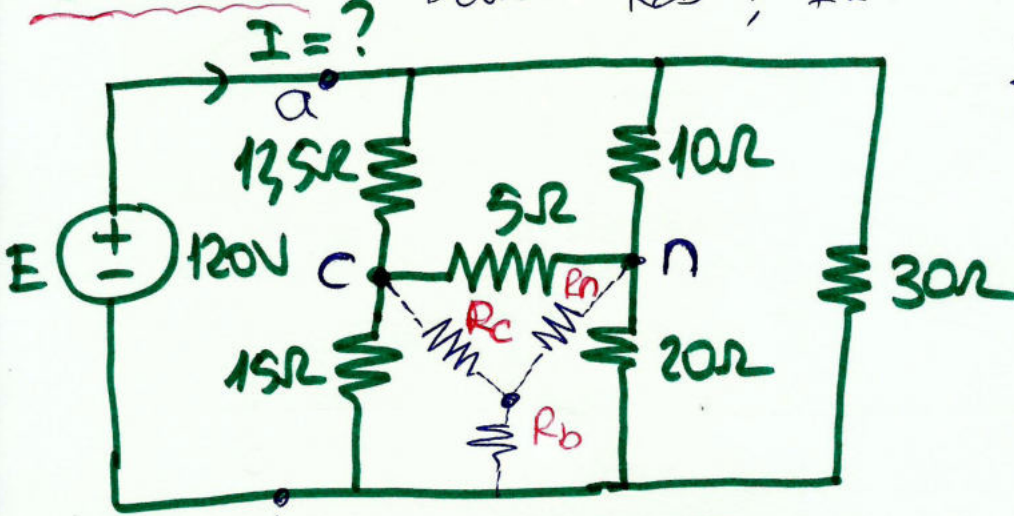
$$R_{ab} = 13 + \left(\frac{39,57 \cdot 85}{39,57 + 85} \right)$$



$$R_{ab} = 40 \Omega$$

Örnek

Devrede $R_{ab}=?$ $I=?$

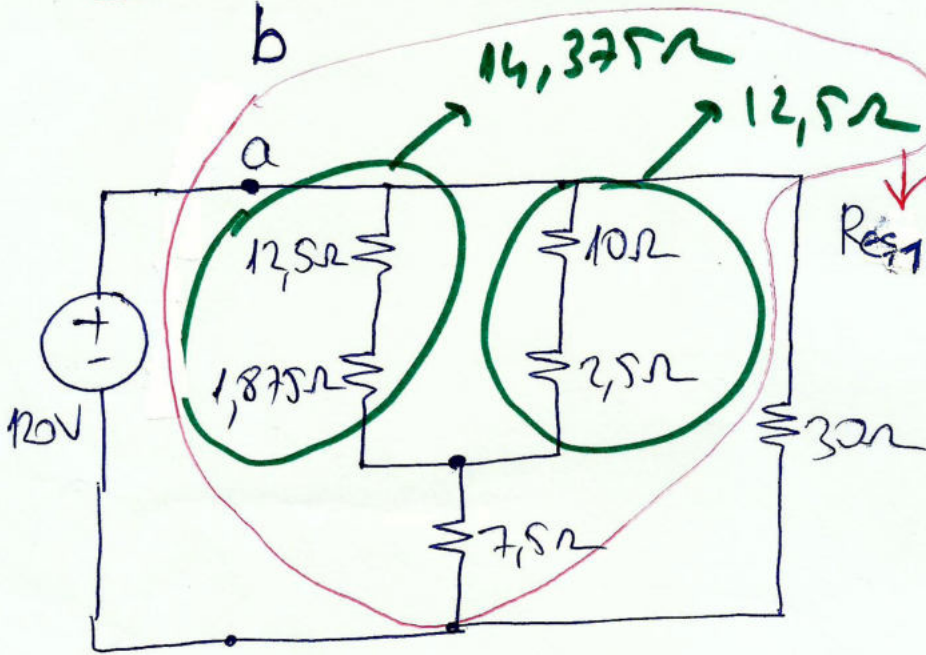


$$\Sigma R_D = 5 + 15 + 20 = 40\Omega$$

$$R_c = \frac{15 \cdot 5}{\Sigma R_D} = \frac{15 \cdot 5}{40} = 1,875\Omega$$

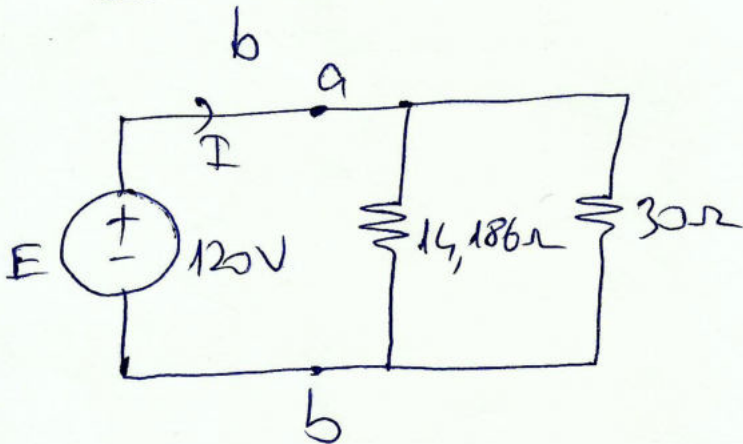
$$R_n = \frac{5 \cdot 20}{40} = 2,5\Omega$$

$$R_b = \frac{15 \cdot 20}{40} = 7,5\Omega$$



$$R_{eq1} = \frac{14,375 \cdot 12,5}{(14,375 + 12,5)} + 7,5$$

$$R_{eq1} = 14,186\Omega$$



$$R_{ab} = \frac{14,186 \cdot 30}{(14,186 + 30)}$$

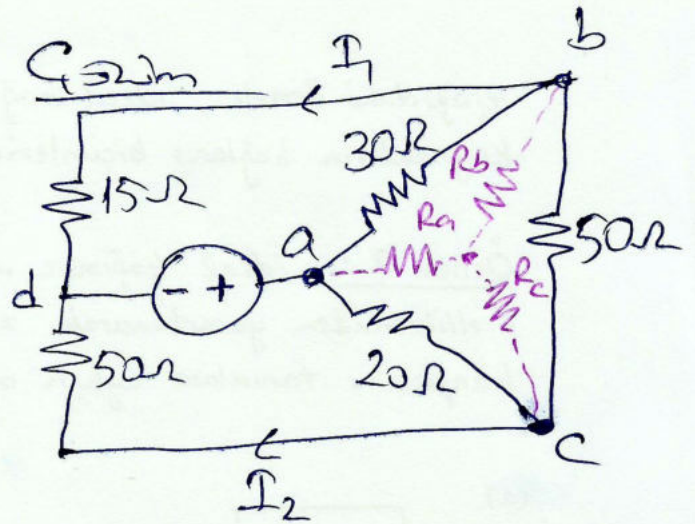
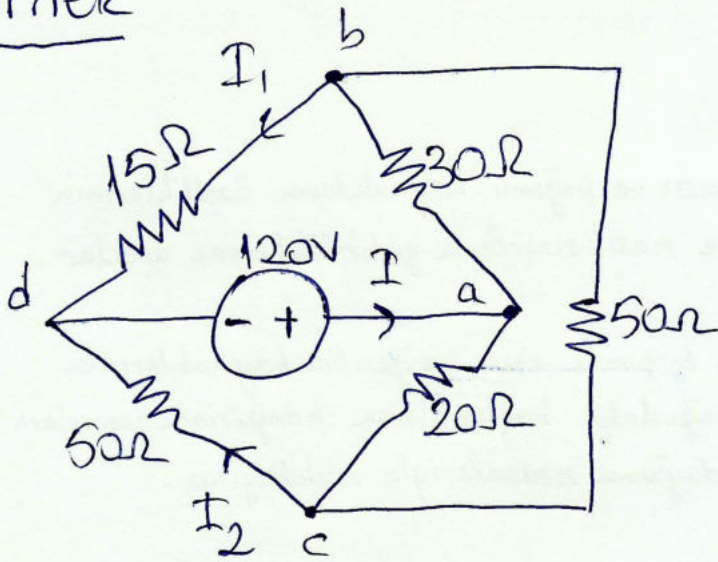
$$R_{ab} = 9,632\Omega$$

$$I = \frac{E}{R_{ab}} = \frac{120}{9,632} \Rightarrow I = 12,458A$$

Örnek

5

$I_1 = ? , I_2 = ? , I_3 = ?$



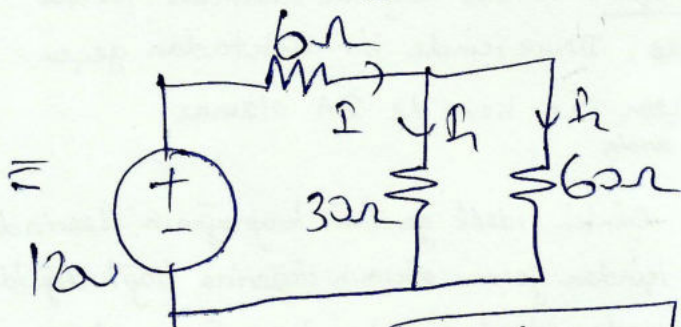
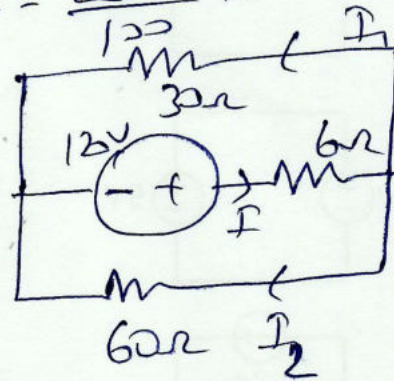
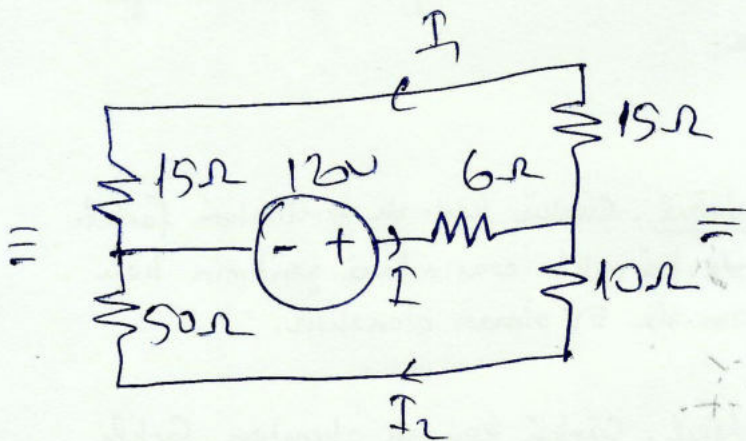
Δ-Y Dönüşümü

$$\sum R_{\Delta} = 30 + 20 + 50 = 100 \Omega$$

$$R_a = \frac{30 \cdot 20}{100} = 6 \Omega$$

$$R_b = \frac{30 \cdot 50}{100} = 15 \Omega$$

$$R_c = \frac{20 \cdot 50}{100} = 10 \Omega$$



$$R_T = 6 + \frac{30 \cdot 60}{30 + 60} = 26 \Omega$$

$$R_T = 26 \Omega$$

$$I = \frac{120}{26} \Rightarrow I = \frac{60}{13} \text{ A}$$

Altın bölücüden: $I_1 = \frac{60}{30+60} \cdot I = \frac{60}{90} \cdot \frac{60}{13} \Rightarrow I_1 = \frac{40}{13} \text{ A}$

$$I_2 = \frac{30}{90} \cdot \frac{60}{13} \Rightarrow I_2 = \frac{20}{13} \text{ A}$$