

# Chapter 3

## Random Variables and Probability Distributions

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# Section 3.1

## Concept of a Random Variable

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# Definition 3.1



A **random variable** is a function that associates a real number with each element in the sample space.

# Definition 3.2



If a sample space contains a finite number of possibilities or an unending sequence with as many elements as there are whole numbers, it is called a **discrete sample space**.

# Definition 3.3



If a sample space contains an infinite number of possibilities equal to the number of points on a line segment, it is called a **continuous sample space**.

# Section 3.2

## Discrete Probability Distribution

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# Definition 3.4



The set of ordered pairs  $(x, f(x))$  is a **probability function**, **probability mass function**, or **probability distribution** of the discrete random variable  $X$  if, for each possible outcome  $x$ ,

1.  $f(x) \geq 0$ ,

2.  $\sum_x f(x) = 1$ ,

3.  $P(X = x) = f(x)$ .

# Definition 3.5

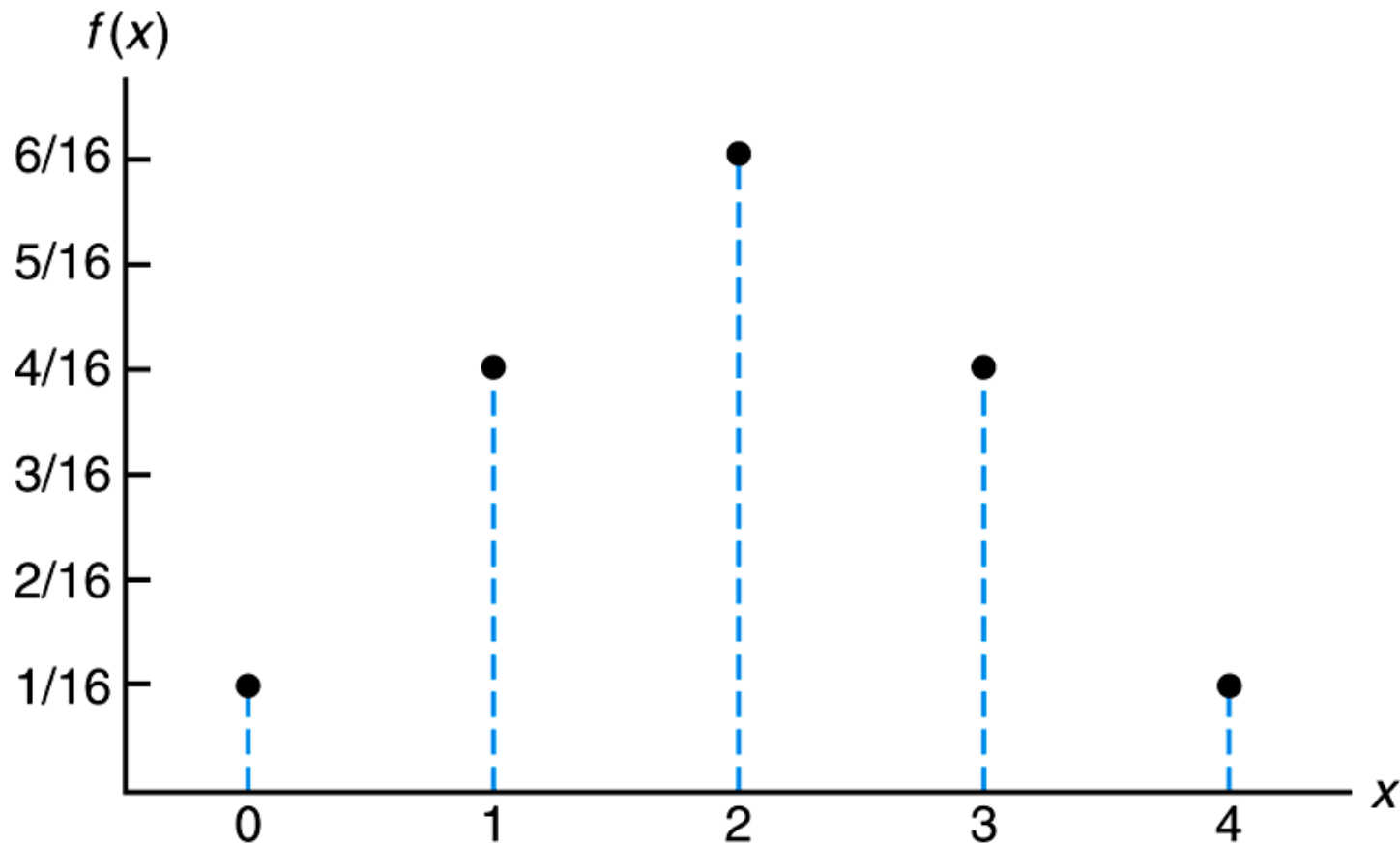


The **cumulative distribution function**  $F(x)$  of a discrete random variable  $X$  with probability distribution  $f(x)$  is

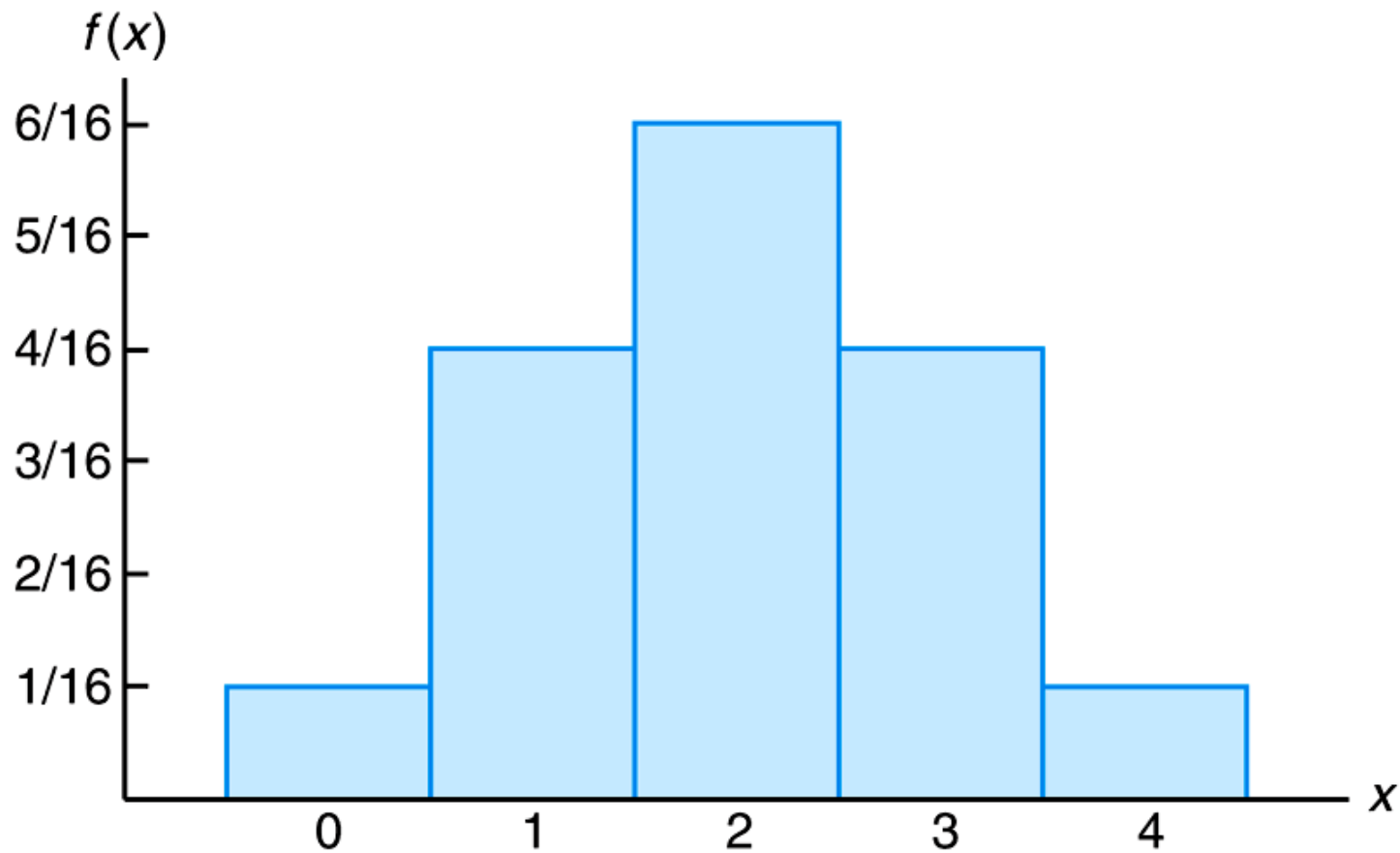
$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -\infty < x < \infty.$$



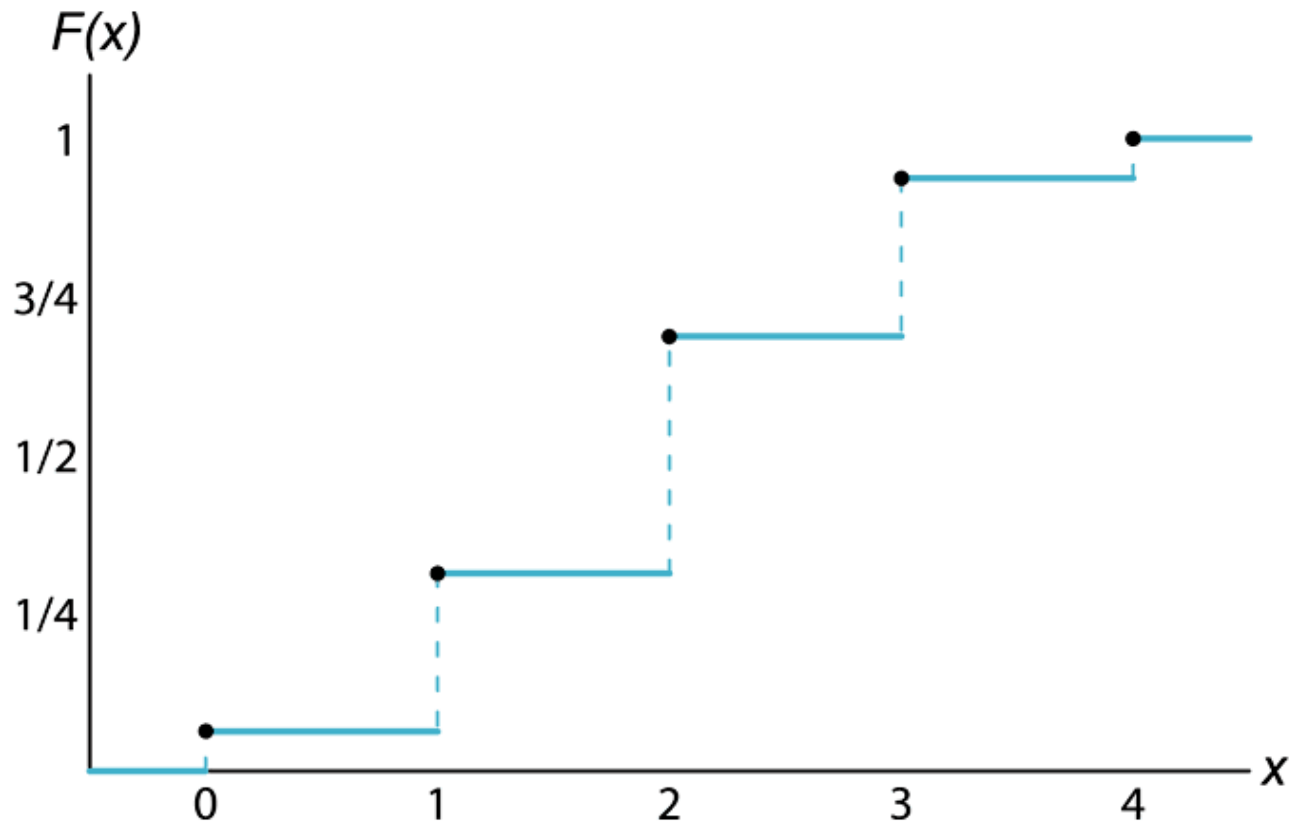
# Figure 3.1 Probability mass function plot



# Figure 3.2 Probability histogram



# Figure 3.3 Discrete cumulative distribution function



# Section 3.3

## Continuous Probability Distributions

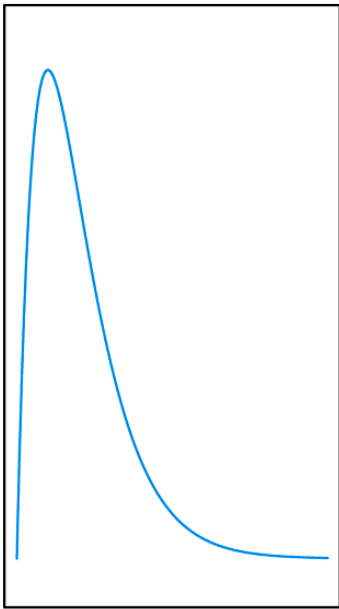
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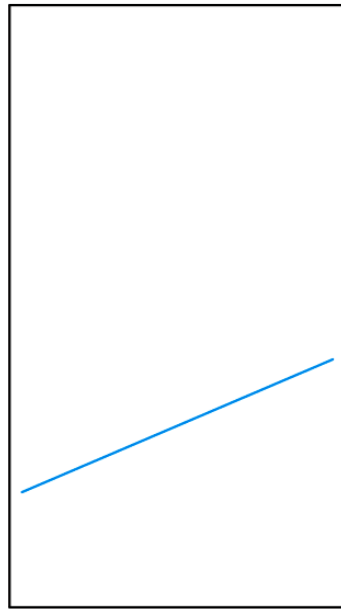


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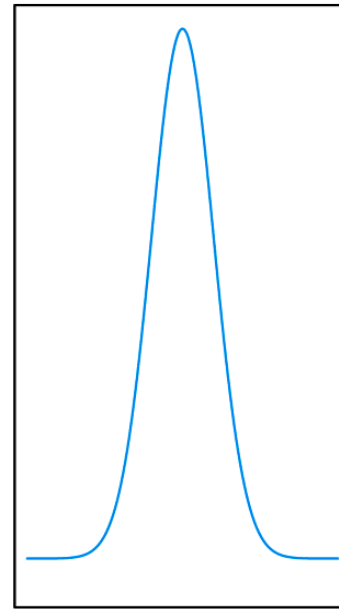
# Figure 3.4 Typical density functions



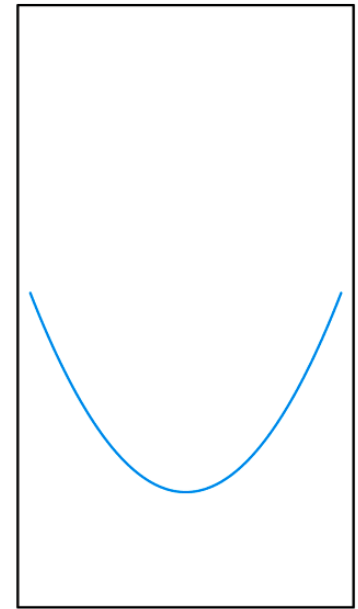
(a)



(b)

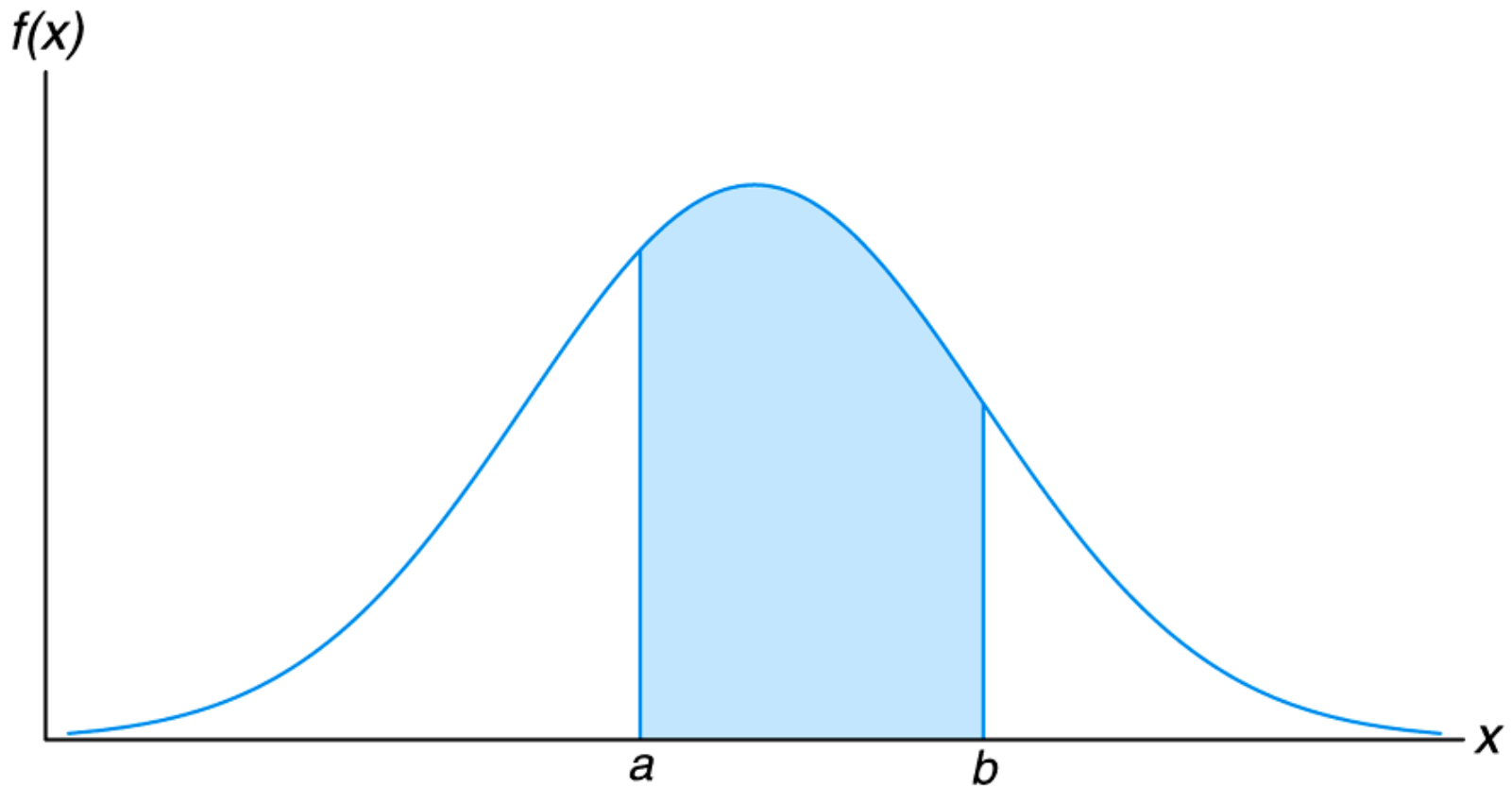


(c)



(d)

# Figure 3.5 $P(a < X < b)$



# Definition 3.6



The function  $f(x)$  is a **probability density function** (pdf) for the continuous random variable  $X$ , defined over the set of real numbers, if

1.  $f(x) \geq 0$ , for all  $x \in R$ .
2.  $\int_{-\infty}^{\infty} f(x) dx = 1$ .
3.  $P(a < X < b) = \int_a^b f(x) dx$ .

# Definition 3.7

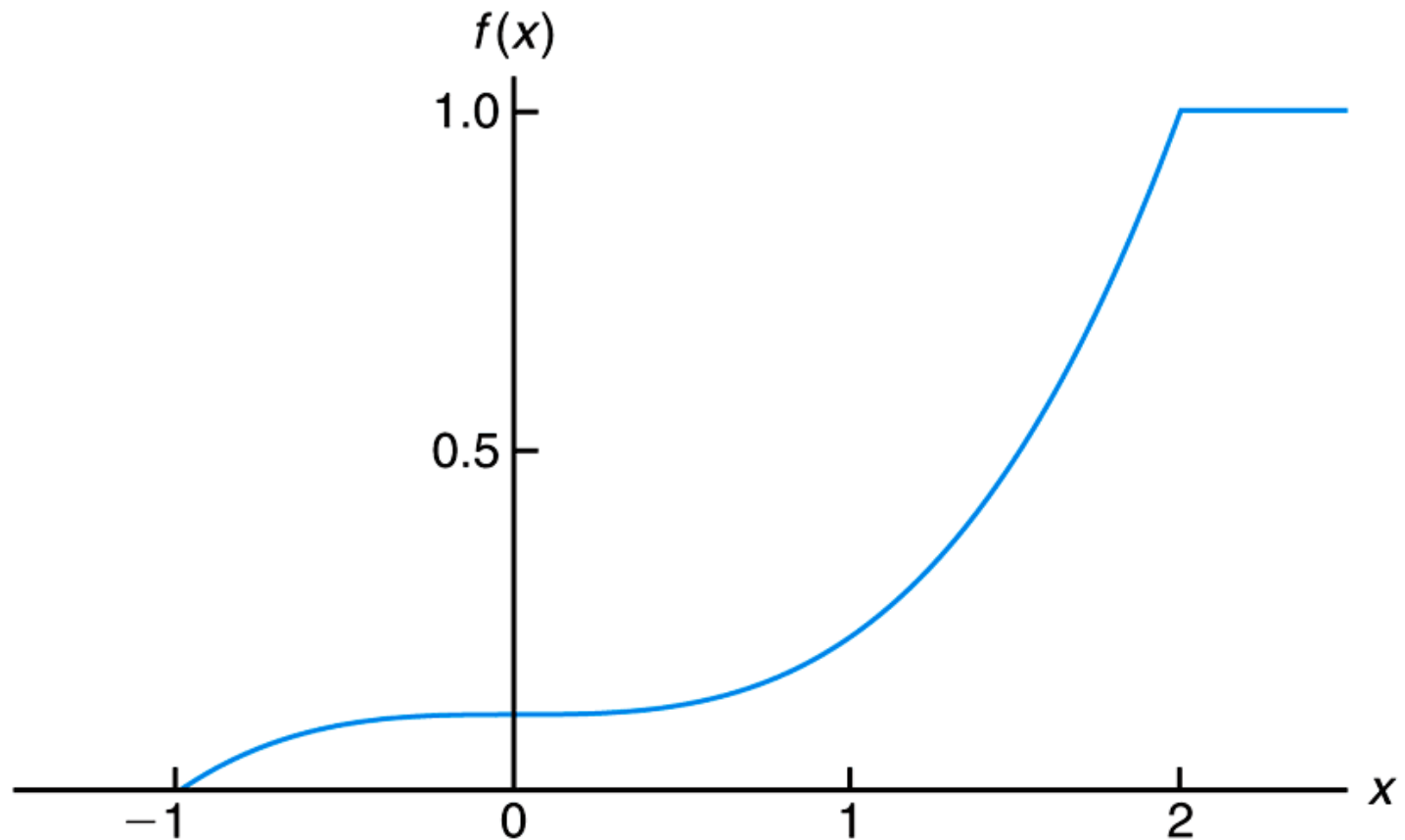


The **cumulative distribution function**  $F(x)$  of a continuous random variable  $X$  with density function  $f(x)$  is

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt, \quad \text{for } -\infty < x < \infty.$$



# Figure 3.6 Continuous cumulative distribution function



# Section 3.4

## Joint Probability Distributions

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# Definition 3.8



The function  $f(x, y)$  is a **joint probability distribution** or **probability mass function** of the discrete random variables  $X$  and  $Y$  if

1.  $f(x, y) \geq 0$  for all  $(x, y)$ ,
2.  $\sum_x \sum_y f(x, y) = 1$ ,
3.  $P(X = x, Y = y) = f(x, y)$ .

For any region  $A$  in the  $xy$  plane,  $P[(X, Y) \in A] = \sum_A f(x, y)$ .

# Table 3.1 Joint Probability Distribution for Example 3.14



$f(x, y)$		$x$			Row Totals
		0	1	2	
$y$	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\frac{3}{14}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
Column Totals		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

# Definition 3.9



The function  $f(x, y)$  is a **joint density function** of the continuous random variables  $X$  and  $Y$  if

1.  $f(x, y) \geq 0$ , for all  $(x, y)$ ,
2.  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$ ,
3.  $P[(X, Y) \in A] = \int \int_A f(x, y) dx dy$ , for any region  $A$  in the  $xy$  plane.

# Definition 3.10



The **marginal distributions** of  $X$  alone and of  $Y$  alone are

$$g(x) = \sum_y f(x, y) \quad \text{and} \quad h(y) = \sum_x f(x, y)$$

for the discrete case, and

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad \text{and} \quad h(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

for the continuous case.

# Definition 3.11



Let  $X$  and  $Y$  be two random variables, discrete or continuous. The **conditional distribution** of the random variable  $Y$  given that  $X = x$  is

$$f(y|x) = \frac{f(x, y)}{g(x)}, \text{ provided } g(x) > 0.$$

Similarly, the conditional distribution of  $X$  given that  $Y = y$  is

$$f(x|y) = \frac{f(x, y)}{h(y)}, \text{ provided } h(y) > 0.$$

# Definition 3.12



Let  $X$  and  $Y$  be two random variables, discrete or continuous, with joint probability distribution  $f(x, y)$  and marginal distributions  $g(x)$  and  $h(y)$ , respectively. The random variables  $X$  and  $Y$  are said to be **statistically independent** if and only if

$$f(x, y) = g(x)h(y)$$

for all  $(x, y)$  within their range.



# Definition 3.13



Let  $X_1, X_2, \dots, X_n$  be  $n$  random variables, discrete or continuous, with joint probability distribution  $f(x_1, x_2, \dots, x_n)$  and marginal distribution  $f_1(x_1), f_2(x_2), \dots, f_n(x_n)$ , respectively. The random variables  $X_1, X_2, \dots, X_n$  are said to be mutually **statistically independent** if and only if

$$f(x_1, x_2, \dots, x_n) = f_1(x_1)f_2(x_2) \cdots f_n(x_n)$$

for all  $(x_1, x_2, \dots, x_n)$  within their range.

# Section 3.5

Potential  
Misconceptions  
and Hazards;  
Relationship to  
Material in Other  
Chapters

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