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A simple formation-tracking controller of mobile robots based on a "spanning-tree" communication

Janset Dasdemir and Antonio Loria

Abstract— **We solve the formation-tracking control problem for mobile robots via linear control. As in the classical tracking control problem for two nonholonomic systems, the swarm is driven by a fictitious robot which moves about freely. Only one "leader" robot communicates with the reference vehicle and in turn, acts as a leader to a second robot hence forming a fixed spanning tree. We show that a simple condition on the reference angular velocity (persistency of excitation) suffices to achieve consensus tracking.**

I. Introduction

In the context of consensus and synchronization, coordinated control of autonomous mobile robots has received much attention in thelast decade. In [1], [2], desired behaviors such as obstacle-avoidance or target-seeking are assigned to each vehicle and formation control action is determined by a weighted average of them. However, these works rely on an all-to-all communication among agents. In [3], [4] the entire formation is treated as a single body which can evolve in a given direction and orientation to build a predefined formation shape however, failure in the virtual robot affects the whole swarm of agents. In [5], [6] the authors use graph-theory to describe communication links and stability of the system is ensured by stability of each individual system and the connectivity of the graph. It is important to mention that the papers mentioned above are restricted to linear systems.

There also exist various articles on leader-follower based formation tracking control of mobile robots. In [7], an adaptive leader-follower based formation control without the need of leaders' velocity information is prescribed. It is assumed that two robots are leaders hence, they know the prescribed reference velocity while the others considered as followers with single integrator dynamics. Stability analysis shows that the triangular formation is asymptotically stable while the collinear one is not. In [8], the authors present a three-level hybrid control architecture based on feedback linearization and the analysis relies on graph theory; it shows that position error system is asymptotically stable with a bounded orientation error. In [9], a virtual vehicle is designed to eliminate velocity measurement of the leader then using backstepping and Lyapunov's direct method position tracking control problem of the follower is solved. The proposed method guarantees asymptotic stability of the closed loop error system dynamics.

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In [10] feedback linearization with sliding mode control are employed for two robots under leader-follower based formation. They exhibit robustness to bounded disturbances and unmodeled dynamics. In [11], leader's influence on the trajectory tracking error dynamics is taken as an unknown but bounded, observable disturbance and is eliminated by the local controllers of the followers. Using adaptive dynamic programming with NN, it is shown that the kinematic tracking error, the velocity tracking error and the parameter estimation errors are all uniformly ultimately bounded. In [12], three different formation control methods are proposed. Two of them are developed by using virtual robot path tracking techniques. One is based on approximate linearization of the unicycle dynamics and other is formed using Lyaponov-based nonlinear time varying design. The third controller is developed through dynamic feedback linearization.

In this paper, we follow a leader-follower approach.We assume that the swarm of n vehicles has only one leader which communicates with the virtual reference vehicle that is, only one robot "knows" the reference trajectory. The formation is ensured via unilateral communication that is, each robot except for the leader and the tail, communicates only with two neighbors: one follower and one leader. To the former the robot gives information of its full state, from the latter it receives full state information which is taken by the decentralized controller as a reference. The tail robot has no followers[1].

Loosely speaking formation control is ensured following the simple intuition that a recursive leader-follower approach is sufficient. From an analytical viewpoint we establish that as for the leader-follower tracking problem it is sufficient that the virtual robot's angular velocity is persistently exciting. More precisely, we establish uniform global exponential stability of the consensus-tracking error system.

For its simplicity, our controller is an original contribution to the problem. For the generality of the result (uniform global exponential stability) our main result supersedes others which establish weaker properties such as asymptotic stability and convergence.

The rest of the paper is organized as follows. In Section II we recall the kinematic model of the mobile robot and formulate the formation tracking control problem. In Section III, we present our main result. In Section IV we present some illustrative simulation results and we conclude with some remarks in Section V.

[1]The names "leader" and "tail" are used in a graph theory sense to denote the root and the leaf nodes, they do not determine any physical relative posture of the robots.

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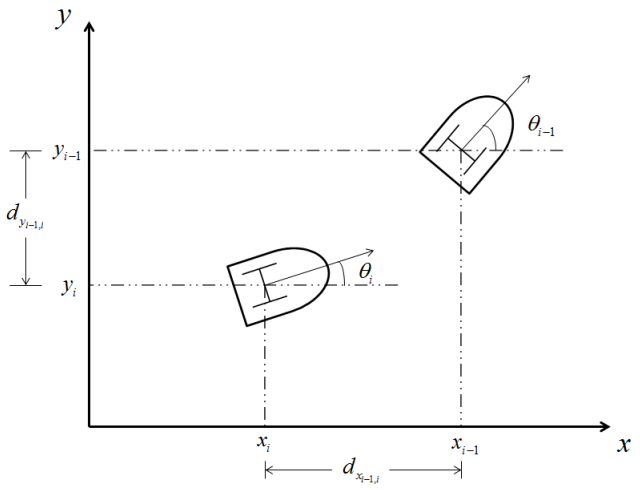

Fig. 1. Generic representation of a leader-follower configuration. For a swarm of n vehicles, any geometric topology may be easily defined by determining the position of each vehicle relative to its leader. This does not affect the kinematic model.

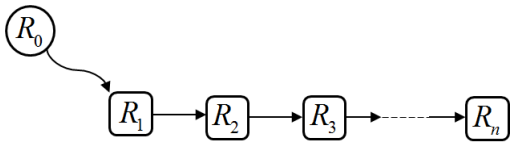

Fig. 2. Communication topology: a spanning directed tree with permanent communication between R_i and R_{i+1} for all $i \in [0, n-1]$. R_0 denotes the virtual leader.

II. Problem Statement and its Solution

Consider a group of n nonholonomic mobile robots with the following kinematic model

$$\begin{aligned} \dot{x}_i &= v_i \cos(\theta_i) \\ \dot{y}_i &= v_i \sin(\theta_i) \qquad i = 1, ..., n \\ \dot{\theta}_i &= w_i \end{aligned} \tag{1}$$

where the coordinates x_i and y_i represent the center of the ith mobile robot with respect to a global fixed frame and θ_i is the heading angle of the ith robot, see Fig 1. It is assumed that each vehicle is velocity-controlled that is the decentralized control inputs are v_i and w_i which correspond respectively to the linear and angular velocities of the ith robot.

The control objective is to make the n robots take specific positions, determined by the topology designer, and make the swarm follow a virtual reference vehicle. Note that any geometrical configuration may be achieved and one can choose any point in the Cartesian plane to follow the virtual reference vehicle.

The swarm has only one 'leader' robot named R_1 which knows the reference trajectory, this is the child of the root node in the graph. The other robots are intermediate robots labeled R_2 to R_{n-1} that is, R_i acts as leader for $R_{i+1}$and follows R_{i-1}. The 'tail' robot R_n has no followers (no children in the graph). It is important to observe that the notation R_{i-1} refers to the *graph* topology as illustrated in Figure 2 but it does not determine any physical formation.

The reference vehicle R_0 describes the reference trajectory defined by

$$\begin{aligned} \dot{x}_0 &= v_0 \cos(\theta_0) \\ \dot{y}_0 &= v_0 \sin(\theta_0) \\ \dot{\theta}_0 &= w_0 \end{aligned} \tag{2}$$

that is, v_0 and w_0 are respectively, the desired linear and angular velocities communicated to the 'leader' robot R_1.

For the sake of analysis and control design, we follow the steps of the seminal paper [14] and write the error dynamics of any two pairs of leader-follower robots. For the leader robot R_1 and the reference virtual vehicle we define

$$\begin{aligned} p_{1x} &= x_0 - x_1 \\ p_{1y} &= y_0 - y_1 \\ p_{1\theta} &= \theta_0 - \theta_1. \end{aligned} \tag{3}$$

Then, we transform the error coordinates $[p_{1x},\ p_{1y},\ p_{1\theta}]$ of the leader robot from the global coordinate frame to local coordinates fixed on the robot that is,

$$\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} \\ p_{1y} \\ p_{1\theta} \end{bmatrix}. \tag{4}$$

In these new coordinates, the error dynamics between the reference vehicle and the leader of the swarm becomes

$$\begin{aligned} \dot{e}_{1x} &= w_1 e_{1y} - v_1 + v_0 \cos e_{1\theta} \\ \dot{e}_{1y} &= -w_1 e_{1x} + v_0 \sin e_{1\theta} \\ \dot{e}_{1\theta} &= w_0 - w_1. \end{aligned} \tag{5}$$

As is observed in a large body of literature that followed [14], the leader-follower tracking control problem boils down to the stabilization of the origin of (5) –see [15] and references therein. In this paper we follow the simple linear time-varying controller originally proposed in [17], where uniform global exponential stability was first established. Define

$$\begin{aligned} v_1 &= v_0 + c_2 e_{1x} \\ w_1 &= w_0 + c_1 e_{1\theta} \end{aligned} \tag{6}$$

then, the closed-loop dynamics is given by

$$\dot{e}_{1x} = [w_0 e_{1y} - c_2 e_{1x}] + [c_1 e_{1\theta} e_{1y} + v_0(\cos e_{1\theta} - 1)] \tag{7a}$$

$$\dot{e}_{1y} = [-w_0 e_{1x}] + [-c_1 e_{1\theta} e_{1x} + v_0 \sin e_{1\theta}] \tag{7b}$$

$$\dot{e}_{1\theta} = -c_1 e_{1\theta}. \tag{7c}$$

The interest of the tracking controller of [17] is that the closed-loop system (7) has a cascaded structure; this is evident if we re-write the first two equations in the compact form

$$\dot{e}_{1xy} = f_1(t, e_{1xy}) + g(t, e_{1xy}, e_\theta) \tag{8}$$

where $g(t, e_{1xy}, 0) \equiv 0$ and $\dot{e}_{1xy} = f_1(t, e_{1xy})$ corresponds to

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \begin{bmatrix} -c_2 & w_0(t) \\ -w_0(t) & 0 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} \tag{9}$$

whose origin is uniformy globally exponentially stable if w_0 is locally integrable, globally Lipschiz and persistently exciting that is if there exist positive constants μ_1, μ_2 and T such that

$$\mu_1 \leq \int_t^{t+T} |w_0(\tau)|^2 \, d\tau \leq \mu_2 \qquad \forall t \geq 0 \tag{10}$$

The latter follows well-established results for adaptive linear control systems -see [16].

Uniform global exponential stability of the origin of (7) follows invoking stability theorems for non-autonomous time varing cascaded systems; roughy speaking, the argument relies on [18], [Lemma 2] which establishes that the origin of a cascaded system is uniformly globally asymptotically stable if so are the respective origins of the disconnected subsystems that is, when the interconnection $g \equiv 0$ and if the solutions of the perturbed dynamics (8) remain bounded -see the appendix for a concrete result.

The main result of this paper consits in showing that the controller of [17] may be used locally on each robot where the reference velocities are replaced by those of the leader vehicle to achieve formation control. The analysis relies on the pbservation that the closed-loop system has a cascaded structure and remarkably, it sufficies for consensus tracking that the virtual vehicle's reference angular velocity w_0 be persistently exciting.

In order to establish our main result, we proceed to write the error dynamics between any pair leader-follower robots starting with the leader R_1. The errors are generally defined by

$$p_{ix} = x_{(i-1)} - x_i - d_{x(i-1),i} \tag{11a}$$
$$p_{iy} = y_{(i-1)} - y_i - d_{y(i-1),i} \tag{11b}$$
$$p_{i\theta} = \theta_{(i-1)} - \theta_i \qquad i \in \{2, ..., n\} \tag{11c}$$

where $d_{x(i-1),i}$ and $d_{y(i-1),i}$ denote the desired distances between any two points on each mobile robot frame; for simplicity but without loss of generality these points are taken to be the origins of the local coordinate frames attached to each robot. Note that any formation topology may be defined by determining the values of d_{ixy}. In addition, one may define differences in the heading angles that is $d_{i\theta}$ however, for simplicity we assume here that all robots are to be aligned with the same heading $d_{i\theta} = 0$ for all $i \in \{2, ..., n\}$.

Using the same transformation given in (4) we obtain

$$\dot{e}_{ix} = w_i e_{iy} - v_i + v_{(i-1)} \cos e_{i\theta} \tag{12a}$$
$$\dot{e}_{iy} = -w_i e_{ix} + v_{(i-1)} \sin e_{i\theta} \tag{12b}$$
$$\dot{e}_{i\theta} = w_{(i-1)} - w_i \tag{12c}$$

hence, following the previous discussion we define the local control inputs

$$v_i = v_{(i-1)} + c_{2i} e_{ix} \tag{13a}$$
$$w_i = w_{(i-1)} + c_{1i} e_{i\theta} \tag{13b}$$

which replaced in (12), lead to

$$\dot{e}_{ix} = \left[w_{(i-1)} e_{iy} - c_2 e_{ix}\right] + \left[c_1 e_{i\theta} e_{iy} + v_{(i-1)} (\cos e_{i\theta} - 1)\right] \tag{14a}$$
$$\dot{e}_{iy} = \left[-w_{(i-1)} e_{ix}\right] + \left[-c_1 e_{i\theta} e_{ix} + v_{(i-1)} \sin e_{i\theta}\right] \tag{14b}$$
$$\dot{e}_{i\theta} = -c_1 e_{i\theta} \tag{14c}$$

for each $i \in \{1, ..., n\}$. That is, each set of equations corresponds to the tracking error dynamics between a leader and a follower robot. For the sake of analysis we remark that these equations may be written in compact form

$$\Sigma_1 : \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} = \begin{bmatrix} -C_2 & W(t, e_\theta) \\ -W(t, e_\theta) & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix} + \Psi(t, e_x, e_y, e_\theta) \tag{15}$$
$$\Sigma_2 : \dot{e}_\theta = -C_1 e_\theta \tag{16}$$

where $W(t, e_\theta) := diag\{w_0(t),\ w_0(t) + c_{11} e_{1\theta}, ..., w_0(t) + \Sigma_{i=1}^n c_{1i} e_{i\theta}\}$, $C_1 := diag\{c_{1i}\}$, $C_2 := diag\{c_{2i}\}$ and the interconnection term

$$\Psi = \begin{bmatrix} c_1 e_{1\theta} e_{1y} + v_0(\cos e_{1\theta} - 1) \\ \vdots \\ c_1 e_{n\theta} e_{ny} + v_{(n-1)}(\cos e_{n\theta} - 1) \\ -c_1 e_{1\theta} e_{1x} + v_0 \sin e_{1\theta} \\ \vdots \\ -c_1 e_{n\theta} e_{nx} + v_{(n-1)} \sin e_{n\theta} \end{bmatrix} \tag{17}$$

is such that $\Psi(t, e_x, e_y, 0) \equiv 0$.

Stability theorems for cascaded time-varying systems may be invoked to establish uniform global exponential stability of the origin. This constitutes our main result, which is presented in the following section.

III. Main result

Our main result implies that consensus tracking is achived by virtue of the local controllers (13) hence,

$$\lim_{t\to\infty} e_{ix}(t) = 0 \quad \lim_{t\to\infty} e_{iy}(t) = 0 \qquad \lim_{t\to\infty} e_{i\theta}(t) = 0. \tag{18}$$

Proposition 1 *Consider the kinematic systems (1) in closed loop with the controllers (13) with $i \in \{1, ...n\}$. Then, the origin of the closed loop system is uniformly globally exponentially stable if* v_0 is bounded, $c_{1i},\ c_{2i} > 0$ *and there exist* $b_\mu,\ \mu_1,\ \mu_2$ and T *such that*

$$\max\left\{\sup_{t\geq 0} |w_0(t)|,\ \sup_{t\geq 0} |\dot{w}_0(t)|\right\} \leq b_\mu \tag{19}$$

and (10) holds.

Proof: The closed loop dynamics is given by (15), (16). Therefore, the proof boils down to showing that the origin of the latter is uniformly globally exponentially stable. To that end we invoke Theorem 2 in the Appendix with the following definitions: $x_1 := [e_x, e_y]^T,\ x_2 := e_\theta$

$$f_1(t, x) := \begin{bmatrix} -C_2 & W(t, 0) \\ -W(t, 0) & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix}, \tag{20}$$

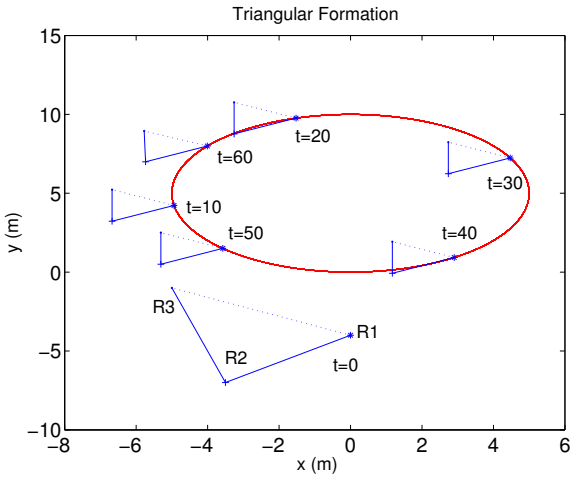

Fig. 3. Motion and relative positioning of the robots in triangular formation on the plane.

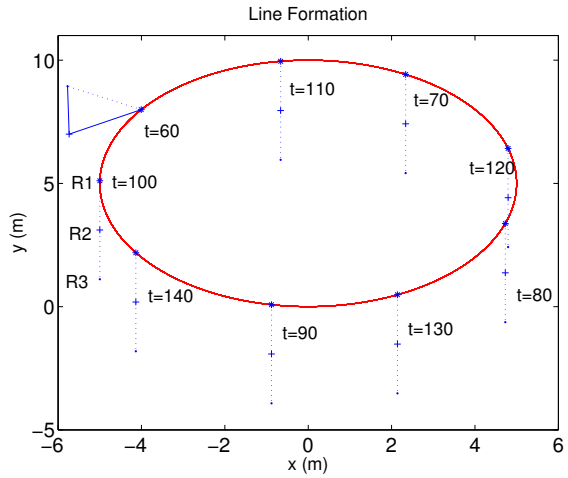

Fig. 4. Motion and relative positioning of the robots in alined formation on the plane.

and

$$g(t,x_1,x_2)=\Psi(t,e_x,e_y,e_\theta)+\left[\begin{array}{cc}0 & W\left(t,e_\theta\right)-W(t,0)\\ -W\left(t,e_\theta\right)+W(t,0) & 0\end{array}\right]\left[\begin{array}{c}e_x\\ e_y\end{array}\right]$$

and $f_2(t,x_2):=-C_1e_\theta$. Then the closed-loop dynamics (15), (16) may be written in compact form as in (22), (23) –see the Appendix. The regularity assumtions on f_1 and f_2 hold in view of (19). Then, the system (24) corresponds to

$$\left[\begin{array}{c}\dot{e}_x\\ \dot{e}_y\end{array}\right]=\left[\begin{array}{cc}-C_2 & W\left(t,0\right)\\ -W\left(t,0\right) & 0\end{array}\right]\left[\begin{array}{c}e_x\\ e_y\end{array}\right]$$

where $W(t,0):=w_0(t)I$. Uniform global exponential stability of the origin of the system follows from standard results on linear time-varying systems, provided that $C_2>0$, $W\left(\cdot,0\right)$ is globally Lipschiz, locally integrable and persistently exciting. All the latter are implied by (19) and (10). On the other hand, uniform global exponential stability of the origin of (16) is evident since C_1 is diagonal positive definite.

It remains to show that Assumptions A1 and A2 in Theorem 2 of the Appendix, hold. Assumption A1 holds with

$$V(t,x_1)=\frac{1}{2}\left[|e_x|^2+|e_y|^2\right]. \tag{21}$$

Its time derivative along the trajectories of (20) yields

$$\dot{V}_{(20)}(t,x_1)=-e_x^TC_2e_x\leq 0$$

and the conditions (26) and (27) hold with $c_2=\eta=1$ and $c_1=2$.

Finally, Assumption A2 holds simply by observig that $x_2=0$ implies that $g=0$ for any $t\geq 0$ and $x_1\in\mathbb{R}^{2n}$ and both Ψ and $W(t,e_\theta)-W(t,0)$ are both linear in $\left[\begin{array}{cc}e_x & e_y\end{array}\right]^T$ and uniformly bounded in t, the latter comes from (19). ■

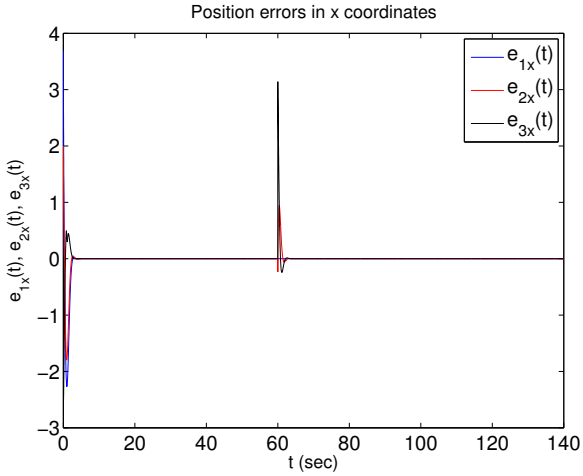

Fig. 5. Position errors in x coordinates.

IV. Simulation Results

To illustrate the feasibility of the proposed control method we performed simulations using Simulink™ of Matlab™. We consider a team of 3 mobile robots where one of them is the leader which knows the reference trajectory and the other two as followers. In the first stage of the simulation, the desired formation shape is in triangular form and after 60s, the topology switches to line formation. The initial states of the robots are $\left[x_1(0),y_1(0),\theta_1(0)\right]^T=[0,-4,3\pi/8]$, $\left[x_2(0),y_2(0),\theta_2\left(0\right)\right]^T=[-3.5,-7,\pi/2]$ and $\left[x_3\left(0\right),y_3(0),\theta_3(0)\right]^T=[-5,-1,\pi/3]$. The desired distance between the robots are $[d_{x1,2},d_{y1,2}]=\left[\sqrt{3},1\right]$ and $[d_{x2,3},d_{y2,3}]=[0,-2]$. In order to obtain the reference trajectory of the leader robot, we set the linear and angular velocities as $[v_0(t),w_0(t)]=[15\text{[m/s]},\ 3\text{[rad/s]}]$.

In Fig 3. we show the motion and relative positioning of the robots in triangular formation. It is easy to see that after a few seconds the formation shape is established and each

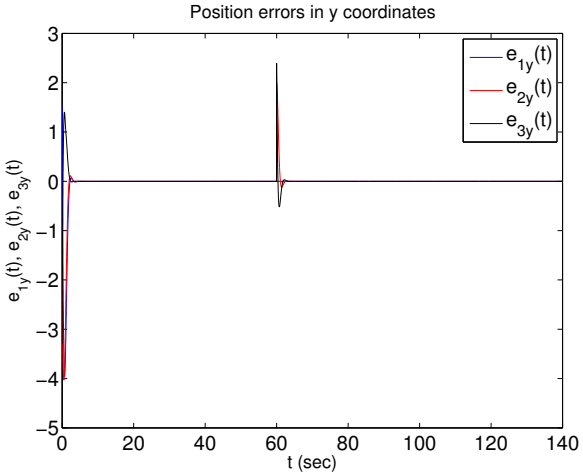

Fig. 6. Position errors in y coordinates.

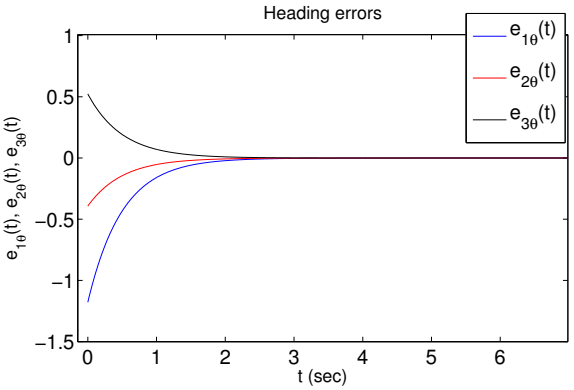

Fig. 7. Heading errors.

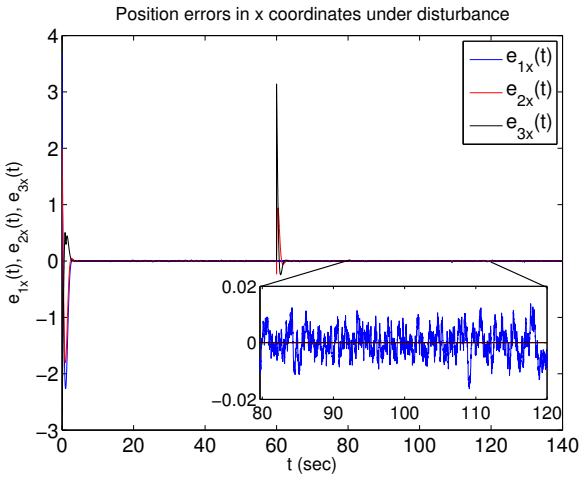

Fig. 8. Position errors in x coordinates under disturbance.

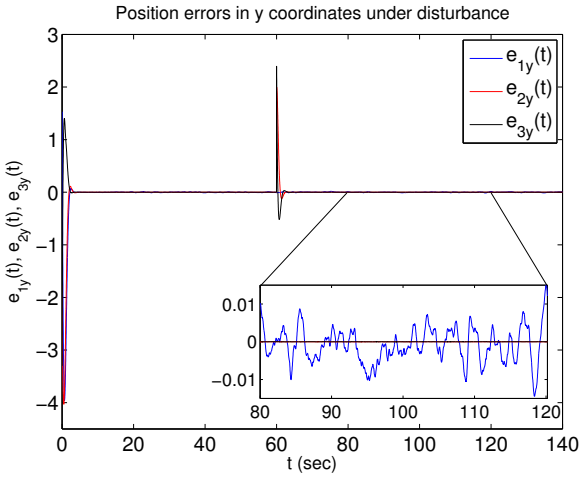

Fig. 9. Position errors in y coordinates under disturbance.

robot tracks its neighbor with its desired off-set, while the leader tracking the reference trajectory with a satisfactory performance.

In Fig 4 we show the formation change from triangular to line at $t = 60\ s$. Because the leader's motion is independent of the formation shape it keeps its tajectory as expected and the followers achieve new formation after a short transient.

In Figs 5-7 the trajectory errors of the robots are depicted. It is clear that with the proposed control method the desired formation tracking is successfully ensured.

To illustrate the robustness of the controller we add a time-varying random signal δ_v as an unknown disturbance to the leader robot R_1; the formation tracking performance is still satisfactory as it is showed in Figs. 9-10, demonstrate that position under disturbance effect converge to a small neighborhood of the origin; the heading is unaffected by the disturbance hence it is not showed.

V. Conclusions

We have presented a simple linear consensus algorithm for formation tracking control of a swarm of nonholonomic robots based on a one-to-one communication. The formation topology is arbitrary and the main assumption is that the angular velocity is persistently exciting. Present research is carried out to consider interconnections and even state-dependent. These extensions are not presented here due to space constraints; indeed, although intuitive, they rely on sharper technical tools which include nonlinear variants of persistency of excitation and corresponding stability results for nonlinear time-varying adaptive control systems.

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VI. Appendix

Consider the system

$$\dot{x}_1 = f_1(t, x_1) + g(t, x_1, x_2) \tag{22}$$

$$\dot{x}_2 = f_2(t, x_2) \tag{23}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, $x \triangleq \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$. The function f_1 is locally Lipschitz in x_1 uniformly in t and $f(\cdot, x_1)$ is continuous, f_2 is continuous and locally Lipschitz in x_2 uniformly in t, g is continuous in t and once differentiable in x. The theorem given below which is reminiscent of the results originally presented in [13] establishes unifom global exponential stability of the cascaded non-autonomous systems.

Theorem 1 *Let the respective origins of*

$$\Sigma_1 : \ \dot{x}_1 = f_1(t, x_1) \tag{24}$$

$$\Sigma_2 : \ \dot{x}_2 = f_2(t, x_2) \tag{25}$$

be uniformly globally exponentially stable and the following assumptions hold.

(A1) There exist a Lyapunov function $V : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ for (24) which is positive definite, radially unbounded,

$$\dot{V}_{(24)}(t, x_1) := \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1} f_1(t, x_1) \leq 0$$

and constants $c_1, c_2, \eta > 0$ such that

$$\left| \frac{\partial V}{\partial x_1} \right| |x_1| \leq c_1 V(t, x_1) \qquad \forall \ |x_1| \geq \eta \tag{26}$$

$$\left| \frac{\partial V}{\partial x_1} \right| \leq c_2 \qquad \forall \ |x_1| \leq \eta \tag{27}$$

(A2) There exist two continuous functions $\theta_1, \theta_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $g(t, x_1, x_2)$ satisfies

$$|g(t, x_1, x_2)| \leq \theta_1(|x_2|) + \theta_2(|x_2|) \ \ |x_1| \tag{28}$$

Then, the origin of the cascaded system (22), (23) is uniformly globally exponentially stable.