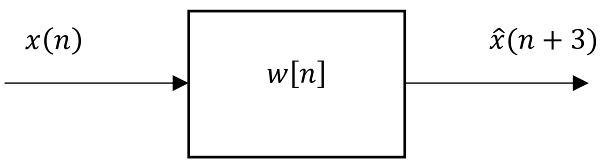
Statistical Signal Processing Homework

1. Consider a three-step predictor using a second-order filter

$$w(n) = \begin{bmatrix} w(0) \\ w(1) \\ w(2) \end{bmatrix}$$

whose diagram is shown below.



If the output is the minimum mean-square estimate of $x[n+3]$.

a) What are the Wiener-Hopf equations for this model.
b) If the sequence $r_x(k)$ is given by

$$\boldsymbol{r}_x = [1.0 \quad 0 \quad 0.1 \quad -0.2 \quad -0.9 \quad -2.2 \quad -6 \quad -15]^T$$

find the Wiener filter coefficients $w[n]$.

2. Let $d(n)$ be an AR(1) process with an autocorrelation sequence

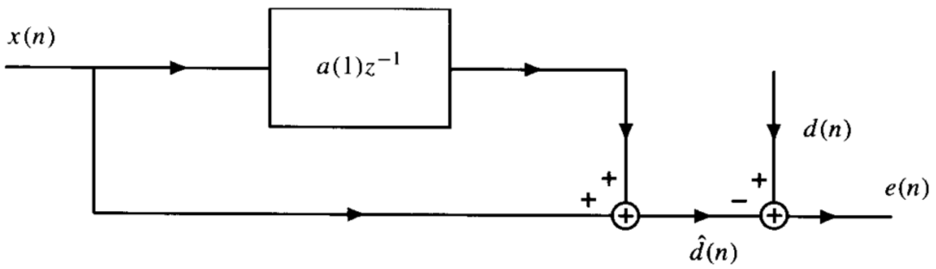
$$r_d(k) = \alpha^{|k|}$$

where $0 < \alpha < 1$. Suppose that $d(n)$ is observed in the presence of uncorrelated white noise, $v(n)$, that has a variance of σ_v^2,

$$x(n) = d(n) + v(n).$$

Compare the minimum errors ξ_{min} obtained by a second-order and a first-order FIR Wiener filters.

3. Consider the system shown in the figure below for estimating a process $d(n)$ from $x(n)$



If $\sigma_d^2 = 4$ and

$$\mathbf{r}_x = [1.0, \quad 0.5, \quad 0.25]; \qquad \mathbf{r}_{dx} = [-1.0, \quad 1.0]$$

find the value of $a(1)$ that minimizes the mean-square error $\xi = E\{|e(n)|^2\}$, and find the minimum mean-square error.

4. Suppose that a signal $d(n)$ is corrupted by noise

$$x(n) = d(n) + w(n)$$

where $r_w(k) = 0.5\delta(k)$ and $r_{dw}(k) = 0$. The signal is an AR(1) process that satisfies the difference equation

$$d(n) = 0.5d(n-1) + v(n)$$

where $v(n)$ is white noise with variance $\sigma_v^2 = 1$. Assume that $w(n)$ and $v(n)$ are uncorrelated.

a) Design a first-order FIR linear predictor $W(z) = w(0) + w(1)z^{-1}$ for $d(n)$ and find the mean-square prediction error $\xi = E\left\{\left[d(n+1) - \hat{d}(n+1)\right]^2\right\}$

b) Design a casual IIR Wiener predictor and compare the mean-square prediction error with that found in part (a).