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DYNAMICAL RESPONSE OF A PRESTRAINED SYSTEM COMPRISING A SUBSTRATE AND BOND AND COVERING LAYERS TO A MOVING LOAD

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The dynamical response of a system consisting of a prestressed substrate and covering and bond layers to a moving load is investigated within the scope of a piecewise-homogeneous body model by using a three-dimensional linearized theory of wave propagation in initially stressed bodies. It is assumed that the materials of the constituents are isotropic and homogeneous, and the subsonic speed of the moving load acting on the covering layer is constant. The investigations are carried out for a plane strain state under complete and incomplete contact conditions. For various values of problem parameters, numerical results for the critical speed are presented and discussed.

1. Introduction

The results of investigations into the dynamical response of bodies to moving loads are of great theoretical and practical significance and can be employed in many branches of modern transportation engineering.

A considerable part of these studies deal with the dynamical response of a layered half-plane to a moving load. In this case, the layered half-plane is taken as a model to simulate track-based systems or various pavements subjected to moving vehicle loads. Relevant theoretical investigations into this field were carried out in [1] and other papers listed therein. However, in the course of time, these studies were improved and developed steadily; the latest results are described in [2-4] and many other reports.

In the aforementioned papers, the dynamical response was studied by employing the classical linear theory of elastodynamics, the motion of the layer was described by the Kirchhoff or Timoshenko beam-plate theory, but the equations of motion of the half-plane were taken from exact linear theory of elastic waves.

The present level of high-speed vehicle engineering requires investigations into these problems on the basis of more accurate theories that take into account nonlinear dynamical effects, too. An interesting and urgent problem that also applies to

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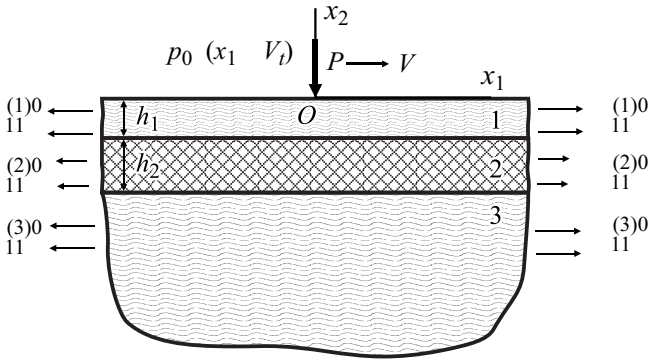


Fig. 1. Geometry of the system considered.

the effects in the aforementioned elastic systems is the elastodynamics problem for initially stressed bodies. The initial stresses occur in the systems after their manufacture and assembly, and, in many cases, they are rather high.

Under certain conditions, these problems can be solved with the use of the three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). The construction of field equations of the TLTEWISB and their application to the wave propagation problem for initially stressed bodies are detailed, e.g., in [5]. Reviews of these investigations can be found in [6-8]. Furthermore, by employing the TLTEWISB, time-harmonic stress fields in layered prestrained bodies were studied in [9-13] and other papers.

Note that, within the framework of TLTEWISB, a few studies have already been performed up to now on the dynamical response of a prestrained layered half-space to a moving load; see, for example, [14, 15]. In [14], the dynamical response of a system consisting of a layer and a prestrained half-plane was considered. The equation of motion for the covering layer was described by the Timoshenko beam theory, but the equation of motion for the half-plane was described by the TLTEWISB. The solution to the corresponding boundary-value problem was found by employing the exponential Fourier integral transformation. Particular numerical investigations were made for the case where the constitutive relations for the half-plane material were described by a potential of harmonic type. It was assumed that the speed of the moving load was subsonic and constant, and the influence of problem parameters on the critical speed of the load was studied. In [15], the foregoing problem was solved through the use of the complex potential of TLTEWISB.

In [16], the results obtained in [14, 15] were extended to the case where the covering layer was also prestrained, and the motion of this layer was also described by the TLTEWISB. The influence of problem parameters on the critical speed was examined.

It should be noted that, in many cases, a system comprising a substrate and a covering layer is not adequate for simulating the track beds of high-speed trains, cars, trucks, etc., because there usually is a bond layer between the covering layer and the substrate. The mechanical and geometrical parameters of this layer can significantly affect the dynamical response of track beds to a moving load. Therefore, in the present paper, the investigation initiated in [16] is continued for a system comprising a substrate and bond and covering layers.

2. Formulation of the Problem

Let us consider a prestressed composite system consisting of a substrate and bond and covering layers. The points of the layers and substrate are specified by Lagrangian coordinates in a Cartesian reference frame $Ox_1x_2x_3$ (Fig. 1). Before assembling, the layers and the substrate have been stressed separately in the Ox_1 axis direction, so that a uniaxial homogeneous initial stress state exists in each of them. The covering and bond layers and the substrate occupy the regions

$$\{ \quad x_1 \quad , \; h_1 \quad x_2 \quad 0, \quad x_3 \quad \},$$

$$\{ \quad x_1 \quad , \ (h_1 \quad h_2) \quad x_2 \quad h_1, \quad x_3 \quad \},$$

and

$$\{ \quad x_1 \quad , \quad x_2 \quad (h_1 \quad h_2), \quad x_3 \quad \},$$

respectively. The Ox_3 axis is perpendicular to the figure plane and is not shown in Fig. 1.

The stresses, strains, and displacements related to the covering and bond layers and the substrate are denoted by the superscripts (1), (2), and (3), respectively. The quantities related to the initial stresses are denoted by the superscript $(m)0$, where $m = 1, 2, 3$.

The linearly elastic materials of the layers and substrate (half-plane) are taken to be homogeneous and isotropic. The initial stresses in the constituents are

$$\overset{(m)0}{ij} \quad \text{const}_m, \quad m = 1, 2, 3; \quad \overset{(m)0}{ij} \quad 0 \ \text{ if } \ ij \quad 11.$$

Since the corresponding strains are small, the coordinates of layers and substrate points in the natural and initial states coincide.

We assume that, to the free upper face of the covering layer, a force P is applied, which moves in the Ox_1-direction with a constant speed V (Fig. 1). It is required to determine the dynamical response of the system considered to the moving load. We will consider only the subsonic state, i.e.,

$$V \quad \min\{c_2^{(1)}, c_2^{(2)}, c_2^{(3)}\}, \quad c_2^{(m)} \quad \sqrt{\ (m) / \ (m)}, \quad m = 1, 2, 3, \tag{1}$$

where $c_2^{(m)}$, $\ (m)$, and $\ (m)$ are the speed of distortion waves, the shear modulus, and the density of constituent materials.

According to [5], the complete system of equations of TLTEWISB in the case of small initial strains is

$$\frac{\overset{(m)}{ij}}{x_1} \quad \overset{(m)0}{11} \frac{\partial^2 u_j^{(m)}}{x_1^2} \quad (m) \frac{\partial^2 u_j^{(m)}}{t^2}, \tag{2}$$

$$\overset{(m)}{ij} \quad (m) \quad (m) \ \overset{j}{i} \quad 2 \ (m) \ \overset{(m)}{ij}, \tag{3}$$

$$(m) \quad \overset{(m)}{11} \quad \overset{(m)}{22}, \quad \overset{(m)}{ij} \quad \frac{1}{2} \frac{u_i^{(m)}}{x_j} \quad \frac{u_j^{(m)}}{x_i}, \tag{4}$$

where $i, j = 1, 2$ and $m = 1, 2, 3$; $\ (m)$ and $\ (m)$ are called the Lamé constants.

Note that Eqs. (2)-(4) are satisfied in the covering and bond layers and in the substrate separately. According to the foregoing statement, on the upper face of the covering layer, the following boundary conditions must be satisfied:

$$\overset{(1)}{12}\Big|_{x_2 \ 0} \quad 0, \quad \overset{(1)}{22}\Big|_{x_2 \ 0} \quad P \ (x_1 \quad Vt), \tag{5}$$

where $\ (x)$ is the Dirac delta function.

As in [16], we will consider two types (complete and incomplete) of contact conditions on the interfaces between the covering and bond layers and between the bond layer and the substrate. For the complete contact conditions, we assume

$$\overset{(1)}{i2}\Big|_{x_2 \quad h_1} \quad \overset{(2)}{i2}\Big|_{x_2 \quad h_1}, \quad u_i^{(1)}\Big|_{x_2 \quad h_1} \quad u_i^{(2)}\Big|_{x_2 \quad h_1}, \tag{6}$$

$$\overset{(2)}{i2}\Big|_{x_2 \quad (h_1 \quad h_2)} \quad \overset{(3)}{i2}\Big|_{x_2 \quad (h_1 \quad h_2)}, \ u_i^{(2)}\Big|_{x_2 \quad (h_1 \quad h_2)} \quad u_i^{(3)}\Big|_{x_2 \quad (h_1 \quad h_2)}, \ i = 1, 2,$$

and for the incomplete ones

$$\begin{gathered}
{}^{(1)}_{22}\Big|_{x_2 \quad h_1} \qquad {}^{(2)}_{22}\Big|_{x_2 \quad h_1}, \quad u_2^{(1)}\Big|_{x_2 \quad h_1} \qquad u_2^{(2)}\Big|_{x_2 \quad h_1}, \\
{}^{(2)}_{12}\Big|_{x_2 \quad h_1} \quad 0, \quad {}^{(3)}_{12}\Big|_{x_2 \quad h_1} \quad 0, \quad {}^{(2)}_{22}\Big|_{x_2 \quad (h_1 \quad h_2)} \qquad {}^{(3)}_{22}\Big|_{x_2 \quad (h_1 \quad h_2)}, \\
u_2^{(2)}\Big|_{x_2 \quad (h_1 \quad h_2)} \qquad u_2^{(3)}\Big|_{x_2 \quad (h_1 \quad h_2)}, \quad {}^{(2)}_{12}\Big|_{x_2 \quad (h_1 \quad h_2)} \quad 0, \quad {}^{(3)}_{12}\Big|_{x_2 \quad (h_1 \quad h_2)} \quad 0.
\end{gathered} \tag{7}$$

In addition, we assume that the following decay conditions are satisfied:

$$\left|u_i^{(3)}\right|, \ \left| \ {}^{(3)}_{ij} \right| \quad 0 \ \text{ as } \ x_2 \qquad , \tag{8}$$

which complete the formulation of the problem. It should be noted that, at ${}^{(m),0}_{11} \quad 0$ ($m = 1, 2, 3$), we return to the corresponding problem of the classical linear theory of elastodynamics.

3. Solution Method

From Eqs. (2)-(4), we obtain the following equations of motion in terms of perturbations of displacements $u_i^{(m)}$:

$$\begin{gathered}
\left({}^{(m)} \quad 2 \ {}^{(m)} \quad {}^{(m)0}_{11} \right) \frac{{}^2 u_1^{(m)}}{x_1^2} \quad {}^{(m)} \frac{{}^2 u_1^{(m)}}{x_2^2} \quad \left({}^{(m)} \quad {}^{(m)} \right) \frac{{}^2 u_2^{(m)}}{x_1 \ x_2} \quad {}^{(m)} \frac{{}^2 u_1^{(m)}}{t^2}, \\
\left({}^{(m)} \quad {}^{(m)} \right) \frac{{}^2 u_1^{(m)}}{x_1 \ x_2} \quad \left({}^{(m)} \quad {}^{(m)0}_{11} \right) \frac{{}^2 u_1^{(m)}}{x_1^2} \quad \left({}^{(m)} \quad 2 \ {}^{(m)} \right) \frac{{}^2 u_2^{(m)}}{x_2^2} \quad {}^{(m)} \frac{{}^2 u_2^{(m)}}{t^2}.
\end{gathered} \tag{9}$$

In the coordinate system

$$x_2 \quad x_2, \ x_1 \quad x_1 \quad Vt,$$

which moves together with the loading force, Eqs. (9) take the form

$$\begin{gathered}
\frac{{}^{(m)}}{{}^{(m)}} \quad 2 \quad \frac{{}^{(m)0}_{11}}{{}^{(m)}} \quad \frac{V^2}{(c_2^{(m)})^2} \quad \frac{{}^2 u_1^{(m)}}{x_1^2} \quad \frac{{}^2 u_1^{(m)}}{x_2^2} \quad \frac{{}^{(m)}}{{}^{(m)}} \quad 1 \quad \frac{{}^2 u_2^{(m)}}{x_1 \ x_2} \quad 0, \\
\frac{{}^{(m)}}{{}^{(m)}} \quad 1 \quad \frac{{}^2 u_1^{(m)}}{x_1 \ x_2} \quad 1 \quad \frac{{}^{(m)0}_{11}}{{}^{(m)}} \quad \frac{V^2}{(c_2^{(m)})^2} \quad \frac{{}^2 u_2^{(m)}}{x_1^2} \quad \frac{{}^{(m)}}{{}^{(m)}} \quad 2 \quad \frac{{}^2 u_2^{(m)}}{x_2^2} \quad 0,
\end{gathered} \tag{10}$$

where the primes of x_1 and x_2 are omitted. In this case, the second boundary condition in (5) is replaced by

$$\left. {}^{(1)}_{22} \right|_{x_2 \quad 0} \quad P \ (x_1). \tag{11}$$

The other conditions remain valid.

In order to solve the system of equations (10), we apply the Fourier transform

$$f_F \ (s, x_2) \qquad f(x_1, x_2) e^{\ isx_1} dx_1$$

to it. As a result, we have

$$s^2 a^{(m)} u_{1F}^{(m)} \quad \frac{d^2 u_{1F}^{(m)}}{dx_2^2} \quad isb^{(m)} \frac{du_{2F}^{(m)}}{dx_2} \quad 0,$$
$$- isb^{(m)} \frac{du_{1F}^{(m)}}{dx_2} \quad s^2 c^{(m)} u_{2F}^{(m)} \quad (b^{(m)} \quad 1) \frac{d^2 u_{2F}^{(m)}}{dx_2^2} \quad 0, \tag{12}$$

where

$$i \quad \sqrt{\quad 1}, \quad a^{(m)} \quad \frac{(m)}{(m)} \quad 2 \quad \frac{m}{(m)} \quad \frac{V^2}{(c_2^{(m)})^2}, \quad b^{(m)} \quad \frac{(m)}{(m)} \quad 1,$$
$$c^{(m)} \quad 1 \quad \frac{m}{(m)} \quad \frac{V^2}{(c_2^{(m)})^2}.$$

After some mathematical manipulations, we obtain from (12) the equations

$$(b^{(m)} \quad 1) \frac{d^4 u_{2F}^{(m)}}{dx_2^4} \quad [s^2 (b^{(m)})^2 \quad s^2 c^{(m)} \quad s^2 a^{(m)} (b^{(m)} \quad 1)] \frac{d^2 u_{2F}^{(m)}}{dx_2^2} \quad s^4 a^{(m)} c^{(m)} u_{2F}^{(m)} \quad 0,$$
$$. \frac{du_{1F}^{(m)}}{dx_2} \quad is \frac{c^{(m)}}{b^{(m)}} u_{2F}^{(m)} \quad i \frac{b^{(m)} \quad 1}{sb^{(m)}} \frac{d^2 u_{2F}^{(m)}}{dx_2^2}. \tag{13}$$

From Eq. (13), we have

$$u_{2F}^{(n)} \quad F_1^{(n)}(s) e^{k_1^{(n)} x_2} \quad F_2^{(n)}(s) e^{\quad k_1^{(n)} x_2} \quad F_3^{(n)}(s) e^{k_2^{(n)} x_2} \quad F_4^{(n)}(s) e^{\quad k_2^{(n)} x_2}, \quad n = 1, 2,$$
$$u_{2F}^{(3)} \quad F_1^{(3)}(s) e^{k_1^{(n)} x_2} \quad F_3^{(n)}(s) e^{k_2^{(n)} x_2}, \tag{14}$$

where

$$k_1^{(m)} \quad \sqrt{\frac{A^{(m)}}{2} \quad \sqrt{\left(\frac{A^{(m)}}{2}\right)^2 \quad B^{(m)}}}, \quad k_2^{(m)} \quad \sqrt{\frac{A^{(m)}}{2} \quad \sqrt{\left(\frac{A^{(m)}}{2}\right)^2 \quad B^{(m)}}},$$
$$A^{(m)} \quad \frac{s^2 b^{(m)} \quad s^2 c^{(m)} \quad s^2 a^{(m)} (b^{(m)} \quad 1)}{b^{(m)} \quad 1}, \quad B^{(m)} \quad \frac{s^4 a^{(m)} c^{(m)}}{b^{(m)} \quad 1}. \tag{15}$$

Thus, from Eqs. (13)-(15), (3), and (4), the Fourier transforms of all sought-for quantities are found. The unknowns $F_1^{(1)}(s),\ldots,F_4^{(1)}(s)$, $F_1^{(2)}(s),\ldots,F_4^{(2)}(s)$, $F_1^{(3)}(s)$, and $F_3^{(3)}(s)$, appearing in these transforms, are obtained from boundary conditions (5) and (11) and contact condition (6) (for a complete contact) or (7) (for an incomplete contact), which form a closed system of algebraic equations. Note that condition (8) was already used for determining $u_{2F}^{(3)}$ in (14).

From the algebraic equations, we find the aforementioned unknowns and, employing the inverse transform

$$f(x_1, x_2) \quad \frac{1}{2} \quad f_F(s, x_2) e^{isx_1} ds, \tag{16}$$

we determine the sought-for stresses and displacements.

Equation (16) can be reduced to the following ones:

$$f(x_1, x_2) \quad \frac{1}{} \quad f_F(s, x_2)\cos(sx_1)ds \text{ for } u_2^{(m)}, \quad {}^{(m)}_{22}, \quad {}^{(m)}_{11}, \quad {}^{(m)}_{22}, \text{ and } {}^{(m)}_{11}, \tag{17}$$

$$f(x_1, x_2) \quad \frac{1}{} \quad f_F(s, x_2)\sin(sx_1)ds \text{ for } u_1^{(m)}, \quad {}^{(m)}_{12}, \text{ and } {}^{(m)}_{12}.$$

In calculating the integrals in Eqs. (17), we used the approximation

$$(.)ds \quad (.)ds, \tag{18}$$

where the value of S in (18) was determined from the corresponding criterion of numerical convergence.

Numerical investigations show that, in general, at fixed values of problem parameters, for each value of V, the quantities $u_{i_F}^{(m)}$ and ${}^{(m)}_{ijF}$ have singular points with respect to sh.

According to the procedure for determining the unknowns $F_1^{(1)}(s), \ldots, F_4^{(1)}(s)$, $F_1^{(2)}(s)$, and $F_3^{(2)}(s)$ (see [9-13, 16]), the singular points coincide with roots of the equation

$$\det \| \quad _{nm}(V(sh)) \| \quad 0, \; n; \; m = 1, 2, \ldots, 6, \tag{19}$$

where $_{nm}(V(sh))$ are coefficients of the unknowns in the system of algebraic equations.

Consequently, the order of singularity (denoted by r) of integrated quantities coincides with the order of roots of equation (19). At $0 \quad r \quad 1$, integral (18) can be calculated by using well-known algorithms. At $r = 1$, the calculation of integral (18) is performed according to the Cauchy principal value. For $r \quad 1$, the integral is meaningless, and the speed corresponding to this case is called the critical speed. At this speed, a resonance-type phenomenon takes place. In this case, the values of all problem parameters are fixed (except that of V), and the speed V at a given value of sh is determined from Eq. (19) as the root of this equation. As a result, a relationship between V and sh_1 is obtained, and the critical speed corresponds to the case where $dV/d(sh_1) \quad 0$.

One of the main issues in problems for layered materials with moving loads in a subsonic state (1) is the determination of the critical speed V_{cr} and the effect of problem parameters on it. Another issue is the determination of stress-strain state in the mechanical systems at $V \quad V_{cr}$. In this case, integral (18) is calculated with the algorithm developed in [9-13].

4. Numerical Results and Discussion

We introduce the designations

$$\frac{V}{c_2^{(1)}}, \; e_1 \quad \frac{E^{(1)}}{E^{(2)}}, \; e_2 \quad \frac{E^{(3)}}{E^{(2)}}, \; H \quad \frac{h_2}{h_1}, \quad k \quad \frac{{}^{(k)}_{11}}{{}^{(k)}}, \quad {}^{(k)} \quad \frac{E^{(k)}}{2(1 \quad {}^{(k)})}$$

and assume that

$${}^{(2)} / {}^{(1)} \quad 0.5, \quad {}^{(3)} / {}^{(2)} \quad 0.5, \quad {}^{(1)} \quad {}^{(2)} \quad {}^{(3)} = 0.3.$$

Let us analyze the relationship between and sh_1. According to the well-known mechanical consideration, with H , this relationship must approach that for the system consisting only of the covering layer and the substrate (shortly, CL+SS), but for $H \quad 0$, it must tend to that for the system consisting of the covering layer and a substrate whose material is the

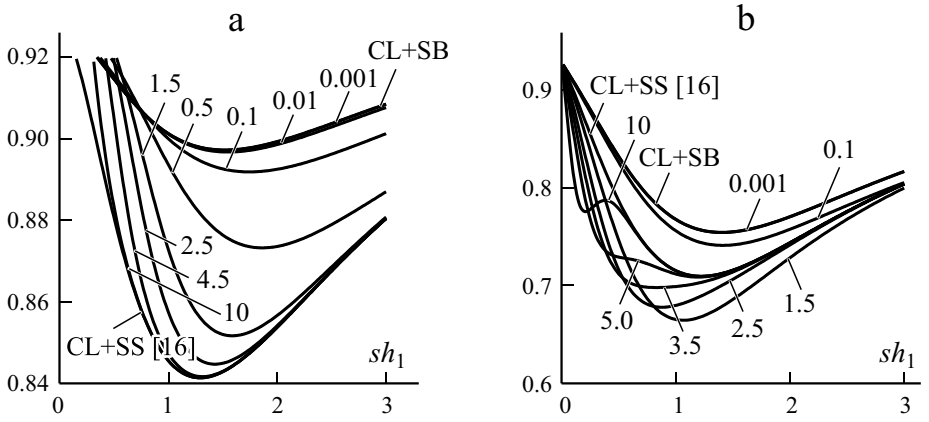


Fig. 2. Graphs of the relationship between and sh_1 for complete (6) (a) and incomplete (7) (b) contact conditions for various H h_2/h_1 (number at the curves).

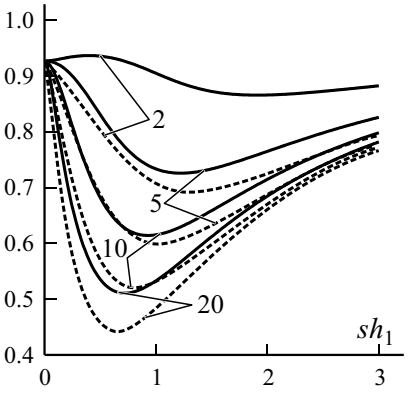


Fig. 3. Graphs of the relationship between and sh_1 for complete (——) and incomplete (- - -) contact conditions at $H = 1$, $e_2 = 1$, and various values of e_1 (numbers at the curves).

same as of the bond layer (shortly, CL+SB). This fact is seen from the graphs given in Fig. 2, which are constructed at $e_1 = 2$, e_2 1.5, $_1$ $_2$ $_3$ 0 and various values of H for complete (6) (Fig. 2a) and incomplete (7) (Fig. 2b) contact conditions. Note that the case $e_1 = 2.0$ for the system CL+SS was also considered in [16]. The coincidence of present results with those obtained in [16] confirms the validity of the algorithm and programs used in the numerical investigations.

Now, let us look at the effect of the parameter e_1 on the relationship between and sh_1. The corresponding graphs are given in Fig. 3 at $H = 1$, $e_2 = 2$, and $_1$ $_2$ $_3$ 0 for complete (6) (solid lines) and incomplete (7) (dashed lines) contact conditions. A comparison of the graphs with those found for the system CL+SS shows that the existence of a bond layer whose elastic modulus is greater than that of the substrate increases the values of (sh_1) which satisfy equation (19).

An analysis of the foregoing and a large number of other numerical results, not illustrated here, allow us to conclude that there are values of problem parameters at which (i) the function (sh_1) has no local maxima or minima; (ii) the function has only one local maximum or minimum at sh_1 ; (iii) the function has one or more local maxima and one or more local minima at sh_1 0 and sh_1 . In case (i), a critical speed does not exist; in case (ii), a critical speed exists and corresponds to the local maximum or minimum of the function (sh_1); in case (iii), a critical speed exists and corresponds to the absolute minimum of .

The values of $_{cr}$ were determined from the graphs given in Figs. 2 and 3 and other similar ones, not shown here. In particular, it follows from the figures that, in the cases where the inequalities e_1 e_2 and e_2 1 are satisfied simultaneously, an increase in thickness of the bond layer decreases $_{cr}$; if e_1 e_2 and e_2 1, $_{cr}$ increases with e_1.

Table 1 shows the values of $_{cr}$ for various values of the pairs (e_1, e_2) and H at $_1$ $_2$ $_3$ 0, i.e., in the absence of initial stresses in constituents of the system considered. The corresponding values of $_{cr}$ for the systems CL+SS and CL+SB

TABLE 1. Values of $_{cr}$ at Complete (Numerator) and Incomplete (Denominator) Boundary Conditions for Various Values of e_1, e_2, and H

(e_1, e_2)	H = 0.001	H = 0.5	H = 1.5	H = 10	H = 30
(2, 4/3)	0.8972	0.8732	0.8517	0.8414	0.8414
	0.7540	0.6970	0.6644	0.7754	0.7093
(5, 5/2)	0.8058	0.7538	0.7100	0.6857	0.6857
	0.6824	0.6303	0.5839	0.5931	0.5931
(10, 5)	0.6856	0.6405	0.5976	0.5664	0.5664
	0.5930	0.5490	0.5041	0.5016	0.5016

TABLE 2. Values of $_{cr}$ for the Systems CL+SS and CL+SB at Various Values of e_1 under Complete (Numerator) and Incomplete (Denominator) Contact Conditions

e_1 = 2	e_1 = 5	e_1 = 10	e_1 = 4/3	e_1 = 5/2
0.8414 [16]	0.6857	0.5664 [16]	0.8972	0.8059
0.7093	0.5931	0.5016	0.7542	0.6825

TABLE 3. Values of $_{cr}$ Obtained for Various e_1 and e_2 at $_1$ $_2$ $_3$ 0 and H = 1 under Complete (Numerator) and Incomplete (Denominator) Contact Conditions

e_1	e_2 = 2	e_2 = 5	e_2 = 10	e_2 = 20
2	0.8661	0.8756	0.8784	0.8798
	0.6930	0.7442	0.7649	0.7760
5	0.7260	0.8669	0.8706	0.8724
	0.5991	0.6523	0.6742	0.6861
10	0.6142	0.8253	0.8325	0.8361
	0.5206	0.5745	0.5973	0.6098
20	0.5106	0.7777	0.7906	0.7970
	0.4431	0.4965	0.5198	0.5329

TABLE 4. Effect of Initial Stretching of Constituent of the System Considered on the Values of $_{cr}$ at e_1 = 2, e_2 = 5, and H = 1

Cases	$_1$, $_2$, $_3$			
	0	0.005	0.01	0.05
$_1$ 0 $_2$ = $_3$ = 0	0.8756 0.7442	0.8821 0.7503	0.8884 0.7563	0.9353 0.7994
$_2$ 0 $_1$ = $_3$ = 0	0.8756 0.7442	0.8764 0.7456	0.8773 0.7469	0.8835 0.7575
$_3$ 0 $_1$ = $_2$ = 0	0.8756 0.7442	0.8758 0.7454	0.8759 0.7465	0.8767 0.7536

are given in Table 2. The numerical results given in these tables agree with mechanical considerations, and in some particular cases, coincide with the corresponding results obtained in [16].

Table 3 shows the effect of the parameters e_1 and e_2 on $_{cr}$ at H = 1.0 and $_1$ $_2$ $_3$ 0. As is seen, an increase in the elastic modulus of the bond layer increases $_{cr}$. This increase is more significant at e_1 5.

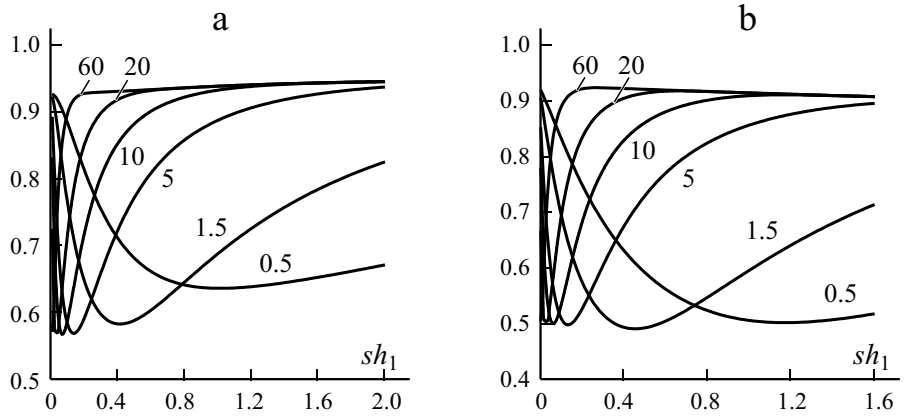


Fig. 4. Graphs of the relationship between and sh_1 for complete (a) and incomplete (b) contact conditions at e_1 $e_2 < 1$ (e_1 $e_2 = 0.1$) and various values of H (numbers at the curves).

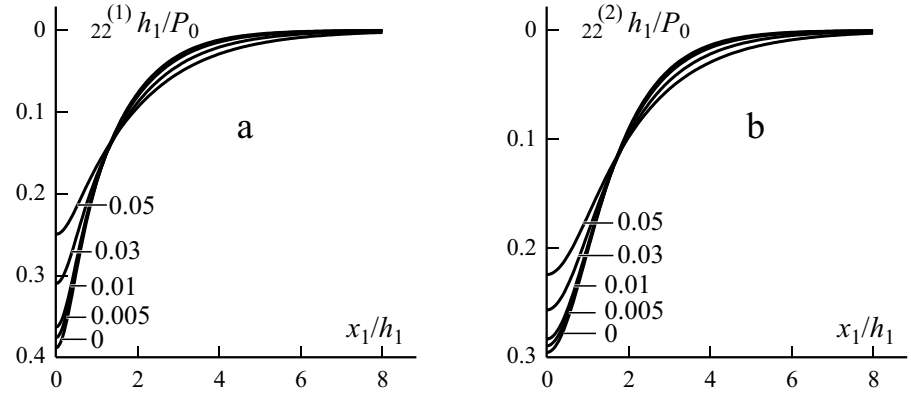


Fig. 5. The quantities $_{22}^{(1)} h_1 / P_0 \big|_{x_2 \quad h_1}$ (a) and $_{22}^{(2)} h_1 / P_0 \big|_{x_2 \quad (h_1 \quad h_2)}$ (b) in relation to x_1/h_1 for complete contact condition (7) at various values of $_1$ ($_{11}^{(1)0} / \ ^{(1)}$) (numbers at the curves).

The influence of initial stretching of constituents of the system considered on $_{cr}$ is illustrated by the data given in Table 4, which were obtained for $e_1 = 2$, $e_2 = 5$, and $H = 1$. As evident from the table, the initial stretching increases $_{cr}$, the effect caused by prestretching the covering layer on the values of $_{cr}$ being the greatest.

Let e_1 $e_2 = 0.1$. As established in [16], in this case, there is no critical speed for the system CL+SB. Consequently, the minima arising in graphs of the relationship between and sh_1 for the system must disappear as H , which is confirmed by the graphs given in Fig. 4 for complete (6) (Fig. 4a) and incomplete (7) (Fig. 4b) contact conditions at various values of H and $_1$ $_2$ $_3$ 0. As is seen, $_{(R)}$ $V_R / C_2^{(1)}$ as sh_1 and H , where V_R is the propagation speed of Rayleght waves in the covering layer. Moreover, these graphs show that, at e_1 e_2 1, there exists such a value of H H that the local minimum of the relationship (sh_1) disappear if H H .

Finally, let us consider some examples for the quantities $_{22}^{(1)} h_1 / P_0$ at x_2 h_1 and $_{22}^{(2)} h_1 / P_0$ at x_2 (h_1 h_2) in relation to x_1/h_1 under complete contact conditions (6). The corresponding graphs are given in Fig. 5 for various values of $_1$ at $H = 1$, $e_1 = 2$, and $e_2 = 5$. As is seen, the absolute values of $_{22}^{(1)} h_1 / P_0$ (Fig. 5a) in the vicinity of the point $x_1 = 0$ are greater than the corresponding values of $_{22}^{(2)} h_1 / P_0$ (Fig. 5b), and the initial stretching of the covering layer decreases the stresses significantly.

5. Conclusions

In the present paper, within the framework of a piecewise-homogeneous body model, with the use of the TLTEWISB, the dynamical response of a system consisting of a substrate and bond and covering layers to a moving load has been investigated.

The numerical results obtained for the critical speed of the load allow us to draw the following conclusions:

in the case $e_1 \quad e_2$ and $e_2 \quad$ l, the presence of the bond layer increases the critical speed;

in the foregoing case, the critical speed decreases with increasing thickness of the bond layer;

in the case $e_1 \quad 1$ and $e_1 \quad e_2$, a critical speed exists if $H \quad H$, i.e., if the bond layer is not too thick;

the prestretching of the layers and substrate increases the critical speed;

the most significant effect on the critical speed is caused by prestretching the covering layer.

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