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On the Condition Number of the Vandermonde Matrix with the Mock-Chebyshev Nodes

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Abstract:
Vandermonde matrices arise frequently in computational mathematics in problems that require polynomial approximation. The polynomial interpolation problem through data given at a set of distinct interpolation points can be expressed in the monomial basis. This gives rise to a linear system of equations with a Vandermonde matrix. A difficulty with a Vandermonde matrix is that this matrix with real nodes is generally quite ill-conditioned [1], even for not very high orders. This ill-conditioning may vary reasonably with the distribution of the points.The general recommendation is to use the highly non-uniform Chebyshev nodes, but the problem remains what to do if experimental data is available only at equally spaced points. In this case, polynomial interpolation has the well-known drawback that polynomial interpolation with equidistant nodes is unreliable due to the Runge phenomenon and is also numerically ill-conditioned. By taking advantage of the optimality of the interpolation processes on Chebyshev-Lobatto nodes, the mock-Chebyshev subset interpolation is one of the best strategies to defeat the Runge phenomenon [2, 3, 4].

In this study, we investigate numerically the condition number of the Vandermonde matrix and find out that the condition number can be reduced by using mock-Chebyshev nodes, as in the case of using Chebyshev-Lobatto nodes. Moreover, we extend our investigation to a Vandermonde-like matrix by using a basis of Chebyshev polynomials and show numerically that, in this case, the condition number of this matrix can be reduced significantly.
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