END4400 – System Dynamics

Week 7 – 20/4/2021

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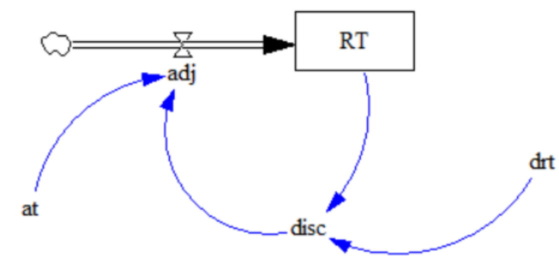
Outline

- Basic Stock – Flow Dynamics
  - Last Time: Goal-Seeking Behavior
- S-Shaped Growth
- Overshoot and Collapse

- Midterm schedule was announced!

  April 28, 2021, 13:30

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)



- model equations
  - differential equation

$$\frac{d(RT)}{dt} = adj$$

  - approximate integral equation

$$RT(t + dt) = RT(t) + adj(t) \times dt$$

  - plug in all variables and rewrite the differential equation

$$\frac{d(RT)}{dt} = \frac{disc}{at}$$

$$\frac{d(RT)}{dt} = \frac{drt - RT}{at}$$

analytical solution? → homework

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- let us try to obtain the analytical solution:

$$\frac{d(RT)}{dt} = \frac{drt - RT}{at}$$

$$drt - RT = u$$

$$d(RT) = -du$$

$$\frac{d(RT)}{drt - RT} = \frac{1}{at} \times dt$$

$$\frac{-du}{u} = \frac{1}{at} \times dt$$

$$\int \frac{du}{u} = \int -\frac{1}{at} \times dt$$

$$lnu = -\frac{1}{at} \times t + C$$

$$u = e^{-\frac{1}{at} \times t + C}$$

$$drt - RT = e^{-\frac{1}{at} \times t} \times C$$

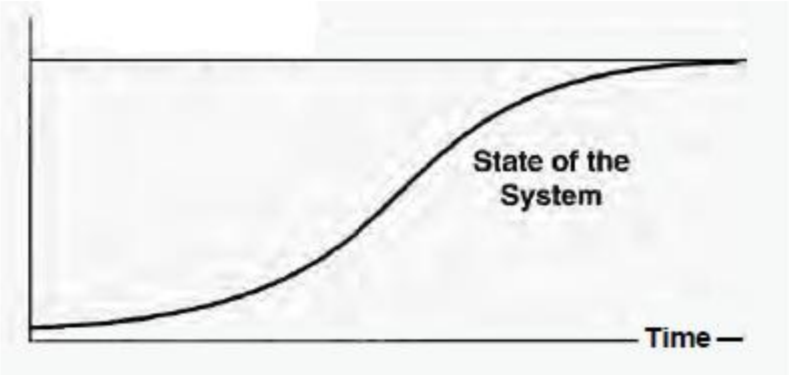
$$RT(t) = drt - e^{-\frac{1}{at} \times t} \times C$$

$$C = drt - RT(0)$$

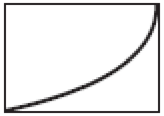
$$RT(t) = drt - \left(drt - RT(0)\right) \times e^{-\frac{1}{at} \times t}$$

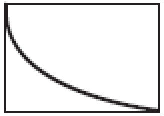
Basic Stock – Flow Dynamics (S-Shaped Growth)

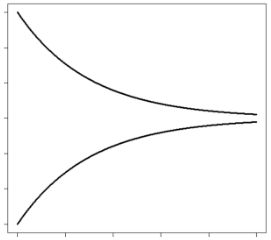
- we will work on the structures exhibiting the behavior given below:



- so far
  - positive feedback loops → exponential growth
  - negative feedback loops → exponential decay
  - adjustment processes → goal-seeking

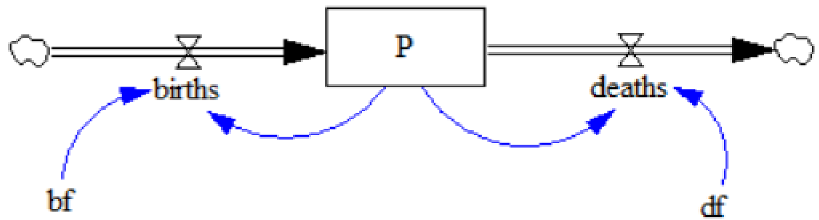






Basic Stock – Flow Dynamics (S-Shaped Growth)

- consider the population model once again
  - linear coupling of a positive and a negative feedback loop

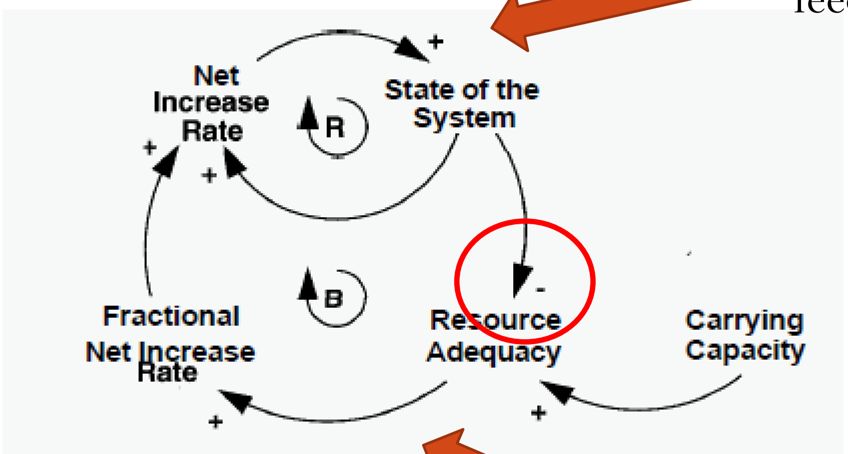


- recall
  - bf > df → births > deaths → exponential growth
    positive feedback loop dominates
  - as a result, population will grow indefinitely
    - realistic?
    - *limits*; food, water, area, etc.

Basic Stock – Flow Dynamics (S-Shaped Growth)

- population model
  - carrying capacity of the nature
- generic structure:

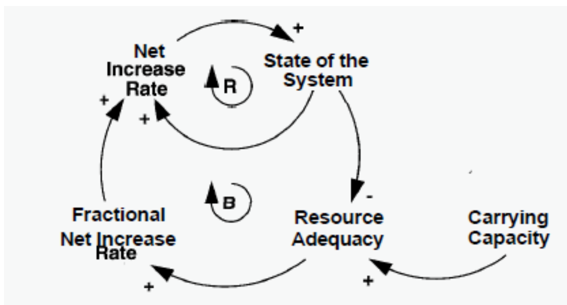
positive (reinforcing) feedback loop

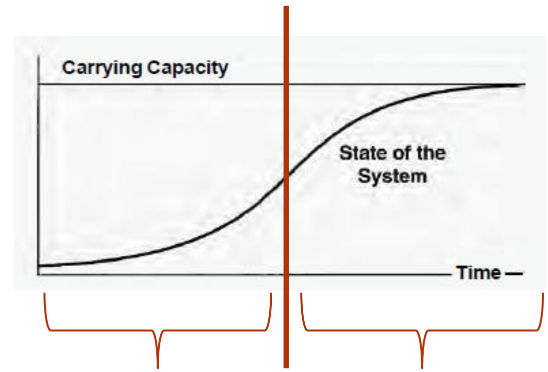


negative (balancing) feedback loop

Basic Stock – Flow Dynamics (S-Shaped Growth)

- recall
  - structure generates the behavior!





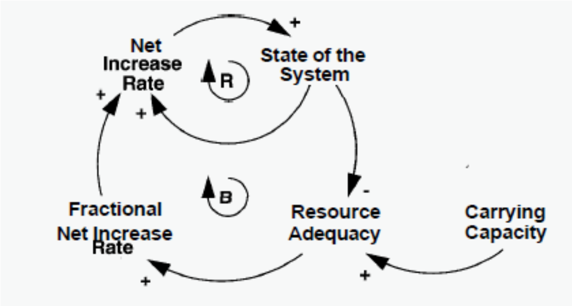
exponential growth goal-seeking

- initially, positive feedback loop dominates
  - exponential growth
- after some time, negative feedback loop dominates
  - goal-seeking

Basic Stock – Flow Dynamics (S-Shaped Growth)

- explanation
  - initially, *resource adequacy* is high, because the *state of the system* is low
  - as the *state of the system* grows, *resource adequacy* drops because the *carrying capacity* (of the environment) is constant
  - a reduction in the *resource adequacy* results in a drop in the *net increase rate*

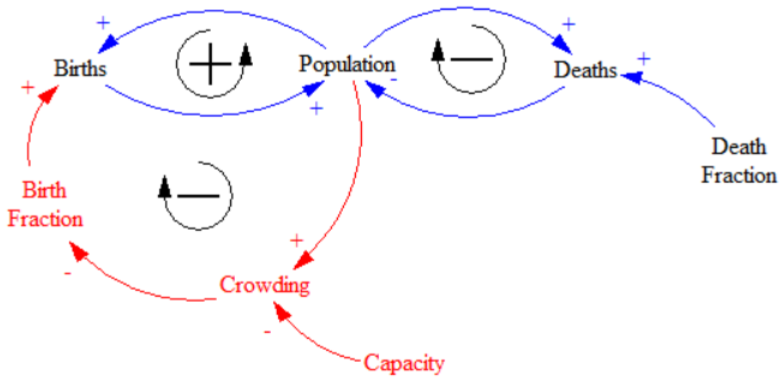
Basic Stock – Flow Dynamics (S-Shaped Growth)



- carrying capacity (of the environment)
  - maximum number of individuals that can be sustained by the environment
  - limiting factor (food, water, area, etc.)
  - cannot be altered quickly and indefinitely
    - technology, culture
  - mostly assumed to be constant

Basic Stock – Flow Dynamics (S-Shaped Growth)

- let us redesign the population model with the carrying capacity
- first consider the causal loop diagram below

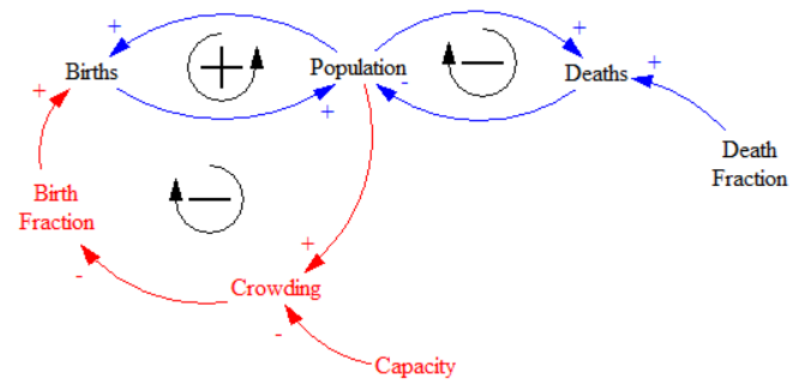


- *crowding* has an effect on the *birth fraction*
- *population* → *crowding* → *birth fraction* → *births* → *population*
  - negative (balancing) feedback loop
  - balances the indefinite growth of the population

- count the number of negative links in the loop
  - if odd → negative feedback loop
  - if even → positive feedback loop

Basic Stock – Flow Dynamics (S-Shaped Growth)

- let us write down the model equations



$$Deaths = Population \times Death\ Fraction$$

$$Births = Population \times Birth\ Fraction$$

$$Crowding = \frac{Population}{Capacity}$$

- what is the formulation for *Birth Fraction*?
  - we know that $Birth\ Fraction = f(Crowding)$
  - when *Crowding* increases, *Birth Fraction* should decrease
  - not formulated analytically

Basic Stock – Flow Dynamics (S-Shaped Growth)

- solution → using table (lookup) functions

$$Birth\ Fraction = f(Crowding)$$

↓

$$Birth\ Fraction = Effect\ of\ Crowding\ on\ Birth\ Fraction$$

- we will specify pairs of points defining this effect

$$(Crowding_1, Birth\ Fraction_1)$$

$$(Crowding_2, Birth\ Fraction_2)$$

$$\vdots \qquad \vdots$$

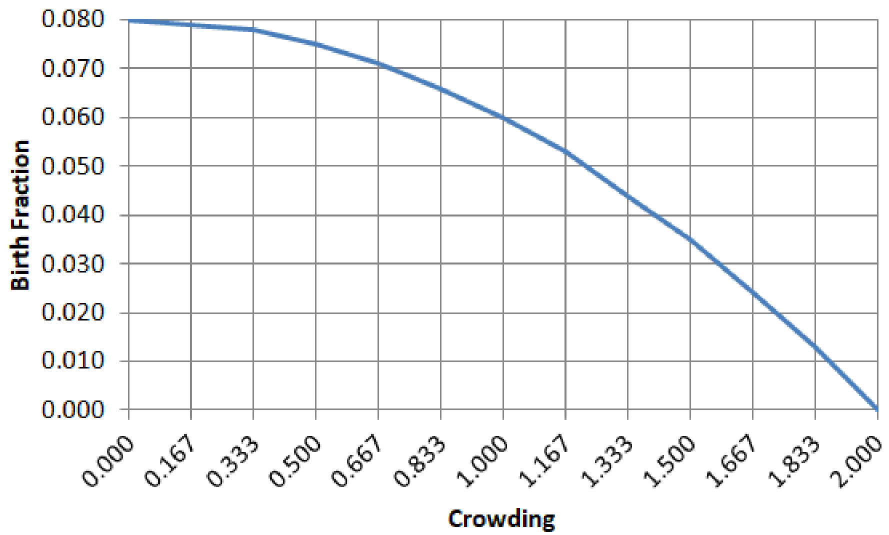
$$(Crowding_n, Birth\ Fraction_n)$$

- we will use linear interpolation for values between the specified points

Basic Stock – Flow Dynamics (S-Shaped Growth)

- Numerical relationship between *Crowding* and *Birth Fraction*:

Crowding	Birth Fraction
0.000	0.080
0.167	0.079
0.333	0.078
0.500	0.075
0.667	0.071
0.833	0.066
1.000	0.060
1.167	0.053
1.333	0.044
1.500	0.035
1.667	0.024
1.833	0.013
2.000	0.000

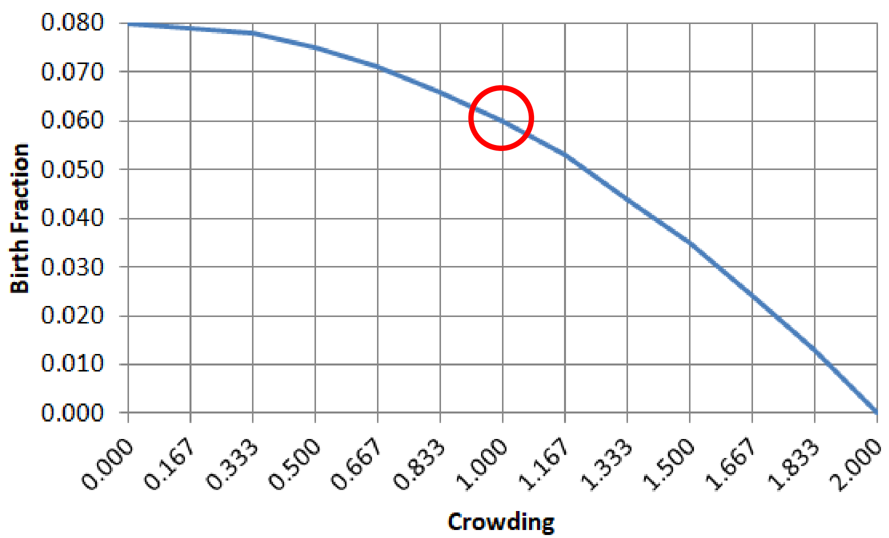


- how to specify these values?
  - experience, literature survey, etc.

Basic Stock – Flow Dynamics (S-Shaped Growth)

$$Crowding = \frac{Population}{Capacity}$$

- when $Crowding = 1.0$, the function yields $Birth\ Fraction = Death\ Fraction$
- when $Crowding > 1.0$ → $Birth\ Fraction < Death\ Fraction$
- when $Crowding < 1.0$ → $Birth\ Fraction > Death\ Fraction$



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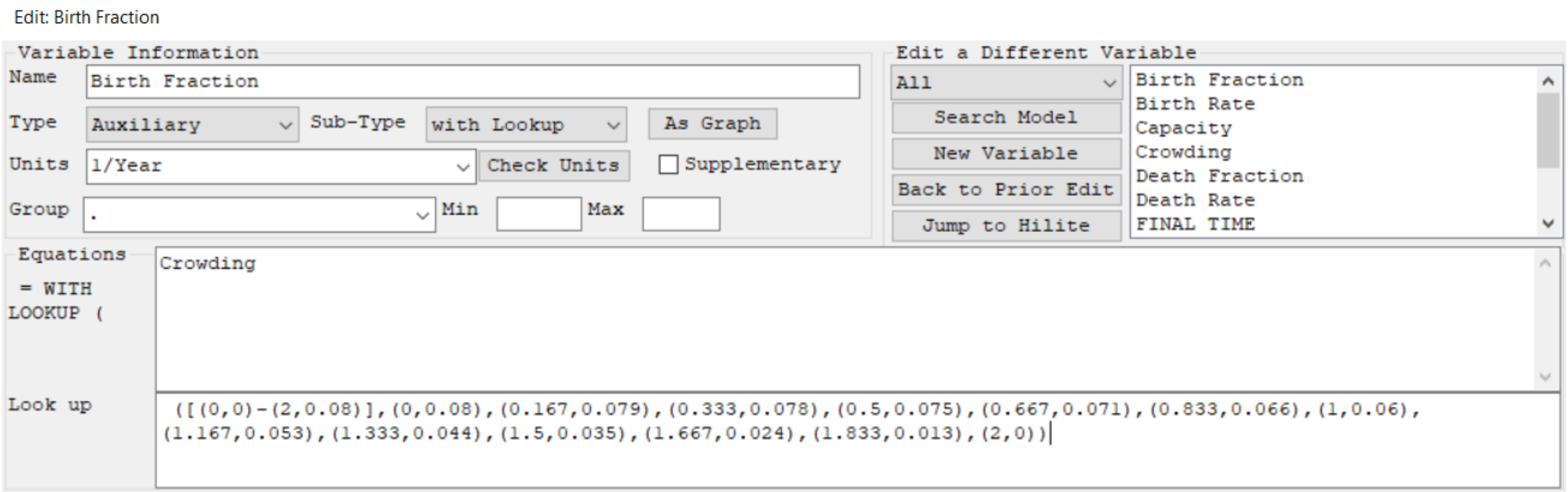
Basic Stock – Flow Dynamics (S-Shaped Growth)

- model parameters:
  - Capacity = 200 [people]
  - Population(0) = 10 [people]
  - Death Fraction = 0.06 [1/Year]
  - Birth Fraction = f(Crowding) [1/Year]
  - Final Time = 300 [Year]
- model equations:
  - Deaths = Death Fraction × Population [people/Year]
  - Births = Birth Fraction × Population [people/Year]
  - Crowding = Population / Capacity [dimensionless]

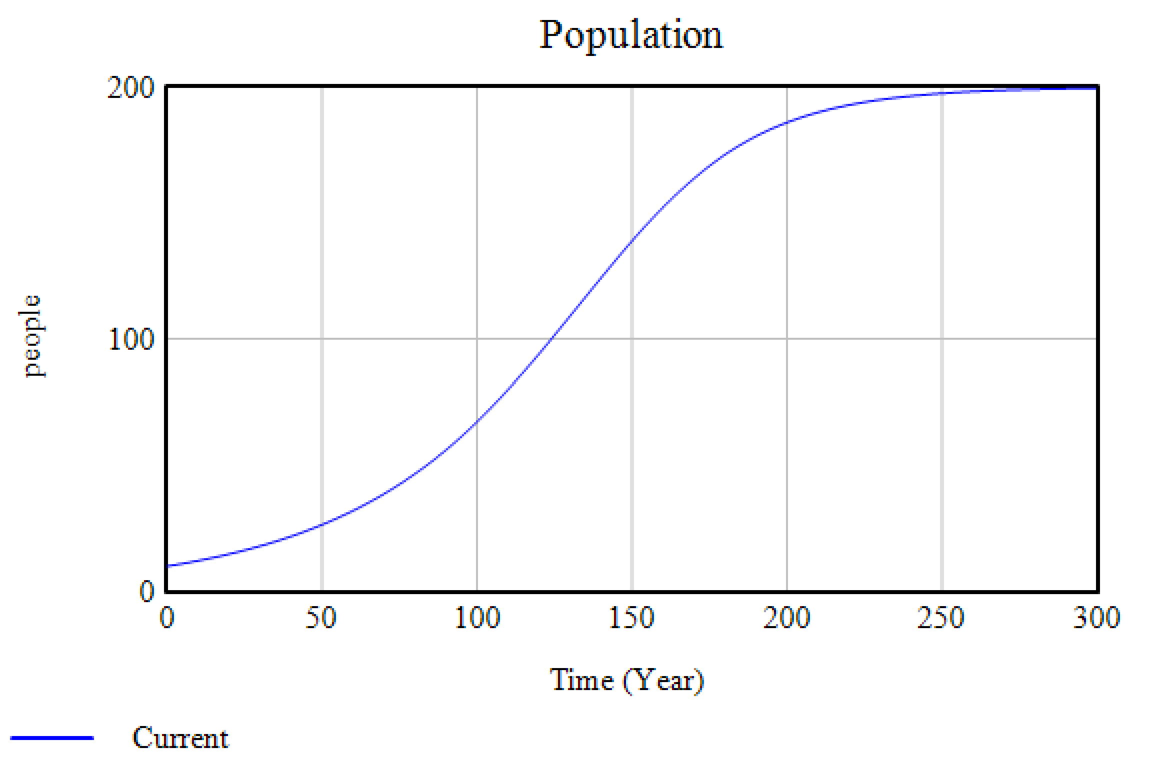
Basic Stock – Flow Dynamics (S-Shaped Growth)

- we need to use "with lookup" function in Vensim



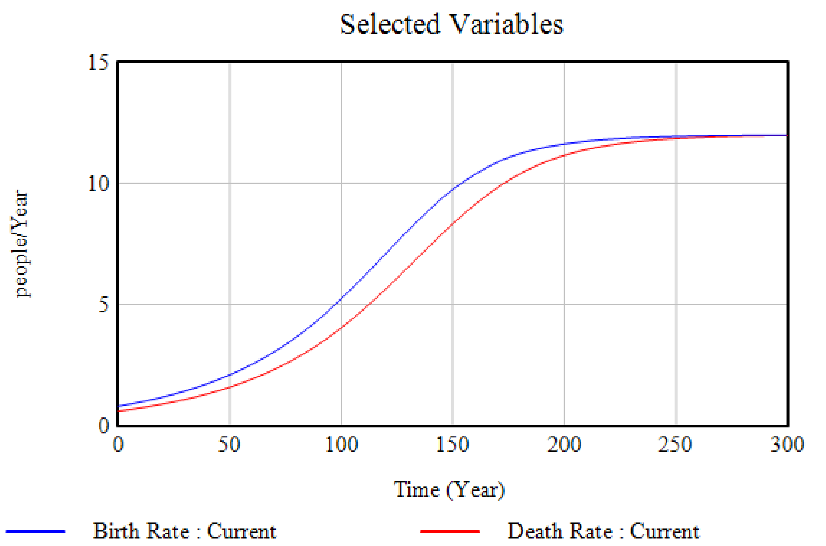
Basic Stock – Flow Dynamics (S-Shaped Growth)

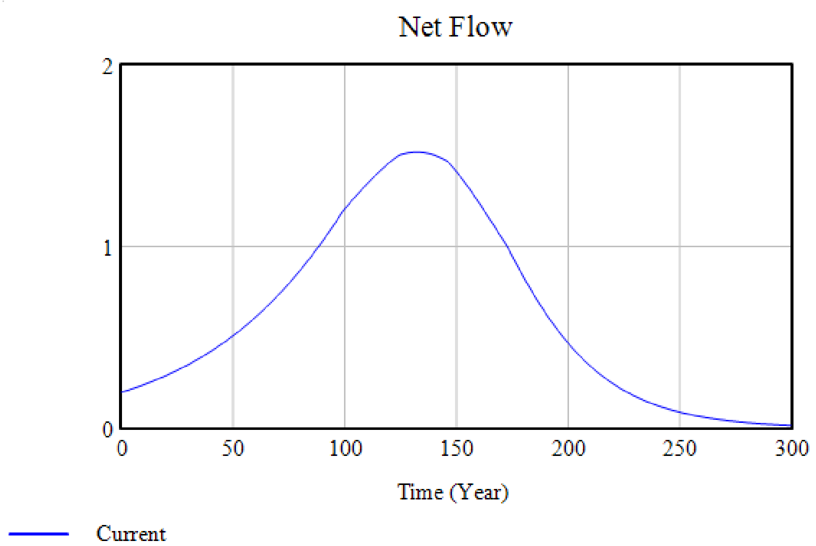
- Population dynamics:



Basic Stock – Flow Dynamics (S-Shaped Growth)

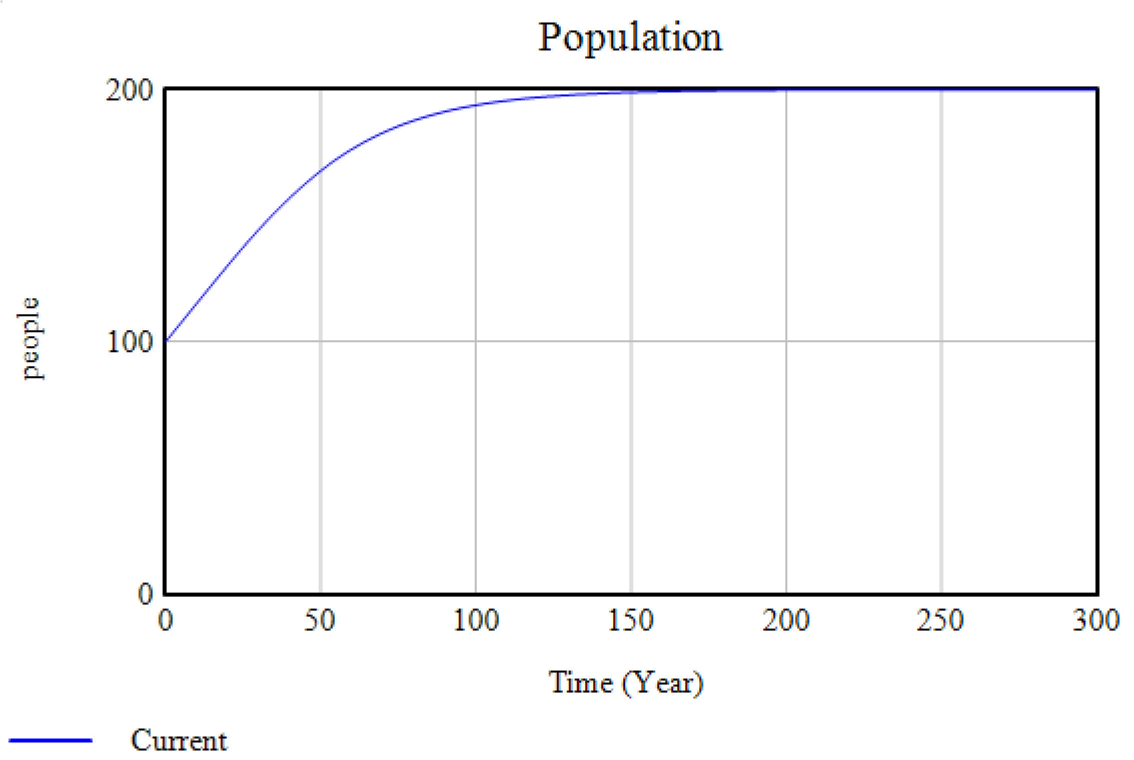
- inflow and outflow dynamics
  - net flow = births - deaths





Basic Stock – Flow Dynamics (S-Shaped Growth)

- if initial population is close to the *Capacity*, we do not observe the exponential growth phase!
- e.g., Population(0) = 100   [people]

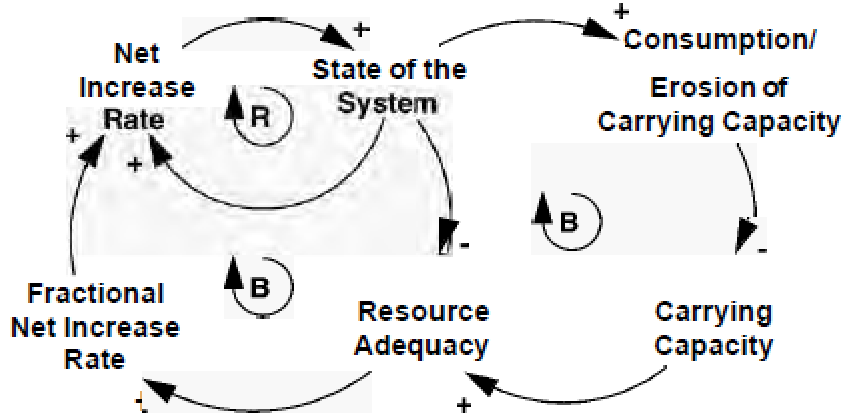


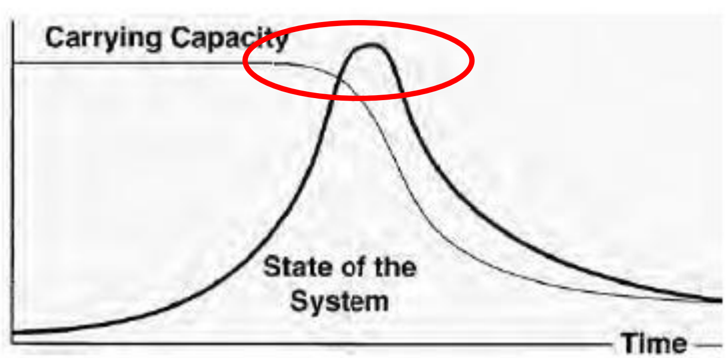
Basic Stock – Flow Dynamics (Overshoot and Collapse)

- in S-Shaped Growth structure, the *carrying capacity* is **constant**
- however, the *carrying capacity* is also reduced through time
- the population of deer in a forest can grow so large that they overbrowse the vegetation → starvation and a sharp decline in the population

Basic Stock – Flow Dynamics (Overshoot and Collapse)

- generic structure and behavior:





Basic Stock – Flow Dynamics

- References

Barlas, Y. "System Dynamics: Systemic Feedback Modeling for Policy Analysis" in Knowledge for Sustainable Development - An Insight into the Encyclopedia of Life Support Systems, UNESCO-EOLSS Publishers, Paris, Oxford, UK. 2002, pp.1131-1175.

Sterman, J. Business Dynamics. Systems Thinking and Modeling for a Complex World. McGraw-Hill, U.S.A., 2000.