END4400 – System Dynamics

Week 5 – 6/4/2021

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Outline

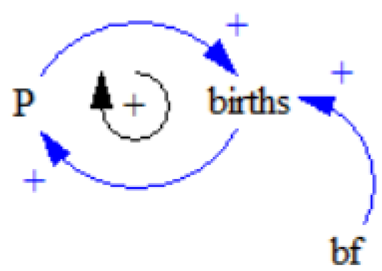
- Basic Stock – Flow Dynamics
- Coupling a Negative and a Positive Feedback Loop
  - Vensim
- Goal-Seeking Behavior

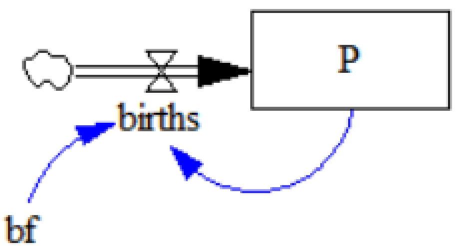
Basic Stock – Flow Dynamics

- consider an SD model with a positive feedback loop
- assume that *births* are proportional to the population (*P*)

  $births = P * bf$

  - *bf*: birth fraction



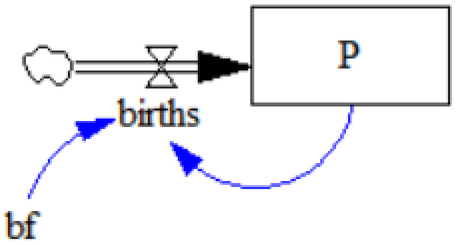


- units:
  - *P*: people
  - *births*: people/time
  - *bf*: 1/time

Basic Stock – Flow Dynamics

- write down the differential equation

$$\frac{dP}{dt} = births$$



- write down the integral equation

$$P(t) = P(0) + \int_0^t births(s)ds$$

- plug in all variables

$$\frac{dP}{dt} = P * bf$$

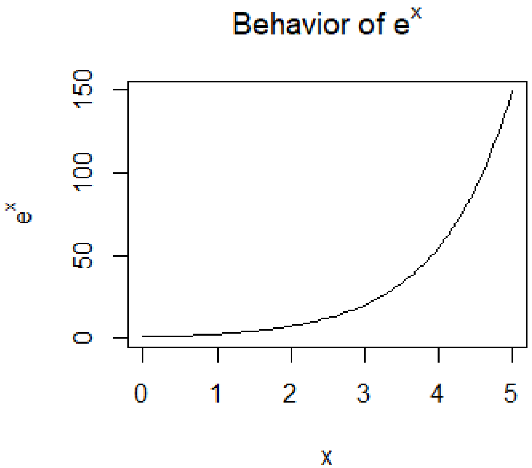
$$\frac{dP}{P} = bf * dt$$

$$\int \frac{dP}{P} = \int bf * dt$$

$$lnP = bf * t + C$$

$$P(t) = e^{bf*t+C} = e^{bf*t}e^{C} = e^{bf*t}C$$

behavior?



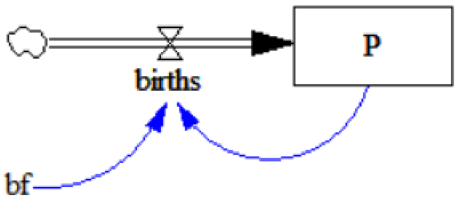
Basic Stock – Flow Dynamics

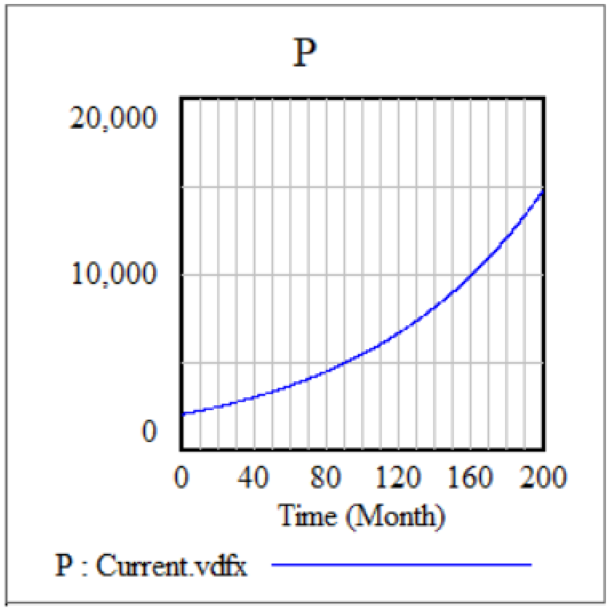
- assume
  - P(0) = 2000 (people)
  - bf = 0.01 (1/time)

$$P(0) = e^{0.01*0}C = 2000$$

$$C = 2000$$

$$P(t) = 2000 * e^{0.01*t}$$





Basic Stock – Flow Dynamics

- what kind of behavior do you expect to see when we have *births* on the *x*-axis and *P* on the *y*-axis?
  - reconsider the differential equation again;

$$\frac{dP}{dt} = P * bf$$

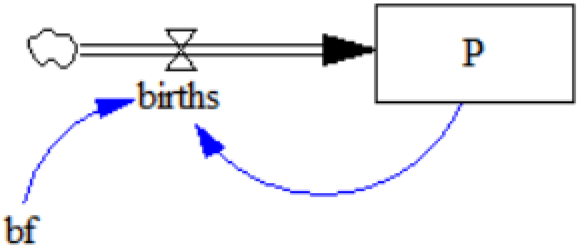
the rate of change (inflow) is a linear function of the state of the system

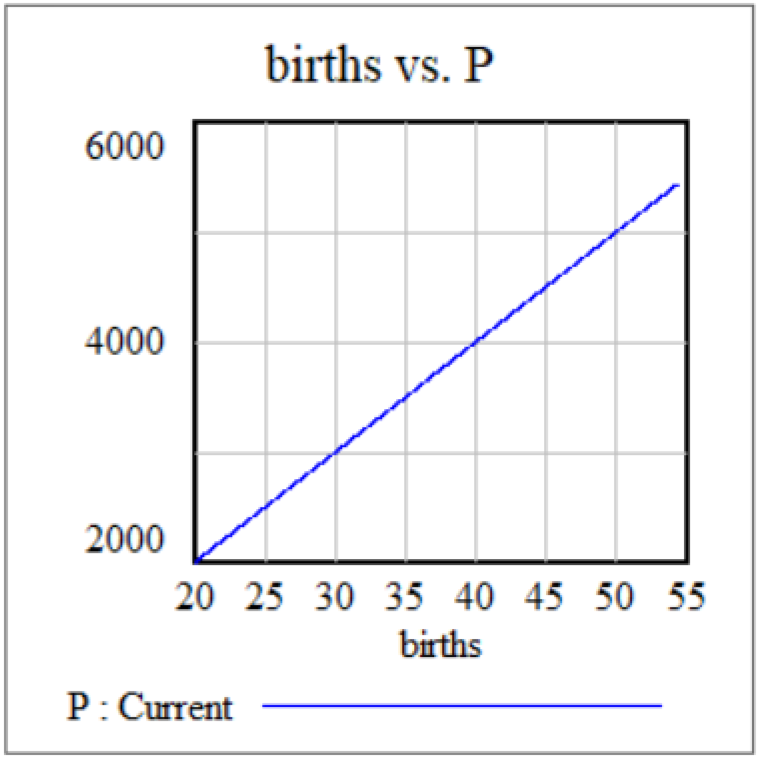
explains how the state of the system changes over time

  - answer will be given next week! try to guess

Basic Stock – Flow Dynamics

- The relationship between *births* and *P* is linear!

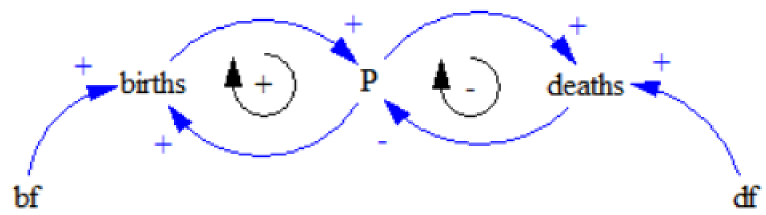




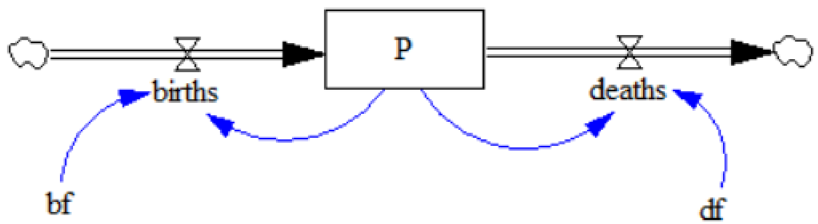
$$births = P * bf$$

Basic Stock – Flow Dynamics

- coupling positive and negative feedback loops
  - population model with births and deaths
- causal loop diagram



- stock-flow diagram



Basic Stock – Flow Dynamics

- write down the differential equation

$$\frac{dP}{dt} = births - deaths$$

- write down the integral equation

$$P(t) = P(0) + \int_0^t \left(births(s) - deaths(s)\right) * ds$$

- write down the approximate integral equation

$$P(t + dt) = P(t) + \underbrace{[births(t) - deaths(t)]}_{\text{net flow}} * dt$$

Basic Stock – Flow Dynamics

- solve the differential equation

$$\frac{dP}{dt} = P * (bf - df)$$

↓

$$\frac{dP}{P} = (bf - df) * dt$$

↓

$$\int \frac{dP}{P} = \int (bf - df) * dt$$

↓

$$lnP = (bf - df) * t + C$$

$$P(t) = e^{(bf-df)*t+C} = e^{(bf-df)*t}e^{C} = e^{(bf-df)*t}C$$

assume

P(0) = 3000

bf = 0.03

df = 0.01

$$P(t) = 3000 * e^{0.02*t}$$

↓

exponential growth

Basic Stock – Flow Dynamics

- Let us consider the more general case

$$\frac{dP}{dt} = P * \underbrace{(bf - df)}_{k}$$

- k < 0 → bf < df → births < deaths → exponential decay (decline)

  negative feedback loop dominates
- k > 0 → bf > df → births > deaths → exponential growth

  positive feedback loop dominates
- k = 0 → bf = df → births = deaths → equilibrium

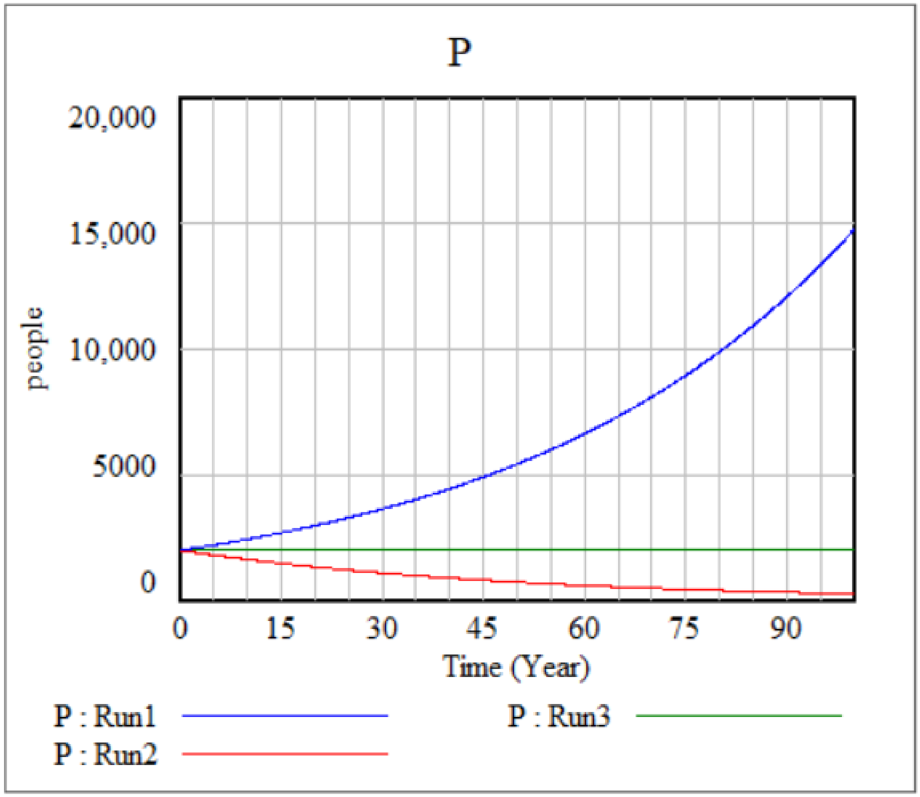
  no loop domination

Basic Stock – Flow Dynamics

- Let us do this as an example on Vensim.
  - P(0) = 2000 people
  - Final Time = 100 years
  - dt = 0.0625
  - Consider the three cases given below;
    1. bf = 0.03, df = 0.01
    2. bf = 0.01, df = 0.03
    3. bf = 0.01, df = 0.01
  - Plot the dynamics of P under these three setting on a single graph

Basic Stock – Flow Dynamics

- At the end, we are going to obtain the following graph:

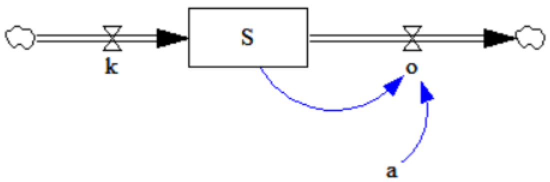


Basic Stock – Flow Dynamics

- positive feedback loop
  - exponential growth
- negative feedback loop
  - exponential decline
- coupled positive and negative feedback loops
  - exponential growth
  - exponential decline
  - constant

Basic Stock – Flow Dynamics

- coupling a constant inflow and a negative feedback loop
  - stock flow diagram



  - differential equation

$$\frac{dS}{dt} = k - o$$

  - plug in all variables and rewrite the differential equation

$$\frac{dS}{dt} = k - a * S$$

Basic Stock – Flow Dynamics

- analytical solution → not very straightforward (but doable)
  - change of variables

$$\frac{dS}{k - a * S} = dt$$

$$k - a * S = u$$

$$-a * dS = du$$

$$dS = \frac{-du}{a}$$

Plug in this to the differential equation

$$-\frac{1}{a} * \frac{du}{u} = dt$$

$$\int -\frac{1}{a} * \frac{du}{u} = \int dt$$

$$-\frac{1}{a} * \int \frac{du}{u} = \int dt$$

$$-\frac{1}{a} * lnu = t + C$$

$$lnu = -a * t + C$$

$$\ln(k - a * S) = -a * t + C$$

$$e^{(-a*t+C)} = k - a * S$$

$$S(t) = \frac{k}{a} - C * \frac{e^{-a*t}}{a}$$

$$C = k - a * S(0)$$

$$S(t) = \frac{k}{a} - \frac{e^{-a*t} * \left(k - a * S(0)\right)}{a}$$

Basic Stock – Flow Dynamics

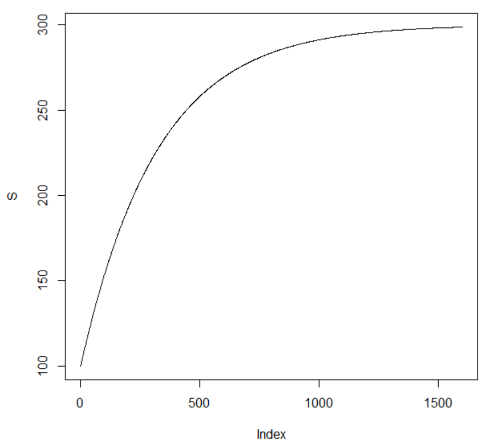
- Write down the solution for the values given below:
  - S(0) = 100
  - k = 15
  - a = 0.05

$$S(t) = 300 - 200 * e^{-0.05*t}$$

- Let us use an R function to simulate

```
S <- function(a, k, S0, t){
    return(k/a - exp(-t*a)*(k-a*S0)/a)
}
```

For a given t, it calculates the value of S

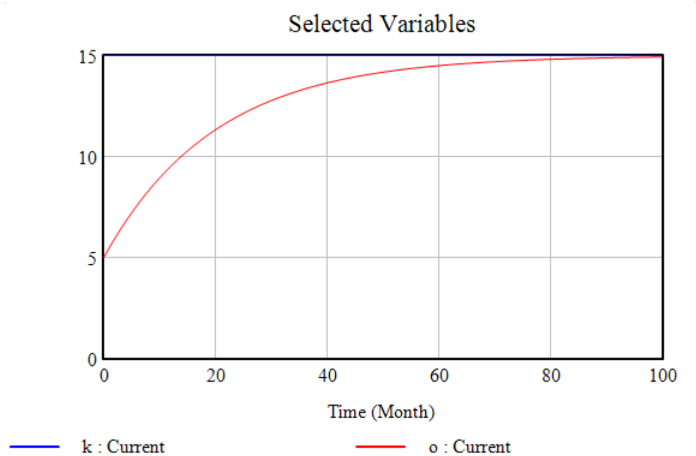


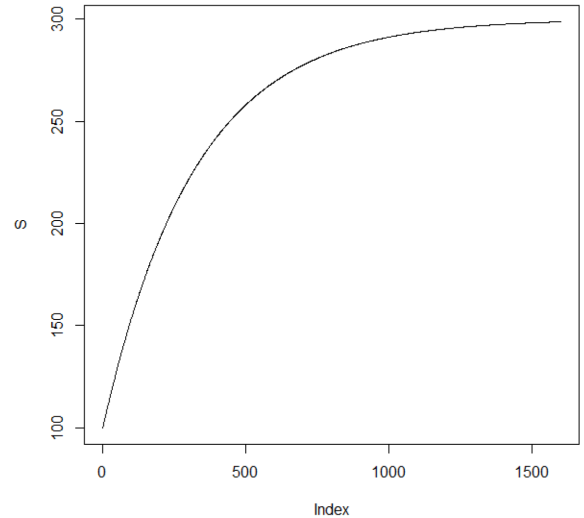
Why are we observing such a dynamic behavior?

Negative feedback loop → balance

Basic Stock – Flow Dynamics

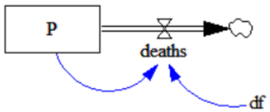
- Consider the values of both inflow and outflow initially;
  - $k = 15$
  - $o = S(0) \times a = 100 \times 0.05 = 5$
- Initially; inflow > outflow → stock increases
- Inflow is constant but outflow exhibits a negative exponential behavior.



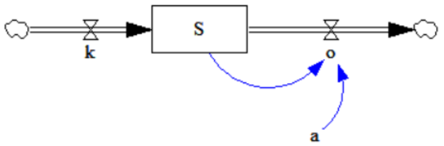


Basic Stock – Flow Dynamics

- moral of the story:
  - consider the stock-flow diagram with a single negative feedback loop
    - easy to solve analytically
    - easily-predictable dynamic behavior (of the stock variable)



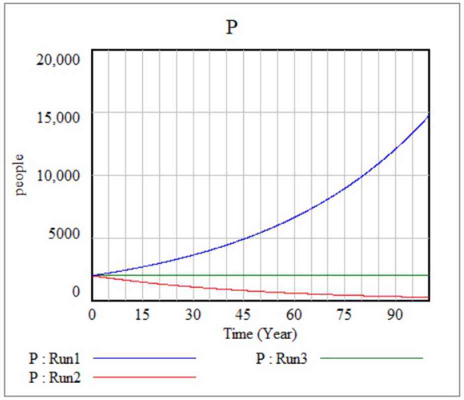
  - we just added a constant inflow to a negative feedback loop
    - not very easy to solve analytically
    - hard to predict the dynamic behavior



  - consider an SD model with many feedback loops
    - (nearly) impossible to guess the potential dynamics

Basic Stock – Flow Dynamics

- homework:
  - please build the stock-flow diagram and experiment with the different values of *a*
  - try to explore the possible dynamics that can be generated by the model
  - similar to the population model with births and deaths, can we write down specific conditions for the values of *a*?
  - *hint*: try to discover the equilibrium case first, then consider other possible cases
  - try to generate a plot like this



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- assume you are in a cool/cold room on a winter day
- the room is equipped with a heating system
- you want to heat the room
- think about your heating system at home
  - it has a "target" temperature value that you set (e.g., 30 °C)
  - it aims to bring the room temperature to the target (desired) temperature level
- a basic control (regulation) problem
  - bring the state of the system to the target (desired) level
  - this does not occur immediately; it takes some time!

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- let us try to define the required variables
  - “room temperature”
  - “desired room temperature”
  - “discrepancy”
  - “adjustment”
  - “adjustment time”

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- think about the meanings of the variables
  - "room temperature"
    current temperature of the room
  - "desired room temperature"
    the target temperature level
  - "discrepancy"
    the difference between "room temperature" and "desired room temperature"
  - "adjustment"
    the corrective action
  - "adjustment time"
    how fast you can close the gap between "room temperature" and "desired room temperature"

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- let us try to draw the causal loop diagram first

room temperature

desired room temperature

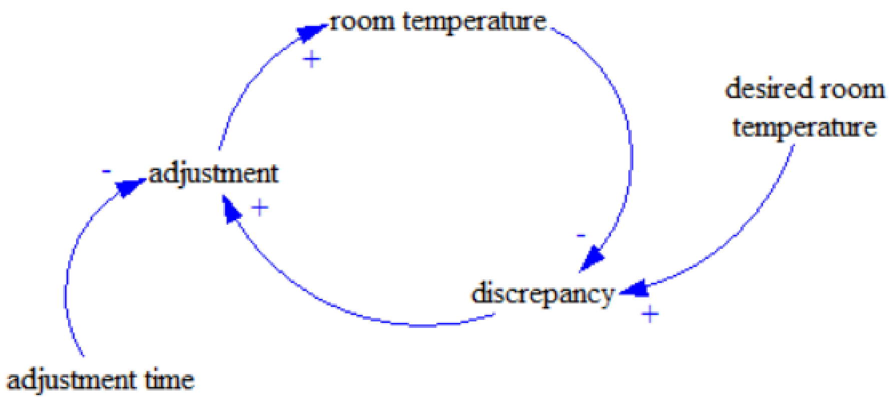
adjustment

discrepancy

adjustment time

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

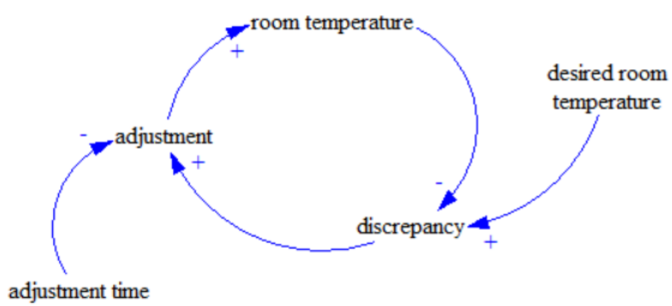
- how to determine the loop polarity?



- count the number of negative links in the loop
  - if odd → negative feedback loop
  - if even → positive feedback loop

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- another way to determine the loop polarity
  - change the value of one of the variables in the loop
  - then trace the effect of this change

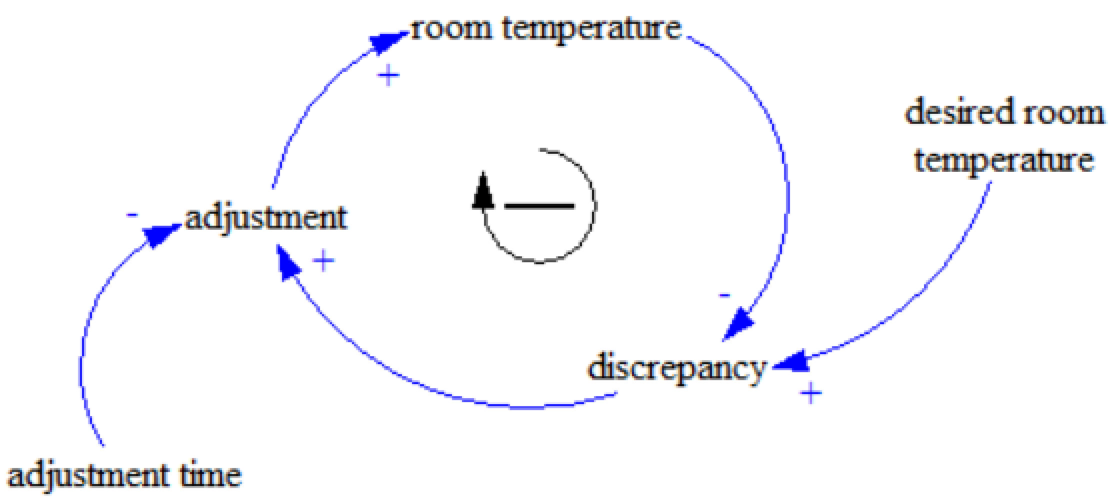


- adjustment ⬆ → room temperature ⬆
- room temperature ⬆ → discrepancy ⬇
- discrepancy ⬇ → adjustment ⬇

} negative feedback loop

Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- final causal loop diagram



Basic Stock – Flow Dynamics (Goal-Seeking Behavior)

- drawing the stock-flow diagram
- stock variable(s)?
  - what are the accumulations in the model?
    - "room temperature"
- flow(s)?
  - how system states change?
    - "adjustment"
- all the remaining variables are auxiliary variables

"room temperature"
"desired room temperature"
"discrepancy"
"adjustment"
"adjustment time"

Basic Stock – Flow Dynamics

- References

Barlas, Y. "System Dynamics: Systemic Feedback Modeling for Policy Analysis" in Knowledge for Sustainable Development - An Insight into the Encyclopedia of Life Support Systems, UNESCO-EOLSS Publishers, Paris, Oxford, UK. 2002, pp.1131-1175.

Sterman, J. Business Dynamics. Systems Thinking and Modeling for a Complex World. McGraw-Hill, U.S.A., 2000.