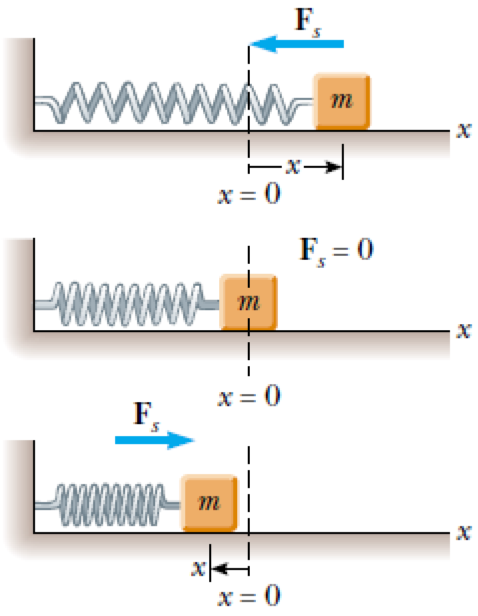
Oscillatory Motion

Periodic motion is motion of an object that regularly repeats—the object returns to a given position after a fixed time interval.

A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium position. If this force is always directed toward the equilibrium position, the motion is called *simple harmonic motion*.



Motion of an Object Attached to a Spring

When the spring is neither stretched nor compressed, the block is at the position called the equilibrium position of the system, which we identify as *x* = 0. We know from experience that such a system oscillates back and forth if disturbed from its equilibrium position.

Hooke's law

this a re $F_s = -kx$ ce because it is always directed toward the equilibrium position and therefore *opposite* the displacement from equilibrium.

$$\Sigma F_x = ma_x \qquad -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

An object moves with ***simple harmonic motion*** whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

Mathematical Representation of Simple Harmonic Motion

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

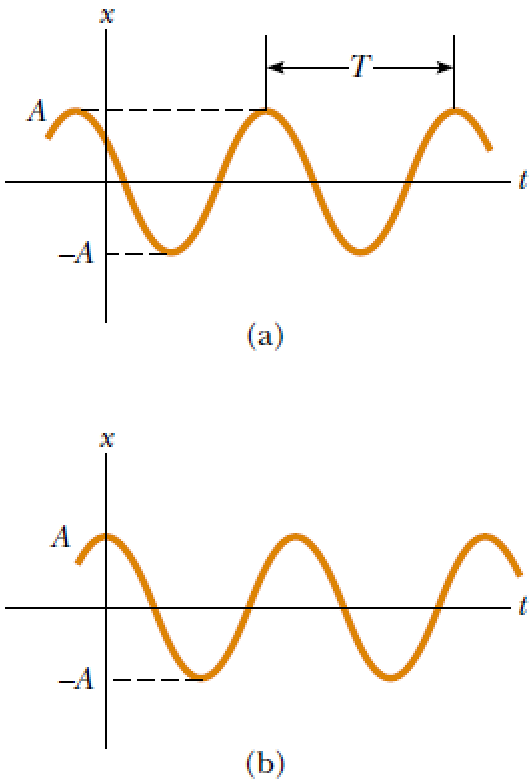
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

The constant angle ϕ is called the **phase constant** (or initial phase angle) and, along with the amplitude A, is determined uniquely by the position and velocity of the particle at $t = 0$. If the particle is at its maximum position $x = A$ at $t = 0$, the phase constant is $\phi = 0$ and the graphical representation of the motion is shown in Figure 15.2b. The quantity $(\omega t + \phi)$ is called the **phase** of the motion. Note that the function $x(t)$ is periodic and its value is the same each time ωt increases by 2π radians.

A, called the amplitude of the motion, is simply the maximum value of the position of the particle in either the positive or negative ***x*** direction. The constant ω is called the angular frequency,



(a) An x-vs.-t graph for an object undergoing simple harmonic motion. The amplitude of the motion is A, the period is T, and the phase constant is ϕ. (b) The x-vs.-t graph in the special case in which $x = A$ at $t = 0$ and hence $\phi = 0$.

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

.

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

A, called the amplitude of the motion, is simply the maximum value of the position

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}\,A$$

$$a_{max} = \omega^2 A = \frac{k}{m}\,A$$

.Ex/ An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation

$$x = (4.00\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

where t is in seconds and the angles in the parentheses are in radians.

(A) Determine the amplitude, frequency, and period of the motion.

(B) Calculate the velocity and acceleration of the object at any time t.

(C) Using the results of part (B), determine the position, velocity, and acceleration of the object at $t = 1.00$ s.

(D) Determine the maximum speed and maximum acceleration of the object.

(A)

$x = A\cos(\omega t + \phi)$, we see that $A =$ 4.00 m and $\omega = \pi$ rad/s. Therefore, $f = \omega/2\pi = \pi/2\pi =$ 0.500 Hz and $T = 1/f =$ 2.00 s.

. **(B)**

$$v = \frac{dx}{dt} = -(4.00\ \text{m/s})\sin\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi\ \text{m/s})\sin\left(\pi t + \frac{\pi}{4}\right)$$

$$a = \frac{dv}{dt} = -(4.00\pi\ \text{m/s})\cos\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi^2\ \text{m/s}^2)\cos\left(\pi t + \frac{\pi}{4}\right)$$

(C)

$$x = (4.00\ \text{m})\cos\left(\pi + \frac{\pi}{4}\right) = (4.00\ \text{m})\cos\left(\frac{5\pi}{4}\right)$$

$$= (4.00\ \text{m})(-0.707) = -2.83\ \text{m}$$

$$v = -(4.00\pi\ \text{m/s})\sin\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi\ \text{m/s})(-0.707) = 8.89\ \text{m/s}$$

$$a = -(4.00\pi^2\ \text{m/s}^2)\cos\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi^2\ \text{m/s}^2)(-0.707) = 27.9\ \text{m/s}^2$$

. **(D)**

$$v_{\text{max}} = 4.00\pi\ \text{m/s} = 12.6\ \text{m/s}$$

$$a_{\text{max}} = 4.00\pi^2\ \text{m/s}^2 = 39.5\ \text{m/s}^2$$

(E)

The position at $t = 0$ is

$$x_i = (4.00\ \text{m})\cos\left(0 + \frac{\pi}{4}\right) = (4.00\ \text{m})(0.707) = 2.83\ \text{m}$$

In part (C), we found that the position at $t = 1.00$ s is -2.83 m; therefore, the displacement between $t = 0$ and $t = 1.00$ s is

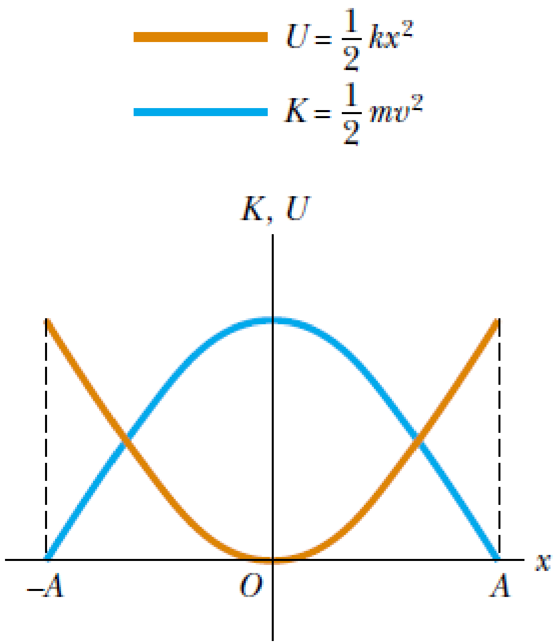
$$\Delta x = x_f - x_i = -2.83\ \text{m} - 2.83\ \text{m} = -5.66\ \text{m}$$

. Energy of the Simple Harmonic Oscillator

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 =$$

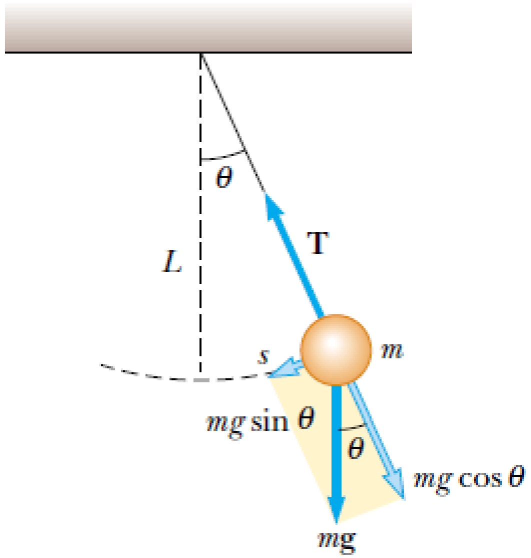
$$\tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}m\omega^2 A^2 = \tfrac{1}{2}m\frac{k}{m}A^2 = \tfrac{1}{2}kA^2$$

$$E = \tfrac{1}{2}kA^2$$



. The Pendulum

- ***simple pendulum***



$$F_t = -mg\sin\theta = m\frac{d^2s}{dt^2}$$

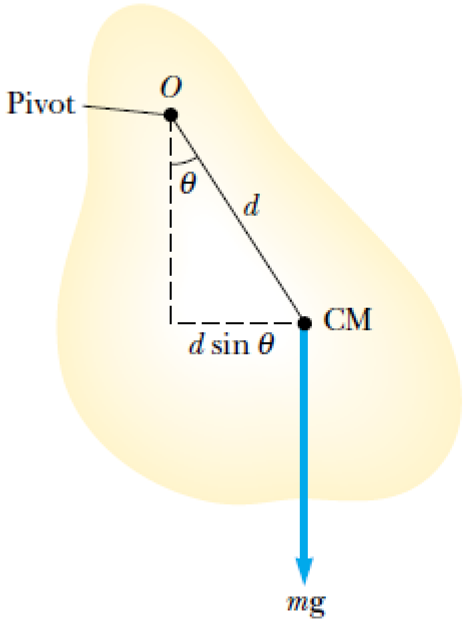
$$s = L\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

For small angles $\sin\theta \approx \theta$; $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

. - ***Physical Pendulum***



$$\Sigma\tau = I\alpha$$

$$-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

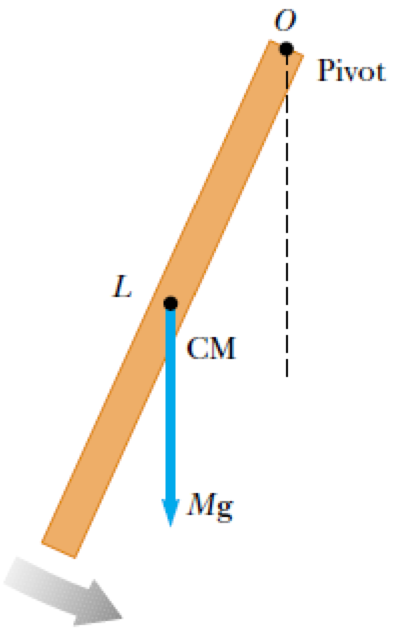
For small angles $\sin\theta \approx \theta$;

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

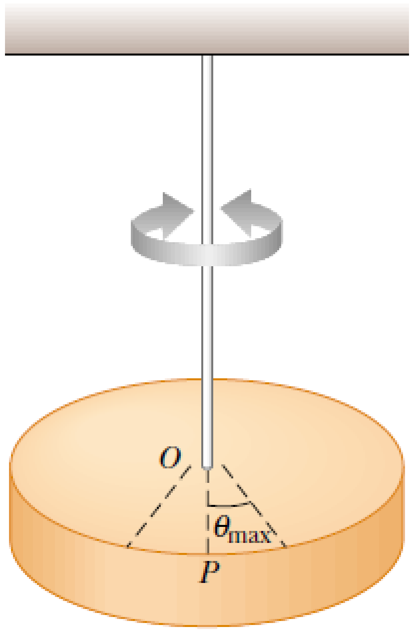
· A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane (Fig. 15.19). Find the period of oscillation if the amplitude of the motion is small.



$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

. - ***Torsional Pendulum***



$$\tau = -\kappa\theta$$

κ *torsion constant*

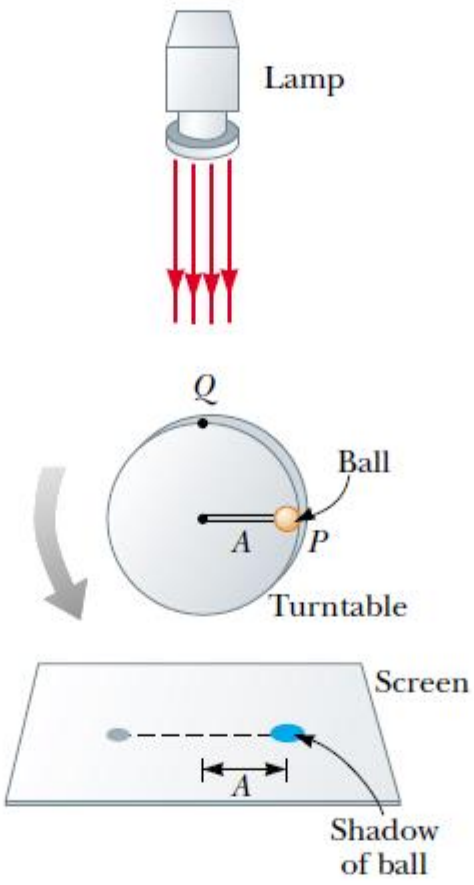
$$\tau = -\kappa\theta = I\frac{d^2\theta}{dt^2}$$

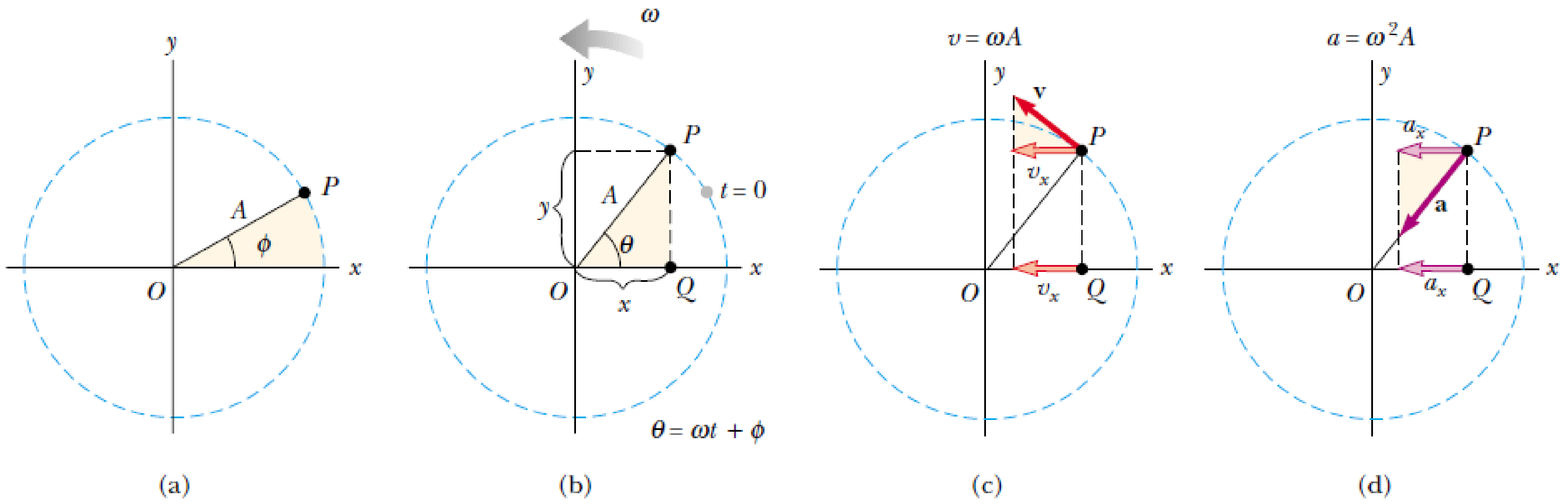
$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

$$\omega = \sqrt{\kappa/I}$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

. Comparing Simple Harmonic Motion with Uniform Circular Motion





simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.