1) **(15 pts) D is a region on the complex plane.**

   **Let $D^* = \{\bar{z} \mid \forall z \in D\}$. If $f(z): D \to \mathbb{C}$ is a differentiable function, $f^*: D^* \to \mathbb{C}$, show**

   **that $f^*(z) = \overline{f(\bar{z})}$ is differentiable.**

2) **Let $u(x,y), v(x,y)$ harmonic functions in region D on the plane. And let**

   $v$ is a harmonic conjugate function of $u$.

   a) **(10 pts) Show that $u^2 - v^2$ is a harmonic function in D.**

      **And show that $2uv$ is a harmonic conjugate of $u^2 - v^2$ .**

   b) **(10 pts) Find the harmonic conjugate function of $u^3 - 3uv^2$.**

3) a) **(8 pts) Find the values of $\left(-1 - \sqrt{3}i\right)^{1/4}$.**

   b) **(7 pts) Sketch the following region on the complex plane**

   $|z - 1 + i| < 2, Arg(z) > \dfrac{\pi}{2}$

*I wish you success.*

**4) (15 pts) Recall that** $\cos z$ **for** $z \in C$ **is defined by** $\cos z = \dfrac{e^{iz} + e^{-iz}}{2}$

Find all complex numbers $z$ satisfying the equation $\cos z = 3$.

**5) (15 pts) Compute the principle value of** $\left(\sqrt{3} - i\right)^{i}$.

**6) Let** $\gamma$ **be the positively oriented circle with radius 1 and center** $i$.

Evaluate the following counter integrals

**a) (10 Pts)** $\displaystyle\oint_{\gamma} \overline{z}\, dz = ?$

**b) (10 Pts)** $\displaystyle\oint_{\gamma} \dfrac{dz}{z^2 - 2} = ?$

*I wish you success.*