1

A rod of length 30.0 cm has linear density (mass-perlength) given by

$$\lambda = 50.0\ \mathrm{g/m} + 20.0x\ \mathrm{g/m^2},$$

where x is the distance from one end, measured in meters.

(a) What is the mass of the rod? (b) How far from the $x = 0$ end is its center of mass?

a) $$M = \int dm = \int_0^{0,3} \lambda\, dx = \int_0^{0,3} [50 + 20x]\, dx$$

$$M = \left[50x + 20\frac{x^2}{2}\right]_0^{0,3} = 15,9\ g.$$

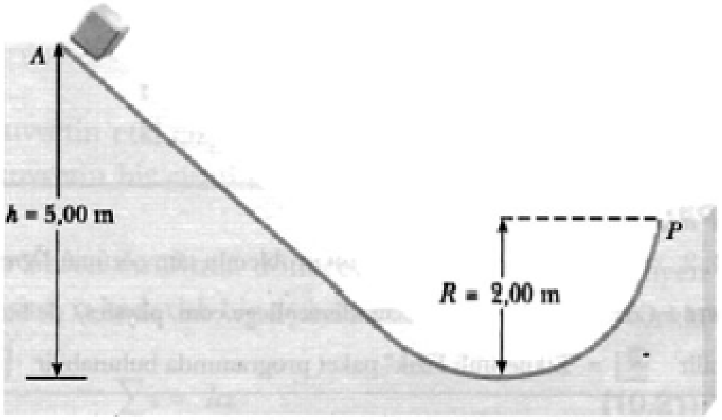
b) $$x_{km} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^{0,3} x(\lambda\, dx)$$

$$x_{km} = \frac{1}{M}\int_0^{0,3} x(50 + 20x)\, dx = \frac{1}{M}\int_0^{0,3} (50x + 20x^2)\, dx$$

$$x_{km} = \frac{1}{15,9}\left[50\frac{x^2}{2} + 20\frac{x^3}{3}\right]_0^{0,3} = 0,153\ m.$$

2

A 6.0 kg block is released from A on the frictionless track shown in Figure. Determine the radial and tangential components of acceleration for the block at P.



$h = 5\ m$

$R = 2\ m$

$m = 6\ kg$

$\mu = 0 \Rightarrow \Delta E = 0$

$E_A = E_P$

$\cancel{K_A} + U_A = K_P + U_P$

$mgh = \frac{1}{2} m V_P^2 + mgR$

$V_P^2 = 2g(h-R) = 60\ m^2/s^2$

Centripetal acceleration at point P: $a_r = \frac{V_P^2}{R} = \frac{60}{2} = 30\ m/s^2$

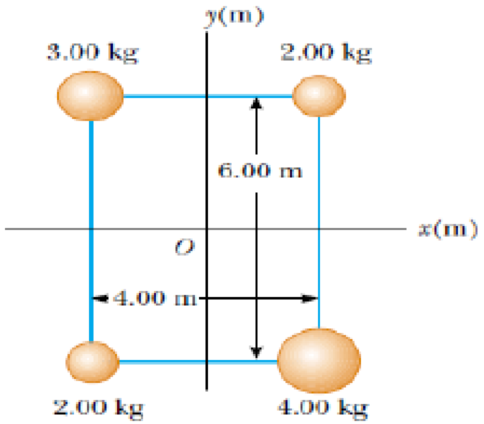
Tangential acceleration at point P: $a_t = g = 10\ m/s^2$

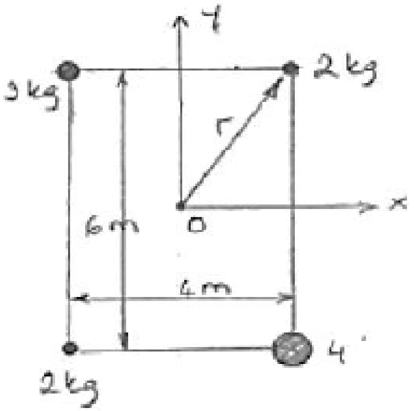
3

The four particles in Figure are connected by rigid rods of neglibigle mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate

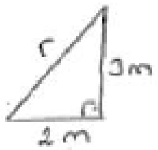
a) The moment of inertia of the system about the z axis and

b) The rotational energy of the system.





a) The distances of the masses to the rotation axis is same. $r_1=r_2=r_3=r_4=r$



$r = \sqrt{2^2+3^2} = \sqrt{13}$ m

$I_{sistem} = \Sigma m_i \cdot r_i^2$ (bağlantı çubuklarının kütleleri ihmal ediliyor)

$= r^2 (m_1+m_2+m_3+m_4) = 13(2+2+3+4)$

$= \underline{143}$ kg m^2 The moment of inertia of the system about the z axis.

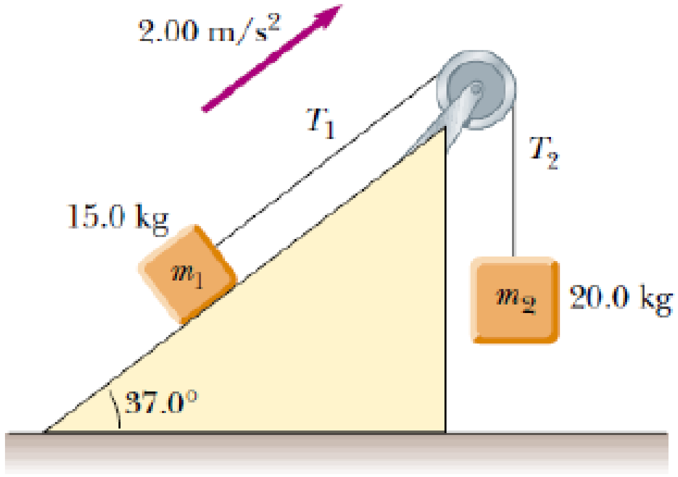
b) $K_d = \frac{1}{2} I w^2 = \frac{1}{2} I w^2 = \frac{1}{2}(143 \text{ kgm}^2)(6 \text{ rad/s}^2) = 2574$ Joule

4

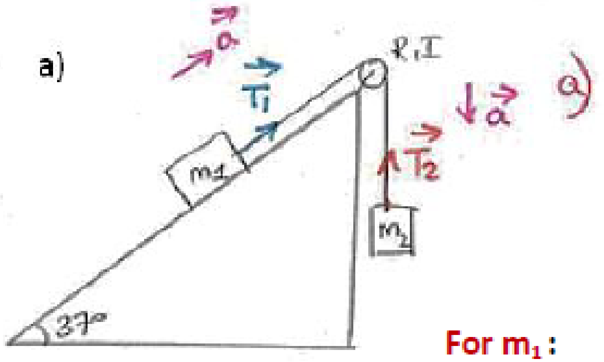
Two blocks, as shown in Figure, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia *I*. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s^2.

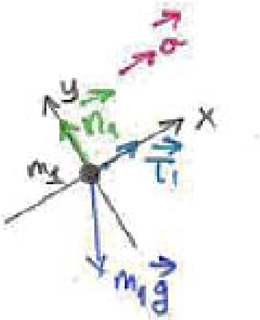
(a) Determine T_1 and T_2, the tensions in the two parts of the string.

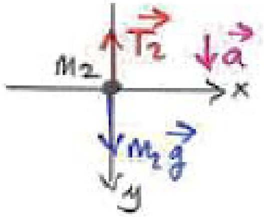
(b) Find the moment of inertia of the pulley.



9. a)







$m_1 = 15$ kg

$m_2 = 20$ kg

$a = 2$ m/s^2

$R = 0{,}25$ m

For m₁ :

$\Sigma F_x = m_1 . a$

$T_1 - m_1 g \sin 37° = m_1 a$

$T_1 = m_1 (a + g \sin 37°)$

$T_1 = 15 [2 + 10 \sin 37°]$

$\boxed{T_1 = 120 (N)}$

For m₂ :

$\Sigma F_y = m_2 a$

$m_2 g - T_2 = m_2 a$

$T_2 = m_2 (g - a)$

$T_2 = 20 (10 - 2)$

$\boxed{T_2 = 160 N}$

b)

$$\Sigma \tau_o = I . \alpha$$

$$T_2 . R - T_1 R = I . \alpha$$

$$I = \frac{(T_2 - T_1) R}{\alpha}$$

$$I = \frac{(160 - 120) . 0{,}25}{8}$$

$$I = 1{,}25 \ (kg . m^2)$$

$$a_t = \alpha R$$

$$\alpha = \frac{a_t}{R} = \frac{2}{0{,}25} = 8 \ (rad/s^2)$$

5 A particle of mass $m = 1$ kg starts its motion from the origin at $t = 0\ s$ then it moves on x-y plane with an instantaneous position vector given by $\vec{r} = (t^2 + t)\hat{\imath} + 3t\hat{\jmath}$.

a) Calculate the linear momentum and angular momentum relative to the origin at $t = 1$ s.

$\vec{r} = (t^2+t)\hat{i} + 3t\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt}$

$\boxed{\vec{v} = (2t+1)\hat{i} + 3\hat{j}}$ 1

$t = 1$ s de $\Rightarrow$ $\begin{cases} \boxed{\vec{r}_1 = (2\hat{i} + 3\hat{j})\ m} & 1 \\ \boxed{\vec{v}_1 = (3\hat{i} + 3\hat{j})\ m/s} & 1 \end{cases}$

$\vec{P} = m\vec{v}_1 = (1\,kg)(3\hat{i} + 3\hat{j})\frac{m}{s} = (3\hat{i} + 3\hat{j})\,kg\frac{m}{s}$

$P = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ kgm/s 2

$\boxed{\vec{L} = m\,\vec{r}_1 \times \vec{v}_1}$ 1 $= (1\,kg)(2\hat{i} + 3\hat{j}) \times (3\hat{i} + 3\hat{j})\frac{m}{s}$

$\hat{k}$ $\quad -\hat{k}$

$\vec{L} = (6 - 9)\hat{k}$

$\boxed{\vec{L} = -3\hat{k}\ J.s}$ 2 $|\vec{L}| = 3\ J.s$

b) Calculate the force and torque acting on the particle at $t = 1$ s.

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \boxed{\vec{a} = 2\hat{i}\ m/s^2}$ 2

$\vec{F} = m\vec{a} \Rightarrow \boxed{\vec{F} = 2N\,\hat{i}}$ 1

$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$ 1 $= (2\hat{i} + 3\hat{j}) \times (2\hat{i})$

$\boxed{\vec{\tau} = -6\hat{k}\ J}$ 1

c) Calculate the rate of change of angular momentum at $t = 1$ s.

$$\frac{d\vec{L}}{dt} = \vec{\tau} \Rightarrow \boxed{\left.\frac{d\vec{L}}{dt}\right|_{t=1} = -6\hat{k}\ J}$$

d) Verify the work and kinetic energy theorem for this particle between the time interval of $t = 1$ s and $t = 2$ s.

$$\boxed{W_{\text{Toplam}} = \Delta K} \; 1 \Rightarrow \vec{F} \cdot \Delta\vec{r} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

$t=2$ s de $\begin{cases} \vec{r}_2 = (6\hat{i} + 6\hat{j})\,m \\ \vec{V}_2 = (5\hat{i} + 3\hat{j})\,m/s \end{cases}$

$$\boxed{W = \vec{F} \cdot \Delta\vec{r} = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)} \; 1$$

$$W = 2\hat{i} \cdot \left[6(\hat{i}+\hat{j}) - (2\hat{i}+3\hat{j})\right]$$

$$\boxed{W = 2\hat{i} \cdot (4\hat{i} + 3\hat{j})} \; 1 \Rightarrow \boxed{W = 8\,J} \; 2$$

$$\boxed{\Delta K = \frac{1}{2}m(V_2^2 - V_1^2)} \; 1$$

$V_1^2 = 18\ \frac{m^2}{s^2}$ ve $V_2^2 = 34\ m^2/s^2$

$$\Delta K = \frac{1}{2} \cdot 1\,(34 - 18) \Rightarrow \boxed{\Delta K = 8\,J} \; 2$$

Equal