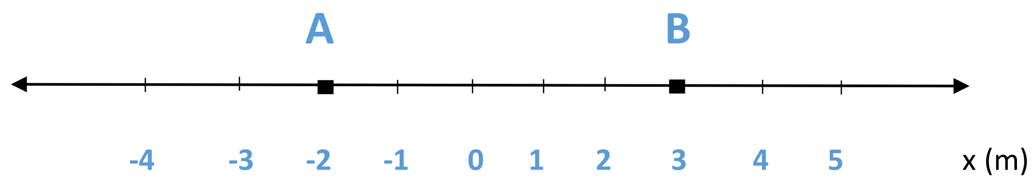
Motion

Position, Velocity, and Speed



Assume that an object is moving form A to B

initial position of the object is X_i=-2 m

Final position of the object is x_f=3 m

The ***displacement*** of the particle is defined as its change in position in some time interval.

$$\Delta x \equiv x_f - x_i$$

The average velocity of the particle

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t}$$

The average speed of the particle

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Distance is the total length of the path followed by the particle.

displacement and velocity are vector quantities

distance and speed are scalar quantities

Ex/ A car is moving from x=30 m to x=- 53 m in 50 s.

Find the displacement, average velocity of the car.

$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = \boxed{-83\text{ m}}$$

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}}$$

$$= \boxed{-1.7\text{ m/s}}$$

Instantaneous Velocity and Speed

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous speed of a particle is the magnitude of its instantaneous velocity.

Acceleration

Average acceleration

$$\overline{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Instantaneous acceleration

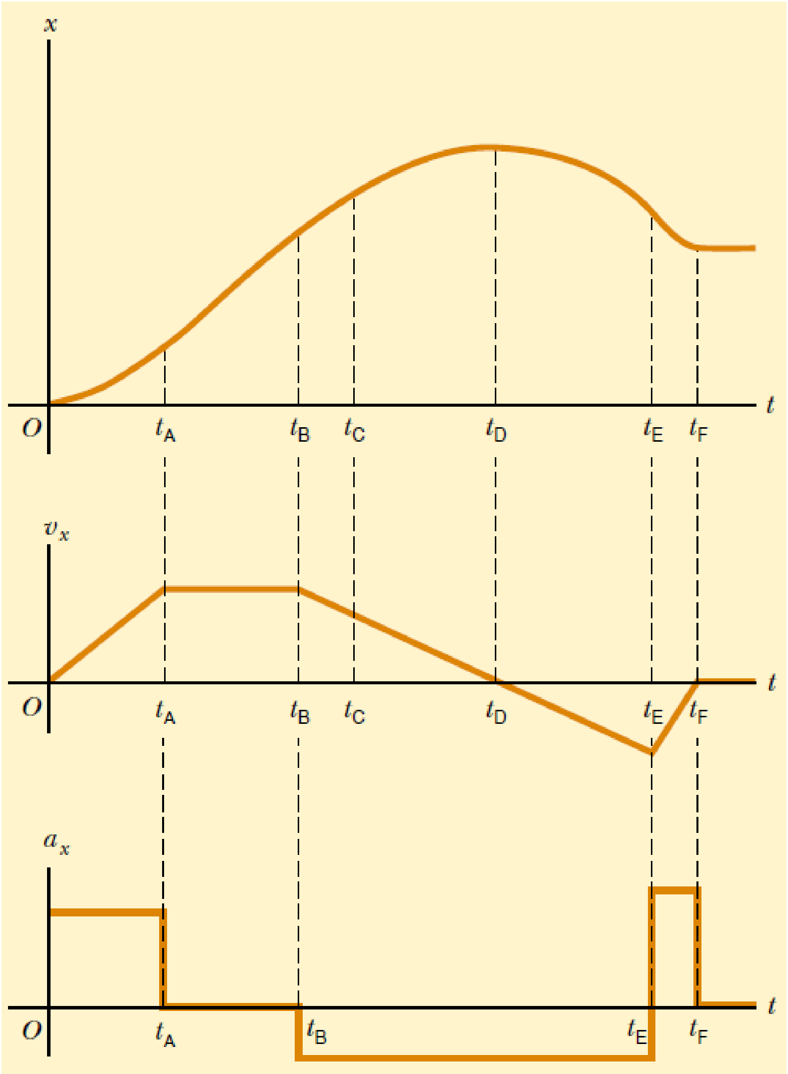
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Ex/ *$x(t) = 3t^2 - 2t + 4$ (m)*

$v(t) = 6t - 2$ (m/s)

$a(t) = 6$ (m/s^2)



Ex/ The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

(B) Determine the acceleration at $t = 2.0$ s.

(A)
$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A}$$

$$v_{xA} = (40 - 5t_A^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

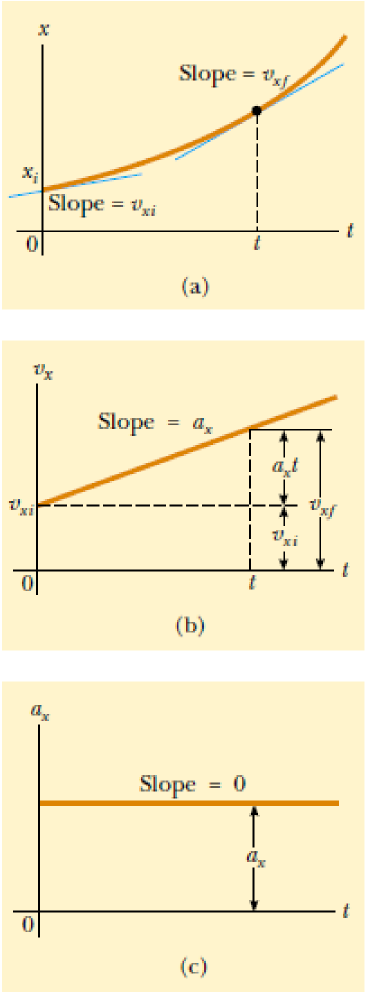
$$v_{xB} = (40 - 5t_B^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}} = -10\text{ m/s}^2$$

(B)
$$a_x = -10t\text{ m/s}^2$$

$$a_x = (-10)(2.0)\text{ m/s}^2 = -20\text{ m/s}^2$$

One-Dimensional Motion with Constant Acceleration



$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$v_{xf} = v_{xi} + a_x t$$

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$$

$$x_f - x_i = \bar{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})\,t$$

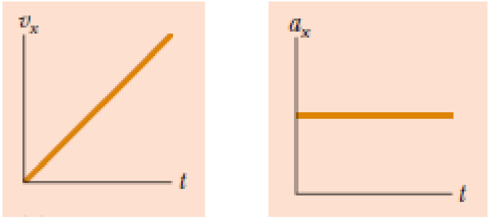
$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\,t$$

$$x_f = x_i + \tfrac{1}{2}\left[v_{xi} + (v_{xi} + a_x t)\right]t$$

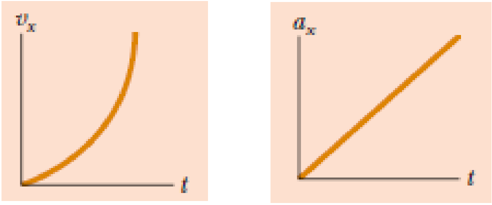
$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

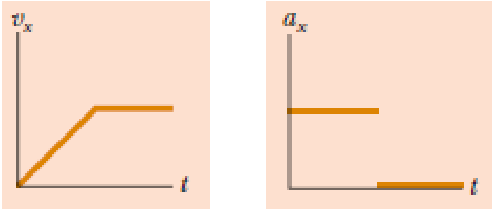
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$



(1)



(2)



(3)

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Freely Falling Objects

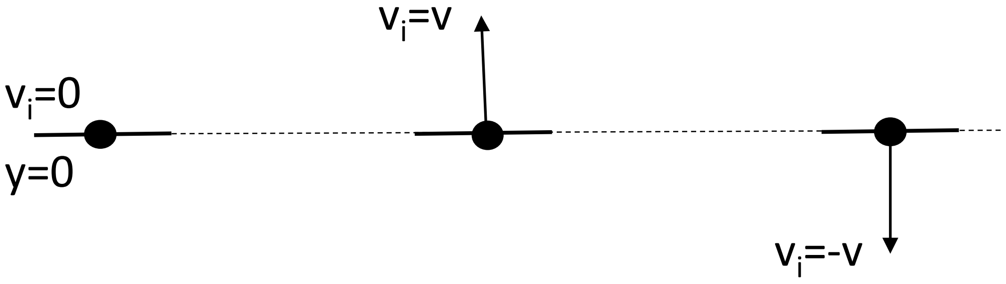
A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects ***thrown*** *upward* or *downward* and those ***released from rest*** are all falling freely once they are released. Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

Magnitude of the *free-fall acceleration denoted* by the symbol «*g*».

The value of g is approximately 9.80 m/s^2.

Generally we use use g=10 m/s^2.

Freely Falling Objects



v_1= -gt $\quad$ v_2= v-gt $\quad$ v_3= -v-gt

y_1= -(1/2)gt^2 $\quad$ y_2=vt-(1/2)gt^2 $\quad$ y_3=vt-(1/2)gt^2

Ex/ A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure. Using t_A=0 as the time the stone leaves the thrower's hand at position «A», determine

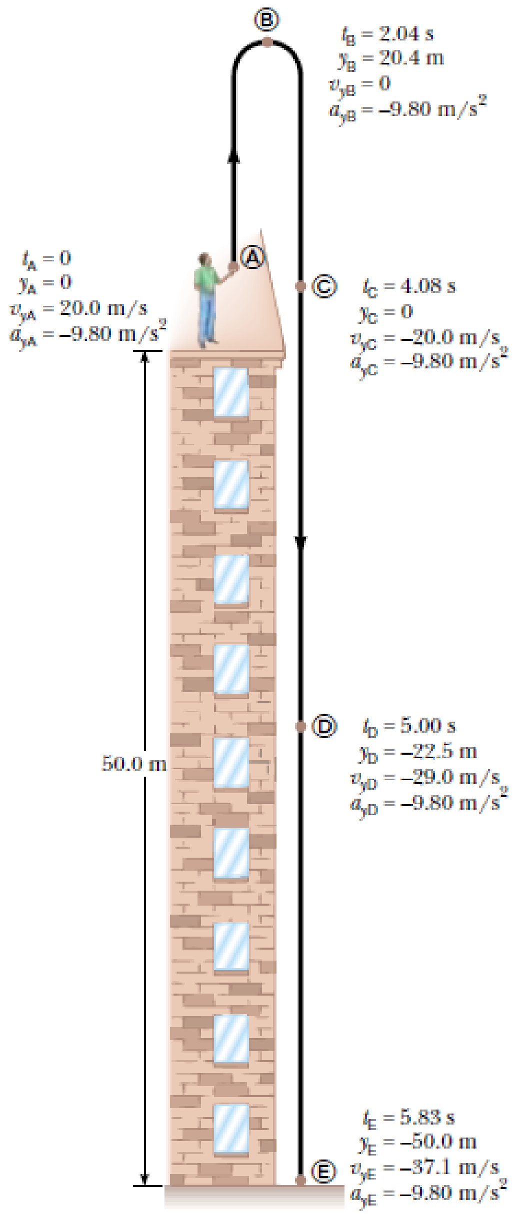
a) the time at which the stone reaches its maximum height,

b) the maximum height,

c) the time at which the stone returns to the height from which it was thrown,

d) the velocity of the stone at this instant,

e) the velocity and position of the stone at *t*=5.00 s.



a)

$$v_{yB} = v_{yA} + a_y t,$$

$$v_{yB} = 0$$

$$0 = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

$$t = t_B = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

b)

$$y_A = 0:$$

$$y_{max} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

c)

$$y_C = 0,$$

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

$$t(20.0 - 4.90t) = 0$$

$$\boxed{t = 4.08 \text{ s},}$$

d)

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$

$$= \boxed{-20.0 \text{ m/s}}$$

e)

$$v_{yD} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s})$$

$$= -29.0 \text{ m/s}$$

$$y_D = y_C + v_{yC}t + \tfrac{1}{2}a_y t^2$$

$$= 0 + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s})$$

$$+ \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$

$$= \boxed{-22.5 \text{ m}}$$