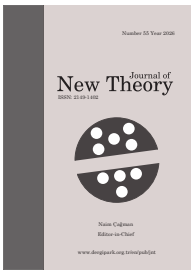
JOURNAL OF NEW THEORY

Journal of New Theory
dergipark.org.tr/tr/pub/jnt
Open Access



ISSN: 2149-1402 (55) (2026) 38-46 doi.org/10.53570/jnt.1912478

On the Diophantine Equation $(a^n-1)(b^n-1)(c^n-1)=x^2$

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Article Info
Received: 18 Mar 2026
Accepted: 11 Jun 2026
Published: 30 Jun 2026
Research Article

Abstract— In this paper, we study the exponential Diophantine equation $(a^n-1)(b^n-1)(c^n-1)=x^2$ in nonnegative integers for certain fixed values of a, b, and c with $1<a<b<c$. Our aim is to extend the classical two-factor framework $(a^n-1)(b^n-1)=x^2$ to the corresponding three-factor setting. We first establish a general nonexistence criterion based on 2-adic valuations and the lifting-the-exponent lemma. As an application, we prove that the equation $(2^n-1)(5^n-1)(7^n-1)=x^2$ has no positive integer solutions. Moreover, we derive a more general result covering a family of triples (a,b,c) under suitable parity conditions. Furthermore, we prove that the equation $(2^n-1)(3^n-1)(5^n-1)=x^2$ has a unique solution $(n,x)=(2,24)$ under a certain congruence restriction on n.

Keywords — *Diophantine equation, lifting-the-exponent lemma, p-adic valuation, quadratic residues, exponential equations*

1. Introduction

In 2000, Szalay [1] proved that the Diophantine equation

$$(2^n-1)(3^n-1)=x^2$$

has no positive integer solutions. This result has become the starting point for the study of exponential Diophantine equations of the form

$$(a^n-1)(b^n-1)=x^2$$

where a and b are fixed positive integers with $1<a<b$. Since then, several authors have investigated this equation and its variants by using different methods, such as elementary congruence considerations, quadratic residues, divisibility arguments, theory of Pell equations, p-adic valuations, and related methods.

Some works focus on specific pairs or explicit families of pairs (a,b). Hajdu and Szalay [2] considered the case $(a,b)=(2,6)$, while Cohn [3] studied further families of fixed pairs. Luca and Walsh [4] investigated products of like-indexed terms in binary recurrence sequences and solved many cases in prescribed ranges. Le [5] proved that if $3\mid b$, then

$$(2^n-1)(b^n-1)=x^2$$

has no positive integer solution. Some other works studied the above equation by taking into account congruence and parity conditions on the bases. In this direction, 2-adic valuations play a particularly

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important role. Guo [6] studied the case in which the highest powers of 2 dividing $a-1$ and $b-1$ have opposite parity. Ishii [7] obtained nonexistence results for the two-factor equation by using related elementary and valuation-based arguments. Keskin [8] proved further nonexistence results, especially for certain cases where a and b have opposite parity and n is even. There are also variants in which the exponents or the right-hand side are modified. Tang [9] considered equations of the form

$$(a^m-1)(b^n-1)=x^2$$

where the exponents are allowed to be different. Other related studies replace the right-hand side by a multiple of a square or by a higher power, or impose additional restrictions on the bases [10–12]. Recently, He and Liu [13] proved that

$$(2^k-1)(3^k-1)=x^m$$

has no solution in positive integers k, x and $m>2$. For further results see [14–17].

Although the above works provide several important extensions of Szalay's original problem, the left-hand side still consists of two factors. In this paper, we consider the three-factor analogue

$$(a^n-1)(b^n-1)(c^n-1)=x^2 \tag{1.1}$$

This extension is not merely formal. In the two-factor case, the square condition requires the total p-adic valuation of two factors to be even. In the three-factor case, however, the contribution of the third factor may change the parity balance completely. Thus, some obstructions that are effective in the two-factor setting may disappear, while new obstructions based on the combined valuations of three factors may arise.

In Theorem 3.4, we completely solve (1.1) for a family of a, b, and c, in particular, for $(a,b,c)=(2,5,7)$. Besides, in Theorem 3.6, we prove that any possible solution of (1.1) for $(a,b,c)=(2,3,5)$ must be of the form $n\equiv 2 \pmod{60}$. For some similar interesting problems in number theory, see [18].

The rest of the paper is organized as follows: Section 2 presents the lifting-the-exponent lemma required in the proofs of the main results. Section 3 provides the main results on the three-factor Diophantine equation (1.1), including the nonexistence results obtained by using p-adic valuations and the lifting-the-exponent lemma. In the same section, the special case $(a,b,c)=(2,3,5)$ is discussed, and the corresponding congruence restrictions for possible solutions are derived.

2. Preliminaries

This section presents the lifting-the-exponent lemma, a fundamental tool required in the subsequent section.

Lemma 2.1. [19] Let p be a prime number, $a,b\in\mathbb{Z}$, and $k\in\mathbb{N}$. Suppose that $p\mid(a-b)$ and $p\nmid ab$. Then, the p-adic valuation $\nu_p(a^k-b^k)$ is provided by

$$\nu_p(a^k-b^k)=\begin{cases}\nu_p(a-b)+\nu_p(k), & p \text{ is odd}\\ \nu_2(a-b), & p=2 \text{ and } k \text{ is odd}\\ \nu_2(a^2-b^2)+\nu_2\left(\dfrac{k}{2}\right), & p=2 \text{ and } k \text{ is even}\end{cases}$$

3. Main Results

This section presents the main results on the three-factor Diophantine equation (1.1).

Theorem 3.1. Let $n, x \in \mathbb{N}$. Then, the Diophantine equation

$$(2^n-1)(5^n-1)(7^n-1)=x^2$$

has no solution in positive integers.

Proof. Set

$$A := (2^n-1)(5^n-1)(7^n-1)$$

Since 2^n-1 is odd, $\nu_2(2^n-1)=0$. Hence,

$$\nu_2(A)=\nu_2(5^n-1)+\nu_2(7^n-1)$$

Thus, there are two cases:

Case 1. Let n be even. By Lemma 2.1 with the case $p=2$ and k even,

$$\nu_2(5^n-1)=\nu_2(5^2-1)+\nu_2\left(\frac{n}{2}\right)=\nu_2(24)+\nu_2\left(\frac{n}{2}\right)$$

and

$$\nu_2(7^n-1)=\nu_2(7^2-1)+\nu_2\left(\frac{n}{2}\right)=\nu_2(48)+\nu_2\left(\frac{n}{2}\right)$$

Therefore,

$$\nu_2(A)=\nu_2(24)+\nu_2(48)+2\nu_2\left(\frac{n}{2}\right)=7+2\nu_2\left(\frac{n}{2}\right)$$

is odd.

Case 2. Let n be odd. By Lemma 2.1 with the case $p=2$ and k odd,

$$\nu_2(5^n-1)=\nu_2(5-1)=\nu_2(4)=2$$

and

$$\nu_2(7^n-1)=\nu_2(7-1)=\nu_2(6)=1$$

Thus, $\nu_2(A)=3$ is odd.

In either case, $\nu_2(A)$ is odd. If $A=x^2$, then $\nu_2(A)=\nu_2(x^2)=2\nu_2(x)$ must be even, a contradiction. □

Corollary 3.2. Let k be an odd integer. Then, the Diophantine equation

$$(2^n-1)(5^n-1)(7^n-1)=k\,x^2$$

has no solution in positive integers n and x.

Proof. For $A=(2^n-1)(5^n-1)(7^n-1)$, the proof of Theorem 3.1 establishes that $\nu_2(A)$ is odd, for every $n \in \mathbb{N}$. If $A=kx^2$ with k odd, then $\nu_2(A)=\nu_2(k)+\nu_2(x^2)=0+2\nu_2(x)$ is even, a contradiction. □

Remark 3.3. In Corollary 3.2, the restriction that k is odd cannot be dropped. For example, the equation

$$(2^n-1)(5^n-1)(7^n-1)=6x^2$$

has at least two solutions $(n,x)=(1,2)$ and $(n,x)=(2,24)$.

Motivated by Theorem 3.1 and Corollary 3.2, we state the following theorem for a general class of a, b, and c.

Theorem 3.4. Let $a, b, c \in \mathbb{N}$. The equation

$$(a^n-1)(b^n-1)(c^n-1)=x^2$$

has no solution in positive integers (n, x) under either of the following conditions:

i. n and a are even, b and c are odd, and $\nu_2(b^2-1)$ and $\nu_2(c^2-1)$ have opposite parity.

ii. n is odd, and among $\nu_2(a-1)$, $\nu_2(b-1)$, and $\nu_2(c-1)$, either exactly one is odd, and two are even, or all three are odd.

Proof. Assume

$$(a^n-1)(b^n-1)(c^n-1)=x^2$$

Then, the 2-adic valuation of the left-hand side must be even. If condition *i.* holds, then a^n-1 is odd since a is even. Thus, $\nu_2(a^n-1)=0$. Because n is even and b and c are odd, by Lemma 2.1,

$$\nu_2(b^n-1)=\nu_2(b^2-1)+\nu_2\left(\frac{n}{2}\right)$$

and

$$\nu_2(c^n-1)=\nu_2(c^2-1)+\nu_2\left(\frac{n}{2}\right)$$

Hence,

$$\nu_2\big((a^n-1)(b^n-1)(c^n-1)\big)=\nu_2(b^2-1)+\nu_2(c^2-1)+2\nu_2\left(\frac{n}{2}\right)$$

The last term is even, while by assumption $\nu_2(b^2-1)$ and $\nu_2(c^2-1)$ have opposite parity, and thus their sum is odd. Therefore, the total valuation is odd, a contradiction.

Assume that condition *ii.* holds. Then, n is odd and from Lemma 2.1,

$$\nu_2(a^n-1)=\nu_2(a-1)$$

$$\nu_2(b^n-1)=\nu_2(b-1)$$

and

$$\nu_2(c^n-1)=\nu_2(c-1)$$

Thus,

$$\nu_2\big((a^n-1)(b^n-1)(c^n-1)\big)=\nu_2(a-1)+\nu_2(b-1)+\nu_2(c-1)$$

By the parity condition in *ii.*, this sum is odd, again impossible for a square. Thus, both cases lead to contradictions. □

Corollary 3.5. Let a be even and b and c be odd integers. Assume that

$$\nu_2(b-1)+\nu_2(c-1) \qquad \text{and} \qquad \nu_2(b^2-1)+\nu_2(c^2-1)$$

are odd integers. Then, for every odd integer k, the Diophantine equation

$$(a^n-1)(b^n-1)(c^n-1)=k\,x^2$$

has no solution in positive integers n and x.

To demonstrate some particular values of b and c, Table 1 lists all 56 prime pairs (b, c) satisfying the conditions in Corollary 3.5 in the range $3 \le b < c < 100$.

Table 1. All prime pairs (b,c) with $3 \le b < c < 100$ satisfying the parity conditions in Corollary 3.5

b	Corresponding c values
5	$7, 23, 31, 41, 71, 73, 89, 97$
7	$13, 17, 29, 37, 53, 61$
13	$23, 31, 41, 71, 73, 89, 97$
17	$23, 31, 41, 71, 73, 89, 97$
23	$29, 37, 53, 61$
29	$31, 41, 71, 73, 89, 97$
31	$37, 53, 61$
37	$41, 71, 73, 89, 97$
41	$53, 61$
53	$71, 73, 89, 97$
61	$71, 73, 89, 97$

In the preceding theorem, it was sufficient to use only Lemma 2.1 to provide the complete proofs. However, this method does not always suffice when different values of a, b, and c are chosen in (1.1). For example, for $(a,b,c)=(2,3,5)$, Lemma 2.1 provides only partial insight. In this case, different methods are applied.

Theorem 3.6. Let n be a positive integer. If the equation

$$(2^n-1)(3^n-1)(5^n-1)=x^2 \tag{3.1}$$

has a solution in positive integers other than $(n,x)=(2,24)$, then $n \equiv 2 \pmod{60}$.

Proof. If $n \le 5$, then it can be observed that

$$(2^2-1)(3^2-1)(5^2-1)=x^2$$

is the only solution corresponding to the pair $(n,x)=(2,24)$. Assume that $n>5$.

Case 1. Assume that n is odd and that (3.1) holds for some integer x. Since 2^n-1 is odd and 3^n-1 and 5^n-1 are even,

$$\nu_2\big((2^n-1)(3^n-1)(5^n-1)\big)=\nu_2(3^n-1)+\nu_2(5^n-1)$$

For odd n, from Lemma 2.1,

$$\nu_2(3^n-1)=\nu_2(3-1)=1, \quad \nu_2(5^n-1)=\nu_2(5-1)=2$$

Hence,

$$\nu_2\big((2^n-1)(3^n-1)(5^n-1)\big)=3$$

is an odd integer. This contradicts (3.1) since the 2-adic valuation of a square is even. Therefore, (3.1) has no solution with n odd.

Case 2. Assume that $4 \mid n$ and that (3.1) holds for some integer x. Write uniquely

$$n=4\cdot 5^{\alpha-1}k, \quad \text{such that } \alpha \ge 1 \text{ and } 5 \nmid k$$

Set $t=4\cdot 5^{\alpha-1}$. Thus, $n=tk$ and $5 \nmid k$. By Lemma 2.1 with $p=5$, applied to $(2^4)^{5^{\alpha-1}}-1$,

$$\nu_5(2^t-1)=\nu_5(2^4-1)+\nu_5(5^{\alpha-1})=1+(\alpha-1)=\alpha$$

Applying Lemma 2.1 to $(2^t)^k-1$,

$$\nu_5(2^n-1)=\nu_5\big((2^t)^k-1\big)=\nu_5(2^t-1)+\nu_5(k)=\alpha$$

The same argument, with 3 in place of 2, provides

$$\nu_5(3^n-1)=\alpha$$

as $\nu_5(3^4-1)=\nu_5(80)=1$. Since $\nu_5(2^n-1)=\nu_5(3^n-1)=\alpha$, it follows that

$$2\alpha=\nu_5((2^n-1)(3^n-1)(5^n-1))=2\nu_5(x)$$

This implies that $\nu_5(x)=\alpha$. Write $x=5^\alpha y$ with $y\in\mathbb{Z}, 5\nmid y$. Dividing (3.1) by $5^{2\alpha}$,

$$\left(\frac{2^n-1}{5^\alpha}\right)\left(\frac{3^n-1}{5^\alpha}\right)(5^n-1)=y^2,\quad \text{where } 5\nmid y \tag{3.2}$$

The normalized factors are evaluated modulo 5. From $\nu_5(2^t-1)=\alpha$, write

$$2^t=1+u5^\alpha,\quad \text{such that } 5\nmid u$$

Since $2^4=16=1+15$,

$$2^t=(1+15)^{5^{\alpha-1}}$$

Reducing modulo $5^{\alpha+1}$ and keeping only the linear binomial term, for $r\geq 2$, each term $\binom{5^{\alpha-1}}{r}15^r$ is divisible by $5^{\alpha+1}$, and

$$2^t\equiv 1+15\cdot 5^{\alpha-1}=1+3\cdot 5^\alpha \pmod{5^{\alpha+1}}$$

Thus,

$$u=\frac{2^t-1}{5^\alpha}\equiv 3 \pmod 5$$

Raising to the kth power,

$$2^n=(1+u5^\alpha)^k\equiv 1+ku5^\alpha \pmod{5^{\alpha+1}}$$

and therefore

$$\frac{2^n-1}{5^\alpha}\equiv ku\equiv 3k \pmod 5$$

Similarly, from $\nu_5(3^t-1)=\alpha$, write $3^t=1+v5^\alpha$ with $5\nmid v$. Since $3^4=81=1+80=1+16\cdot 5$, and similarly higher binomial terms are divisible by $5^{\alpha+1}$,

$$3^t=(1+80)^{5^{\alpha-1}}\equiv 1+80\cdot 5^{\alpha-1}\equiv 1+16\cdot 5^\alpha \pmod{5^{\alpha+1}}$$

Thereby, $v\equiv 1 \pmod 5$. Hence,

$$\frac{3^n-1}{5^\alpha}\equiv kv\equiv k \pmod 5$$

Thus,

$$\frac{2^n-1}{5^\alpha}\equiv 3k \pmod 5 \qquad \text{and} \qquad \frac{3^n-1}{5^\alpha}\equiv k \pmod 5 \tag{3.3}$$

Moreover, $5^n-1\equiv -1\equiv 4 \pmod 5$. Taking Legendre symbols modulo 5 in (3.2) and using (3.3),

$$1=\left(\frac{y^2}{5}\right)=\left(\frac{3k}{5}\right)\left(\frac{k}{5}\right)\left(\frac{4}{5}\right)=\left(\frac{3}{5}\right)\left(\frac{k^2}{5}\right)\cdot 1=\left(\frac{3}{5}\right)$$

However, $\left(\frac{3}{5}\right)=\left(\frac{5}{3}\right)=\left(\frac{-1}{3}\right)=-1$. This contradiction establishes that (3.1) has no solution with $4\mid n$.

Hence, $n=4N+2$, for some nonnegative integer N. In particular, (3.1) can be rewritten as

$$(4\cdot 2^{4N}-1)(9\cdot 3^{4N}-1)(25\cdot 5^{4N}-1)=x^2 \tag{3.4}$$

By using quadratic residue laws and properties of the Legendre symbol, further restrictions on N are derived. First, $N \equiv 0 \pmod 3$ is established. Assume that $N \equiv 2 \pmod 3$ and write $N = 3t+2$ so that (3.4) takes the form

$$(2^{10} \cdot 2^{12t} - 1)(3^{10} \cdot 3^{12t} - 1)(5^{10} \cdot 5^{12t} - 1) = x^2 \tag{3.5}$$

By using $2^6 \equiv 3^6 \equiv 5^6 \equiv 1 \pmod 7$, from (3.5), $x^2 \equiv 3 \pmod 7$, impossible since the only quadratic residues modulo 7 are 0, 1, 2, and 4. This rules out the case $N \equiv 2 \pmod 3$. For the case $N \equiv 1 \pmod 3$, write $N = 3t+1$ so that $n = 4N+2 = 12t+6$. Thus, (3.4) is of the form

$$(2^{6} \cdot 2^{12t} - 1)(3^{6} \cdot 3^{12t} - 1)(5^{6} \cdot 5^{12t} - 1) = x^2 \tag{3.6}$$

The congruence modulo the prime $q = 241$ is considered. The multiplicative orders of 2^{12}, 3^{12}, and 5^{12} modulo 241 are 2, 10, and 10, respectively. Hence, the evaluation of (3.6) modulo 241 depends only on $t \pmod{10}$. From Table 2, it can be observed that, for each $t \in \{0, 1, 2, \ldots, 10\}$,

$$\left(\frac{(2^n-1)(3^n-1)(5^n-1)}{241}\right) = -1$$

Table 2. Legendre symbol results of (3.6) for $n = 12t+6$ with $q = 241$

t	$2^n-1 \pmod{241}$	$3^n-1 \pmod{241}$	$5^n-1 \pmod{241}$	$x^2 \pmod{241}$	$\left(\frac{x^2}{241}\right)$
0	63	5	200	99	-1
1	176	215	105	74	-1
2	63	63	176	126	-1
3	176	134	24	148	-1
4	63	39	234	153	-1
5	176	234	39	152	-1
6	63	24	134	168	-1
7	176	176	63	111	-1
8	63	105	215	84	-1
9	176	200	5	70	-1

Thus, $\left(\frac{x^2}{241}\right) = -1$, a contradiction. Note that $q = 241$ is the least prime with this property. Consequently, the case $N \equiv 1 \pmod 3$ is also impossible, and hence $N \equiv 0 \pmod 3$. Therefore,

$$n = 4N+2 \equiv 2 \pmod{12}$$

Let $n = 12t+2$, for some positive integer t. The possible values of n are further refined by repeating the last argument, writing $t = 5s+i$, for $i \in \{0, 1, 2, 3, 4\}$. Thus, for every positive integer s,

i. $\left(\frac{(2^n-1)(3^n-1)(5^n-1)}{241}\right) = -1$, such that $n = 12 \cdot (5s+i) + 2$, for $i \in \{1, 4\}$

ii. $\left(\frac{(2^n-1)(3^n-1)(5^n-1)}{61}\right) = -1$, such that $n = 12 \cdot (5s+i) + 2$, for $i \in \{2, 3\}$

a contradiction. Hence, (3.1) has no positive integer solutions except for $i = 0$, that is, $n \equiv 2 \pmod{60}$. □

4. Conclusion

According to the current literature, most works based on the equation

$$(a^n-1)(b^n-1) = x^2$$

introduced by Szalay [1] investigate its various generalizations, while always maintaining two factors on the left-hand side. In this study, we consider the three-factor analogue

$$(a^n-1)(b^n-1)(c^n-1)=x^2$$

for certain families of integers a, b, and c. In particular, we prove that

$$(2^n-1)(5^n-1)(7^n-1)=k\,x^2$$

has no solutions in positive integers n and x when k is odd. Moreover, in Theorem 3.6, we demonstrate that, for $(a,b,c)=(2,3,5)$, any possible solution n must satisfy $n\equiv 2 \pmod{60}$. This naturally leads to the following question: Do there exist positive integer solutions of

$$(2^n-1)(3^n-1)(5^n-1)=x^2$$

for $n=60t+2$ with $t\geq 1$? Experiments indicate that the main problematic case appears when $t=\dfrac{n-2}{60}\in 5\mathbb{Z}$. The above argument reveals that the difficulty of solving the equation depends on the choice of a, b, and c, and that no general method is currently available for solving such equations. Finally, it remains an open problem to determine the nonnegative integer solutions of the equation

$$(a^n+1)(b^n+1)(c^n+1)=x^2$$

Author Contributions

All authors equally contributed to this study. They have all read and approved the final version of the study.

Declaration

This study is derived from the second author's master's thesis supervised by the first author.

Conflicts of Interest

All the authors declare no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

Artificial Intelligence (AI) Use Statement

The authors declare that no artificial intelligence (AI) tools were used in the conception, design, or generation of the main/original results of this study, including data collection, analysis, or interpretation. AI tools were used only for editing assistance, such as improving language, readability, or reference formatting. The authors are solely responsible for the content, accuracy, and integrity of the study.

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